unique local solution $u \in S(r) \subset Y_\infty$.

PROOF. If $c(\bar{t}) = \max \{c_1(\bar{t}), c_2(\bar{t})\}$ then we can choose $\bar{t}$ so that $c(\bar{t}) < 1$. Then, Lemma (7) shows that $P$ maps $S(r)$ into itself and that $P$ is strictly contractive over $S(r)$.

Remark (2). The nonlinear operator $F$ is not Fréchet differentiable, contrary to what happens in the papers [1] and [2], where this fact allowed the assertion that the mild solution was also the strict solution of the problem (see [6]).

The results are so different because in paper [1] the operator $F$ is mollified and in paper [2] we used the space $X = U.C.B.(R^3)$ and
$X_o = \{f : f \in X \ , \ f(x,v,w) = 0 \text{ if } (v,w) \notin \bar{V}^2\}$

4. Positivity of the solution.

In this section we propose to prove that the solution of the problem (13) is positive if the initial condition $u_o$ is positive.

This result is important from a physical point of view, since $u(x,v,w;t)dxdvdw$ gives the expected number of vehicles that, at time $t$, have (i) position between $x$ and $x+dx$
(ii) speed between $v$ and $v+dv$, (iii) desired speed between $w$ and $w+dw$.

Introduce the following closed positive cones:

$X_o^+ = \{f \in X_o : f(x,v,w) \geq 0 \quad \text{for a.e. } (x,v,w) \in R \times \bar{V}^2\}$

$Y^+ = \{u \in Y : u(t) \in X_o^+ \quad \text{for } t \in [0,\bar{t}]\}$

and the relatively closed subsets:
$s^+(r) = s(r) \cap X_o^+$
$S^+(r) = S(r) \cap Y^+$ .

Moreover define:

$Y_\infty^+ = Y_\infty \cap Y^+$.

Note that $Z_0(t)\ [X_0^+] \subset X_0^+$ but $F$ does not map $D(F) \cap X_0^+$ into $X_0^+$. If this last condition was satisfied it would easily follow that $u(\cdot) \in S^+(r)$ locally, when $u_0 \in X_0^+ \cap X_\infty$ and $r > ||u_0||_\infty$.

In order to prove that the solution is positive it is sufficient to prove that:

(14) there exists $a > 0$ such that $F_1(u) = (a\ I + F)(u) \in X_0^+$ for $u \in s^+(r)$ and that if we define

$$T(t) = e^{-at} Z_0(t) \qquad \text{and}$$

$$(P_1 g)(t) = T(t)u_0 + \int_0^t T(t-s)F_1(g(s))ds$$

(15) $P_1$ maps $S(r)$ into itself.

These sufficient conditions are in [7], but for the reader's convenience we prove them in the appendix and we seize the opportunity to generalize some results.

Hence we have

<u>Lemma</u> (8). The assertions (14) and (15) are true.

<u>PROOF</u>

$au + F(u) = q\ J_1 u, J_2 u + (a - q\ J_3 J_1 u)u$, so if $u \in s^+(r)$ in order to prove $a\ u + F(u) \geq 0$ a.e. it is sufficient to prove $q\ J_3\ J_1 u \leq a$. Note that $q\ J_3 J_1 u \leq ||u||_\infty \frac{d}{2} q \leq r \frac{d}{2} q$ for $u \in s(r)$ so the condition (14) follows if we take $a \geq r \frac{d}{2} q$.

To prove the condition (15) we put $b = \frac{1}{T} - a$ then, if $g \in S(r)$, we have

$$||(P_1 g)(t)||_\infty \leq e^{bt}\ ||u_0||_\infty + d\ r^2 \int_0^t e^{b(t-s)} ds$$

and thus

$$||P_1 g)(t)||_\infty \leq c'(\bar{t})r$$

where

$$c'(\bar{t}) = \begin{cases} \dfrac{||u_o||_\infty}{r} + \left( \dfrac{||u_o||_\infty}{r} b + d\, r \right) \dfrac{e^{b\bar{t}}-1}{b} & \text{if} \quad b \neq 0 \\ \dfrac{||u_o||_\infty}{r} + d\, r\, \bar{t} & \text{if} \quad b = 0 \end{cases}$$

In each case we can have $c'(\bar{t}) < 1$ provided $r > ||u_o||_\infty$.

$\blacksquare$

Hence we can conclude with the following

<u>THEOREM</u> (2). If $u_o \in X_o^+ \cap X_\infty$ and $r > ||u_o||_\infty$ then the equation (13) has a unique local solution $u \in S^+(r)$.

<u>Remark</u> (3)For fixed $\bar{t}$ and $r = ||u_o||_\infty$ we can always choose a so as $P_1$ maps $S(r)$ into itself. In fact if $g \in S(r)$ we have

$$||P_1 g)(t)||_\infty \leq r + r(b+dr) \frac{e^{b\bar{t}}-1}{b} \qquad \text{for} \quad t \in [0,\bar{t}]$$

and so

$$||(P_1 g)\ (t)||_\infty \leq r \qquad \text{for} \quad t \in [0,\bar{t}]$$

when

$$b+dr \leq 0 \quad , \text{ i.e.} \qquad a \geq \frac{1}{T} + dr \quad .$$

Nevertheless this result does not enable us to improve theorem (2) by removing the condition $r > ||u_o||_\infty$, because it was used in theorem (1), from which the theorem (2) comes.

In other words given $[0,\bar{t}]$ and $r = ||u_o||_\infty$, $P_1$ maps $S(r)$ into itself but $P_1$ can be noncontractive.

## 5. <u>Global mild solution</u>.

As in [1] we introduce the functional

$$J\, f = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dv \int_{v_1}^{v_2} f(x,v,w)dw \qquad \text{for} \quad f \in \mathcal{X}.$$

We have: