$\mathcal{U}$-proper set representation $((f(S),\subseteq),f)$ of $(S,\leq)$[2] $f(c)$ is a point closure.

N. 2. A BRIEF REVIEW OF PREORDERED SETS.

Let S be a set and $\lesssim$ a preorder relation for S(i.e. $\lesssim$ exhibits the transitive and reflexive properties). All the most important notions about a poset can be extended to a preordered set (e.g. upper bound, lower bound, maximum, minimum, ℓ.u.b., g.ℓ.b., etc.); thus a right tail of a preordered set $(S,\lesssim)$ will be every $Y \subseteq S$ such that $\forall\, x,y \in S: x \in Y$ and $x \lesssim y \Longrightarrow y \in Y$.

We observe that if $Y_1 \subseteq S$ then the set $r(Y_1) = \{x \in S : x$ is an upper bound of $Y_1\}$ is a rigth tail of $(S,\lesssim)$; in particular the principal filter $r(y) = r(\{y\})$ generated by $y \in S$ is a right tail of $(S,\lesssim)$. Moreover

(ii) $r(X) = \bigcap_{x \in X} r(x)$ and if X is a right tail then $X = \bigcup_{x \in X} r(x)$.

Now let $\mathcal{C}$ be a subset of $\mathcal{P}(S)$ (the power set of S), x an element of S and $\mathcal{C}_x = \{X \in \mathcal{C} : x \in X\}$. Then we define, for every $x,y \in S$

(j) $$x \lesssim y(\mathcal{C}) \text{ iff } \mathcal{C}_x \subseteq \mathcal{C}_y .$$

Clearly the defined relation is a preorder relation. Moreover if $\mathcal{C}'$ is the set of set complements of the elements of $\mathcal{C}$ it follows, since $\mathcal{C}_x \subseteq \mathcal{C}_y$ iff $\mathcal{C}'_y \subseteq \mathcal{C}'_x$,

(jj) $$x \lesssim y(\mathcal{C}) \text{ iff } y \lesssim x(\mathcal{C}') .$$

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(2) We shall prove that there exists at least a $\mathcal{U}$-proper set representation of $(S,\leq)$.

One can easily verify that every element of $\mathcal{C}$ is a right tail of S preordered with respect to relation defined in (j). On the other hand if $(S,\lesssim)$ is a preordered set and $\mathcal{R}$ is the set of the right tails of $(S,\lesssim)$ then for every $x,y \in S$ the following properties hold

(e) $x \lesssim y$ iff $\mathbf{x} \lesssim \mathbf{y}(\mathcal{R})$ according to (j), and $x \lesssim y$ iff $\mathbf{x} \lesssim \mathbf{y}(\mathcal{R}_0)$,

where $\mathcal{R}_0$ is the set of the principal filters of $(S,\leq)$ generated by an elment of S. And hence, as a consequence of (jj), the following property holds:

(ee) $x \leq y$ iff $\mathbf{y} \leq \mathbf{x}\ (\mathcal{R}_0')$,

where $\mathcal{R}_0'$ is the set of all set complements in S of the elements of $\mathcal{R}_0$.

Now we can give the following

THEOREM 1. Let $\mathcal{C} \leq \mathcal{P}(S)$ be, $Y \in \mathcal{C}$ and $y \in Y$. Then Y is a point closure in $\mathcal{C}$ with respect to y iff y is minimum in Y with respect to relation defined in (j).

PROOF. In fact: y is minimum in $Y \Longleftrightarrow \forall x\in Y : \mathcal{C}_y \subseteq \mathcal{C}_x \Longleftrightarrow \forall x\in Y$, $\forall Z\in C_y : Z\in\mathcal{C}_x \Longleftrightarrow \forall x\in Y, \forall Z\in\mathcal{C}_y : x\in Z \Longleftrightarrow \forall Z\in\mathcal{C}_y : Y\subseteq Z$.

Q.E.D.

N. 3. A CHARACTERIZATION OF V-PRIME AND STRONGLY V-PRIME ELEMENTS OF A POSET.

Henceforth let $(S,\leq)$ be a poset. Then we can consider the function $g : S \to \mathcal{P}(S)$ mapping an element $x \in S$ into the set $g(x) = \{y\in S : x \nleq y\} = S - r(x)$. Clearly f is an injective function; moreover $\forall x,y\in S$ $x \leq y$ iff $g(x) \leq$