formula for the n-derivative of the function $e^{-x}K(\alpha,\gamma;x)$.

10 . ANOTHER RECURRENCE FORMULA.

The use of (5.1) and the relation (9.1) allows us to write down another recurrence formula besides (4.1).

In fact, by integrating term by term (9.1) with $-\rho=\mu$ and applying (5.1), we find the following relation:

$$(10.1)\quad \begin{aligned}&(\mu+\alpha-\gamma\beta)\psi(\alpha+\mu,\beta,\gamma;x)-\gamma\psi(\alpha+\mu+1,\beta,\gamma;x)+\gamma\psi(\alpha+\mu+1,\beta,\gamma-1;x)+\\&+\gamma\beta\psi(\alpha+\mu,\beta,\gamma-1;x)+x^{\mu}\Big[(-\alpha+\gamma\beta)\,\psi\,(\alpha,\beta,\gamma;x)+\\&+\gamma\psi(\alpha+1,\beta,\gamma;x)-\gamma\psi(\alpha+1,\beta,\gamma-1;x)-\gamma\beta\psi(\alpha,\beta,\gamma-1;x)\Big]=0.\end{aligned}$$

Remark 10.1

When $\gamma=1$, Eq. (10.1) gives the well-known recursive relation for the incomplete Γ-function:

$$(10.2)\quad \begin{aligned}&(\mu+a)\,\Gamma\,(a+\mu,x)-\Gamma(a+\mu+1,x)+x^{\mu}\Big[\Gamma(a+1,x)-\\&-a\Gamma(a,x)\Big]=0,\end{aligned}$$

where $a=\alpha-\beta$.