b is a fixed element of S, then $S(+,\cdot)$ is a $(2,p)$-semifield and $b^{-1}+b^{-1}=b^{-1}$.

And now we want to prove that if $S(+,\cdot)$ is a $(2,p)$-semifield and $|S|>1$ then $S(\cdot)$ is a direct product of groups of order 3. This is an immediate consequence of the following two theorems

THEOREM 5. $S(+)$ is a group and a is its zero-element.

PROOF. In fact for all $b\in S$ one has $b+b=b^{k+1}\cdot(1+1)=b^{k+1}\cdot a^{-1}$; then, since $a^{-1}=a^2=a^p$, $4b=(b+b)+(b+b)=b^{k+1}\cdot a^p+b^{k+1}\,a^p=(b^{k+1}+b^{k+1})\cdot a=$ $=(b^{k+1})^{k+1}\cdot a^{-1}\cdot a=b^{[(k+1)^2]}$. Now then, since the coset $k+1+(n)$ is invertible in $\frac{z}{(n)}(\cdot)$, the element $m=(k+1)^2$ is such that the coset $m+(n)$ is invertible too. As a consequence an element $h\in N$ exists such that $m^h\equiv 1$ (mod n), then $4^h b=b^{(m^h)}=b$. The conclusion now follows in the same way as in the proof of theorem 2.

Q.E.D.

THEOREM 6. The subset M coincides with S.

PROOF. In fact for all $x\in S$ one has:

$$1+x=a^2\cdot a+a^2\cdot a\cdot x=a(a+a\cdot x)=a\cdot a\cdot x=a^2\cdot x,$$

$$1+x=a\cdot a^2+x\cdot a\cdot a^2=a\cdot a^p+x\cdot a\cdot a^p=(a+x\cdot a)\cdot a=x\cdot a\cdot a=x\cdot a^2$$

Then a^2 is a central element in $S(\cdot)$ and hence $a=(a^2)^2$ is central too.

Q.E.D.

REFERENCE

[1] A. LENZI *Su di una struttura introdotta da J. Szép* to be published.