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THE FIRST NORMALIZATION THEOREM FOR REGULAR HOMOTOPY OF FINITE DIRECTED GRAPHS.(*)

RIASSUNTO.- *Dati uno spazio topologico normale* S *ed un grafo finito ed orientato* G, *si dimostra che ogni funzione regolare di* S *in* G *è omotopa ad una funzione completamente regolare, vale a dire priva di singolarità.*

INTRODUCTION.- Keeping on [2] and using the results obtained there (see Background), we prove that every regular function from a normal (**) topological space S to a finite directed graph G is homotopic to a *completely regular function*, i.e. *without singularities* (see Theorem 12). (The first normalization theorem).

In order to define the singularities, we consider particular subset of the graph G. Precisely, we say that a subset of G is *headed* (resp. *tailed*) if it includes a vertex which is a predecessor (resp. successor) of all the others; while it is *totally headed* (resp. *totally tailed*) if all its subsets are headed (resp. tailed). (See Definition 1).

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(**) Consequently, we distinguish between normal space and T_4-space, according to whether it is a T_2-space or not.

We note that a totally headed set is also totally tailed and vice-versa. (See Proposition 4).

Then, a n-tuple $v_1,\ldots,v_n$ of the graph G is called a *singularity* for an o-regular (resp. o^*-regular) function f, if it is non-headed (resp. non-tailed) and if the intersection $\overline{f^{-1}(v_1)} \cap \ldots \cap \overline{f^{-1}(v_n)}$ is non-empty. (See Definition 5). (In particular, in a finite undirected graph there are only singular couples).

Moreover, we give the first normalization theorem for regular functions from a pair of topological spaces S,S' to a pair of graphs G,G', where S is a normal topological space and S' a closed subspace of S. (See Theorem 15). At least, in similar conditions, we prove that two homotopic completely regular functions are also completely homotopic. (See Theorem 16).

The previous results and, particularly, the first normalization theorem in its different statements will be used in the next papers in order to prove that:

1) *If S is a paracompact topological space, there is a bijection between the sets of homotopy classes* $Q(S,G)$ *and* $Q^*(S,G)$. (Duality theorem).
2) *The homotopy groups of a finite directed graph* G *are isomorphic to the classical homotopy groups of the poljhedron of a suitable simplicial complex associated with* G.

As concerns 1), we note that the first normalization theorem allows us to identify the sets $Q(S,G)$ and $Q^*(S,G)$ of regular homotopy classes with the ones $Q_c(S,G)$ and $Q^*_c(S,G)$ of completely regular homotopy classes. Consequently, the duality theorem follows from a bijection between $Q_c(S,G)$ and $Q^*_c(S,G)$, as we prove in a paper near to appear.

As concerns 2), to obtain the above-mentioned isomorphisms, we can now anticipate that we will associate with G the simplicial complex, whose simplexes are the totally headed subsets of G.