Introduction.

This paper pursues a study devoted to point out geometrical results required by a further structural analysis of physical theories.

In a previous paper [7] we have studied the tangent space of a bundle. Tangent spaces suffice to formulate one - body mechanics, as we are dealing with curves $c : R \to M$, whose differential is a map $dc : R \to T M$. On the other hand, continuum mechanics requires jet spaces, in order to get the derivatives of a field $f : M \to N$ as a map valued on a well structured space $jf : M \to J(M,N)$.

In a way analogous to [7], we show how the affine structure enables us to understand better the nature of jet spaces and of operations on them like Lie and covariant derivatives.

Let M and N be two manifolds. We consider the jet spaces $J^h(M,N)$ and the jet maps $j^h f : M \to J^h(M,N)$ of $f : M \to N$ and the bundles $J^h(M,N)$ on $J^k(M,N)$, with $h > k$. We give an explicit and intrinsic construction of $J'(M,N)$ and $J^2(M,N)$, showing that $J^2(M,N)$ is an affine bundle on $J'(M,N)$. This result can be extendend to higher orders (§ 1).

Let $\eta \equiv (E,p,M)$ be a bundle. We consider the relation between jet of sections JE and jet of maps $J(M,E)$. We give an explicit and intrinsic construction of $J'E$ and J^2E as affine sub bundles of $J'(M,E)$ and $J^2(M,E)$, respectively. This result can be extended to higher orders. We introduce contractions between jet spaces and tangent spaces, which will be used for Lie and covariant derivatives. In the particular case where η is a vector bundle, we show that J^hE is an affine bundle on J^kE, with $h > k$, and we introduce several interesting maps related with tensor product and duality (§2).

Let $\eta = (E,p,M)$ be a bundle. If we endow η with a morphism $B : E \times_M J^k TM \to TE$, affine on hTE and linear on E, we get a Lie operator which unifies the covariant derivatives (k=0), the usual Lie derivatives of tensors (k=1) and of many geometrical objects (§3).

We analyse connections on η in terms of jet bundles and we relate these results with the analogous ones obtained by means of tangent bundles (§4).

In the following, all manifolds and maps are C^∞. We leave to the reader the coordinate expression of formulas and the proof of some propositions.

1 - JETS.

Let M and N be two C^∞ manifolds.

1 DEFINITION.

The JET SPACE, of order i, OF MAPS $M \to N$, is the set

$$J^i(M,N) \equiv \bigsqcup_{p \in M} \mathfrak{F}_{p / \rho^i_p} \quad ,$$

where

a) $\mathfrak{F}_p$ is the set of C^∞ maps $M \to N$ defined in a neighbourhood of p;

b) ρ^i_p is the equivalence relation in $\mathfrak{F}_p$ given by

$$f \, \rho^i_p \, g \quad \Longleftrightarrow \quad T^i_p f = T^i_p g \quad \underline{.}$$

2 DEFINITION.

Let $$f : M \to N$$
be a C^∞ map, perhaps defined locally.

The JET, of order i, of f is the map

$$j^i f : M \to J^i(M,N)$$

given by $$p \to [f]^i_p \qquad \underline{.}$$

3 PROPOSITION.

There is a unique C^∞ structure on $J^i(M,N)$, such that

$$\forall \, f : M \to N \qquad \text{the map} \qquad j^i f \qquad \text{is} \qquad C^\infty.$$

PROOF.

It can be easily seen by means of an atlas of M and N $\underline{.}$

4 PROPOSITIONS.

Let $0 \leq i \leq j$. The natural projection

$$\sigma^{ij} : J^j(M,N) \to J^i(M,N)$$

given by $$[f]^j_p \to [f]^i_p$$

(which is well defined) induces a bundle structure

$$(J^j(M,N),\ \sigma^{ij}\ ,\ J^i(M,N)) \quad \underline{\cdot}$$

5 PROPOSITION.

The map $\qquad J^\circ(M,N) \rightarrow M \times N$

given by $\qquad [f]^\circ_p \rightarrow (p,f(p))$

(which is well defined) is a diffeomorphism $\underline{\cdot}$

Henceforth we will make the identification

$$J^\circ(M,N) \cong M \times N\ .$$

6 PROPOSITION.

The map

$$J'(M,N) \rightarrow T^*M \otimes T\,N$$

$$[f]^i_p \rightarrow T_p f \in T^*_p M \otimes T_{f(p)}N$$

(which is well defined) is a diffeomorphism.

PROOF.

We have $\qquad J'(M,N) = \bigsqcup_{(p,q)\in M\times N} \{\Phi_{(p,q)}\}$

where $\qquad \Phi_{(p,q)} : T_pM \rightarrow T_qN$

is any linear map $\underline{\cdot}$

Henceforth we will make the identification

$$J'(M,N) \cong T^*M \otimes T\,N\ .$$

7 THEOREM.

$(J^2(M,N),\sigma'',J'(M,N))$ is an affine bundle, whose vector bundle is

$$(J'(M,N)\times_{M\times N}(T^*M\ \vee_M T^*M\ \underset{M\times N}{\otimes} T\,N),\ \bar{\sigma}^{12},J'(M,N))\ .$$

(where $\vee$ denotes the symmetrized tensor product).

PROOF.

We have $\qquad J^2(M,N) = \bigsqcup_{\Phi_{(p,q)}\in J'(M\ N)} \{\bar{\Phi}_{\Phi_{(p,q)}}\}$

where $\bar{\Phi}_{\Phi_{(p,q)}} : T_p^2 M \to T_q^2 N$

is any map such that

a) $\bar{\Phi}_{\Phi_{(p,q)}}$ is a linear bundle homomorphism, hence the following diagram is commutative

$$\begin{array}{ccc} T_p^2 M & \xrightarrow{\bar{\Phi}_{\Phi_{(p,q)}}} & T_q^2 N \\ \downarrow \Pi_{TM} & & \downarrow \Pi_{TN} \\ T_p M & \xrightarrow{\Phi_{(p,q)}} & T_q N \end{array}$$

b) $\bar{\Phi}_{\Phi(p,q)} \circ s$ is linear

c) $T\,\Pi_N \circ \bar{\Phi}_{\Phi_{(p,q)}} = \Phi_{(p,q)}$

d) $\coprod \circ \bar{\Phi}_{\Phi_{(p,q)}} \circ \nu = \Phi_{(p,q)} \circ T\,\Pi_M$.

In fact a) ...,d) characterize the jets of maps $M \to N$.
Moreover, if we fix $\Phi_{(p,q)} \in J'(M,N)$, then the conditions a) and b) determine a vector space structure on the set $\{\bar{\Phi}_{\Phi_{(p,q)}}\}$ and the linear functional conditions c) and d) determine an affine subspace.

The associated vector space is obtained taking $\Phi_{(p,q)} = 0$ in the conditions c) and d). Such maps can be identified with a couple constituted by a bilinear symmetric map $TM \times_M TM \to TN$ and a linear map $TM \to TN$ over a same map $M \to N$.

This theorem can be generalized to higer orders.

8 PROPOSITION.

a) We get $J'(R,N) \cong R \times TN$

This isomorphism is the unique map $J'(R,N) \to R \times T\,N$ that makes commutative the following diagram, for each curve $c : R \to N$,

$$\begin{array}{ccc} J'(R,N) & \longrightarrow & R \times T N \\ {\scriptstyle j^1 c}\nwarrow & & \nearrow{\scriptstyle (id_R, dc)} \\ & R & \end{array}$$

b) We get $J'(M,R) \cong R \times T^*M$.

This isomorphism is the unique map $J'(M,R) \to R \times T^*M$ that makes commutative the following diagram, for each function $f : M \to R$,

$$\begin{array}{ccc} J'(M,R) & \longrightarrow & R \times T^*M \\ {\scriptstyle j^1 f}\nwarrow & & \nearrow{\scriptstyle (f, df)} \\ & M & \end{array}$$

c) There is a unique map (which is an isomorphism)

$$J^2(R,N) \to R \times s\, T^2 N$$

such that the following diagram is commutative, for each curve $c : R \to N$,

$$\begin{array}{ccc} J^2(R,N) & \longrightarrow & R \times s\, T^2 N \\ {\scriptstyle j^2 c}\nwarrow & & \nearrow{\scriptstyle (id_R, d^2 c)} \\ & R & \end{array}$$

$\underline{\cdot}$

2 - JETS OF SECTIONS.

Let $\eta \equiv (E,p,M)$ be a bundle.

1 DEFINITION.

The JET SPACE, of order i, OF SECTIONS $M \to E$, is the set

where $$J^i E \equiv \bigsqcup_{p \in M} \mathcal{S}_{p/\rho^i_p} ,$$

a) $\mathcal{S}_p$ is the se of C^∞ sections $M \to E$ defined in a neighbourhood of p;

b) ρ^i_p is the restriction of the equivalence relation defined in (1,1) $\underline{\cdot}$

Let us remark that we get

$$J^i(M,N) = J^i(M \times N),$$

considering $M \times N$ as a trivial bundle on M.

2 PROPOSITION.

a) J^iE is a submanifold of $J^i(M,E)$.

b) The following diagram is commutative, for each $i > j$

$$\begin{array}{ccc} J^iE & \hookrightarrow & J^i(M,E) \\ \sigma^{ji}\downarrow & & \downarrow\sigma^{ji} \\ J^jE & \hookrightarrow & J^j(M,E) \\ & \sigma^j \searrow \qquad \swarrow \Pi' \circ \sigma^{\circ j} & \\ & M & \end{array}$$

c) The triple $\eta^{ji} \equiv (J^iE, \sigma^{ji}, J^iE)$ is a bundle.

d) The triple $\eta^i \equiv (J^iE, \sigma^i, M)$ is a bundle.

e) If $f : M \to E$ is a section,

then $j^if : M \to J^iE$ is a section .

3 PROPOSITION.

We get a natural isomorphism

$$J^\circ E \cong E \quad .$$

4 THEOREM.

$\eta^{01} \equiv (J'E, \sigma^{01}, J^\circ E)$ is an affine subbundle of $(J'(M,E), \sigma^{01}, J^\circ(M,E))$

over the inclusion $J^\circ E \to J^\circ(M,E)$,

whose vector bundle is $\bar{\eta}^{01} \equiv (T^*M \otimes_E \nu\, TE, \Pi_E, E)$.

PROOF.

We have $$J'E = \bigsqcup_{e \in E} \{\Phi_e\}$$

where $\Phi_e : T_{p(e)}M \to T_eE$

is any linear map such that

a) $Tp \circ \Phi_e = id_{T_{p(e)}M}$ $\dot{-}$

5 THEOREM.

$\eta^{12} \equiv (J^2E, \sigma^{12}, J'E)$ is an affine subbundle of $(J^2(M,E), \sigma^{12}, J'(M,E))$

over the inclusion $J'E \to J'(M,E)$, whose vector bundle is

$$\bar{\eta}^{12} \equiv (J'E \times_E (T^*M \vee_M T^*M \otimes_E \nu TE), \bar{\sigma}^{12}, J'(M,E))$$

PROOF.

We have $J^2E = \bigsqcup_{\phi_e \in J'E} \{\bar{\Phi}_{\phi_e}\}$

where $\bar{\Phi}_{\phi_e} : T^2_{p(e)}M \to T^2_eE$

is any linear map as in (1.7), that satisfies the further condition

a) $T^2p \circ \bar{\Phi}_{\phi_e} = id_{T^2_{p(e)}M}$ $\dot{-}$

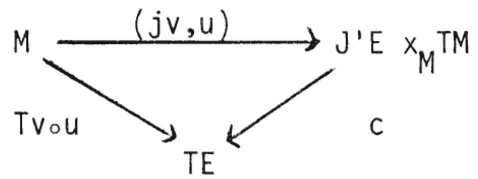
This theorem can be generalized to higher orders.

6 PROPOSITION.

There is a unique map

$$c : J'E \times_M TM \to TE$$

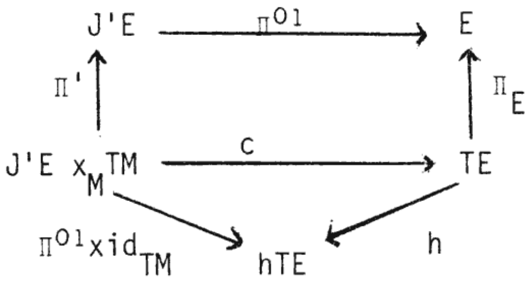
such that, for each section $u : M \to TM$, $v : M \to E$, the following diagram is commutative



Such a map is given by

$$c : (\Phi_e, u_{p(e)}) \quad \to \Phi_e(u_{p(e)}) \in T_e E \; .$$

c is an affine morphism on hTE and a linear morphism on $J'E \to E$.
Hence the following diagram is commutative

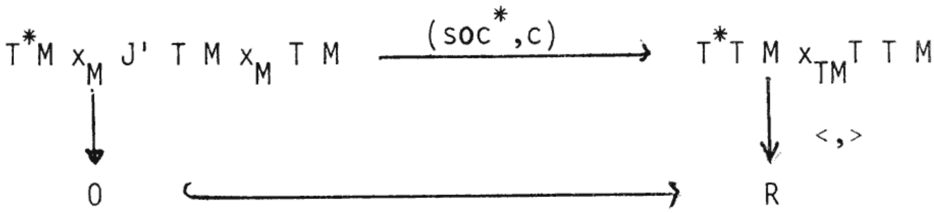


7 PROPOSITION.

There is a unique map

$$c^* : T^*M \; x_M \; J' \; T \; M \quad \to \quad T \; T^*M$$

such that the following diagram is commutative



8 PROPOSITION.

Let $\eta \equiv (E,p,M)$ be an affine (vector) bundle, whose vector bundle is $\bar{\eta} \equiv (\bar{E},\bar{p},M)$.

Then $\eta^i \equiv (J^i \; E, \sigma^i, M)$ is an affine (vector) bundle and

$$\overline{J^i E} = J^i \bar{E} \quad .$$

PROOF.

The affine (vector) operations on E are compatible with respect to the equivalence relations ρ^i_p $\dot{-}$

9 COROLLARY.

Let $k > h > 0$.

Let $\eta \equiv (E,p,M)$ be a vector bundle.

Then $\eta^{hk} \equiv (J^kE, \sigma^{hk}, J^kE)$ is an affine bundle, whose vector bundle $\bar{\eta}^{hk}$ is the pull back bundle of $(\mathrm{Ker}\ \sigma^{hk}, \sigma^k, M)$ with respect to the map σ^h.

Namely the following diagram is commutative

$$\begin{array}{ccc} \bar{J}^{hk}E & \longrightarrow & \mathrm{Ker}\ \sigma^{hk} \\ \downarrow & & \downarrow \sigma^k \\ J^h & \xrightarrow{\sigma^h} & M \end{array} .$$

Moreover, if $h \equiv k-1$, we get

$$\bar{J}^{hk}E = J^hE \times_M (\underset{kM}{\vee} T^*M \otimes_M E) ,$$

where $\underset{kM}{\vee} T^*M$ is the K-symmetrized tensor product of T^*M over M.

Let us remark that, if $E \equiv M \times F$ (i.e. η is a trivial bundle), then we get

$$J^kE = E \oplus_M \mathrm{Ker}\ \sigma^{ok} = F \times \mathrm{Ker}\ \sigma^{ok} .$$

In such a case, we put

$$j'^k u \equiv \Pi^2 \circ j^k u .$$

In particular, for $F \equiv R$, we get

$$j'^1 f = d f$$

10 PROPOSITION.

Let $\eta' \equiv (E',p',M)$ and $\eta'' = (E'',p'',M)$ be vector bundles.

There is a unique linear map $t : J^kE' \otimes_M J^kE'' \to J^k(E' \otimes_M E'')$

such that the following diagram is commutative

$$\begin{array}{ccc} J^kE' \otimes_M J^kE'' & \xrightarrow{t} & J^k(E' \otimes_M E'') \\ & \nwarrow (j^ku', j^ku'') \quad M \quad j^k(u' \otimes u'') \nearrow & \end{array}$$

for each section $u' : M \to E'$, $u'' : M \to E''$.

11 COROLLARY.

Let $\eta \equiv (E,p,M)$ be a vector bundle.

There is a unique linear map on M

$$t : \mathbb{K} \otimes_M J^{k-1}E \to J^k E,$$

where $\mathbb{K}$ is the Kernel of the linear morphism $J^k(M\times R) \to J^\circ(M\times R)$ on M, such that

$$\begin{array}{ccc} \mathbb{K} \otimes_M J^{k-1}E & \xrightarrow{t'} & J^k E \\ j'^k f \otimes j^{k-1}u \nwarrow & M & \nearrow t(j^k f \otimes j^k u) - f\, j^k u . \end{array}$$

As a particular case, we get

$$j^1(fu) = f\, j^1 u + d f \otimes u,$$

being
$$t'(d f \otimes u) = d f \otimes u \quad \dot{}$$

12 PROPOSITION.

Let $\eta \equiv (E,p,M)$ be an affine bundle, whose vector bundle is $\bar{\eta} \equiv (\bar{E},\bar{p},M)$. There is a unique map

$$J'E \times_E J'E \xrightarrow{\text{dif}} T^*M \otimes_M \bar{E}$$

such that, for each vertical curve $c,c' : R \to E$, the following diagram is commutative

$$\begin{array}{ccc} J'E \times_E J'E & \longrightarrow & T^*M \otimes_M \bar{E} \\ (j'c, j^1c^1) \nwarrow & R & \nearrow j^1(c-c') \end{array}$$

13 PROPOSITION.

Let $\eta \equiv (E,p,M)$ be a vector bundle and let $\eta^* \equiv (E^*,p,'M)$ be the dual one. Ther is a unique map

$$J'E \times_M J'E^* \xrightarrow{<,>} T^*M$$

such that, for each section $u : M \to E$ and $v : M \to E^*$, the following diagram is commutative

$$\begin{array}{ccc} J'E \times_M J'E^* & \xrightarrow{<,>} & T^*M \\ {\scriptstyle (j^1u,j^1v)} \nwarrow & & \nearrow {\scriptstyle \Pi^2(j^1<u,v>)} \\ & M & \end{array}$$

Such a map is bilinear .

14 PROPOSITION.

Let $\eta \equiv (E,p,M)$ be a vector bundle and let $g : E \times_M E \to R$ be a pseudo-Riemannian structure.

There is a unique map

$$g : J'E \times_M J'E \to T^*M$$

such that the following diagram is commutative, for each section $u,v : M \to E$

$$\begin{array}{ccc} J'E \times_M J'E & \xrightarrow{g} & T^*M \\ {\scriptstyle (j^1u,j'v)} \nwarrow & & \nearrow {\scriptstyle \Pi^2 \circ j^1(g(u,v))} \\ & M & \end{array} \quad .$$

3 LIE DERIVATIVES.

1 DEFINITION.

A K-LIE-DERIVABLE bundle is a 4-plet

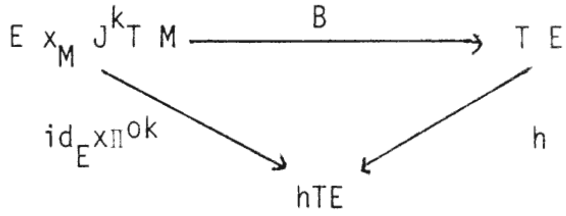
$$\eta \equiv (E,p,M;B),$$

where (E,p,M) is a bundle and

$$B : E \times_M J^kTM \to T\,E$$

is a bundle morphism on hTE and a linear morphism on E .

Hence the following diagram is commutative



and B is an affine morphism on hTE.

2 DEFINITION.

Let η be a K-LIE-derivable bundle.

a) The LIE OPERATOR is the map

$$\check{\sim} : J'E \times_M J^k TM \to \bar{\nu}\, TE$$

given by the composition

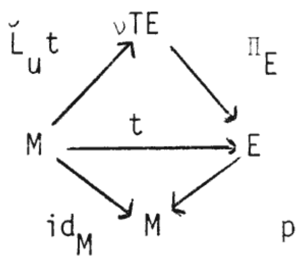
$$J'E \times_M J^k M \to (J'E \times_M TM) \times (E \times_M J^k TM) \xrightarrow{\;C-B\;} \bar{\nu}\, TE \quad .$$

b) Let $u : M \to TM$ and $t : M \to E$ be sections.

The LIE DERIVATIVE of t with respect to u is the section

$$\check{L}_u t \equiv (j^1 t, j^k u) : u \to \bar{\nu}\, TE \; .$$

Hence the following diagram is commutative



.

3 PROPOSITION.

We have

a) $$\check{L}_{(u+u')} v = \check{L}_u + \check{L}_u v$$

b) $$\check{L}_{fu} v = f\, \check{L}_u v - B(v, t \circ (j^k f \otimes j^{k-1} u))$$

4 If η is a vector bundle, we denote by

$$\check{\mathcal{L}} : J'E \times_M J^kTM \to E$$

the map $$\mathcal{L} \equiv \amalg_E \circ \check{\mathcal{L}}$$

and by $$L_u t : M \to E$$

the map $$L_u t \equiv \amalg_E \circ \check{L}_u t \ .$$

5 PROPOSITION.

Let η be a vector bundle and let B be a linear morphism on $J^kTM \to TM$

Then we have

$$L_u(t+t') = L_u t + L_u t'$$

$$L_u(ft) = f\, L_u t + (u.f)t \quad \dot{-}$$

6 PROPOSITION.

Let η' and η'' be vector bundles and let B' and B'' be linear morphisms on $J^kTM \to TM$.

Then there is a unique linear morphisms on $J^kTM \to TM$

$$B : (E' \otimes_M E'') \times_M J^kTM \to T(E' \otimes_M E'')$$

such that the following diagram is commutative

$$\begin{array}{ccc}
E' \times_M E^k \times_M J^kTM & \longrightarrow & (E' \times_M J^kTM) \times (E'' \times_M J^kTM) \\
 & & \downarrow B' \times B'' \\
 & & T E' \times_{TM} T E'' \\
\downarrow & & \downarrow t \\
(E' \otimes_M E'') \times J^kTM & \xrightarrow{\quad B \quad} & T(E' \otimes_M E'')
\end{array}$$

Then $(E' \otimes_M E'', p, M; B)$ results into a Lie derivable bundle.

Furthermore, we get

$$L_u(t' \otimes t') = L_u t' \otimes t'' + t' \otimes L_u t'' \quad \dot{-}$$

7 EXAMPLE.

Let $\eta \equiv (E,p,M;B)$ be a 0-Lie derivable bundle.

Then $$B : E \times_M TM \to TE$$

results into a horizontal section (see [7], §5).

Moreover, if η is a vector bundle and B is a linear morphism on TM, the 0-Lie-derivative coincide with the covariant derivative.

8 EXAMPLE

We get the usual Lie derivative of tensors $M \to T_{(p,q)}M$, taking into account the previous proposition and the 1-Lie-derivable bundles

$\eta \equiv (TM, \Pi_M, M; s\circ c)$ and $\eta \equiv (T^*M, \rho_M, M, C^*)$.

9 EXAMPLE.

Let $\beta \equiv (E,p,M;B)$ a bundle of geometric objects (see [7]) .

Let β be of "order K", i.e. such that the following condition holds: if $v \in J^kTM$, $x',x'' : M \to TM$ are two representative of v and f',f'' are the one parameter groups generated by x',x''

then $$\partial(Bf') = \partial(Bf'') \quad .$$

Then the map
$$B : E \times J^kTM \to TE,$$
given by $$B(e,v) \equiv \partial(Bf)(e)$$

makes $(E,p,M;B)$ a k-Lie derivable bundle.

4 CONNECTION ON A BUNDLE.

Let $\eta \equiv (E,p,M)$ be a bundle.

1 DEFINITION.

A CONNECTION on η is an affine bundle morphism on E

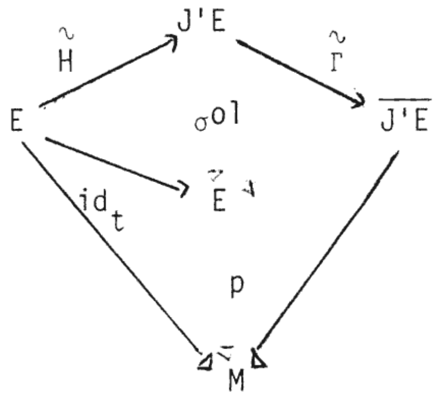
$$\tilde{\Gamma} : J'E \to \overline{J'E} = T^{*}M \otimes_{E} \nu TE$$

whose fiber derivatives are 1.

A HORIZONTAL SECTION is a section

$$\tilde{H} : E \to J'E$$

Hence the following diagram is commutative



2 PROPOSITION.

The maps α and β between the set of connections and the set of horizontal sections, given by

$$\alpha : \tilde{\Gamma} \to \tilde{H} ,$$

where $\check{H}$ is the unique horizontal section such that $\check{\Gamma} \circ \check{H} = 0$,

and $$\beta : \tilde{H} \to \tilde{\Gamma} \equiv id_{J'E} - \tilde{H} \circ \sigma^{01} ,$$

are inverse bijections.

Henceforth we will consider $\tilde{\Gamma}$ and $\tilde{H}$ as mutually related .
Hence giving a connection is the choice of a point for each affine fiber of $J'E$, getting in this way an identification of the affine fibers with their vector spaces.

3 PROPOSITION.

The set $\tilde{\mathcal{J}}$ of all connections is the affine space of the sections of the affine bundle $\eta^{01}E$, whose vector space is the space of sections of the vector bundle $\overline{\eta^{01}E}$.

4 Let us remark that $\mathcal{J}$ [7] and $\tilde{\mathcal{J}}$ have the same vector space.

PROPOSITION.

Each one of the following commutative diagrams determine the same isomorphism, whose derivative is 1, between the two affine spaces $\mathcal{J}$ and $\tilde{\mathcal{J}}$:

a)

$$\begin{array}{ccc} TM \times_M J'E & \xrightarrow{id_{TM} \times \tilde{\Gamma}} & TM \times_M (T^*M \otimes_E \nu TE) \\ \Big\downarrow C & & \Big\downarrow <,> \\ T\,E & \xrightarrow{\Gamma} & \nu\, T\, E \end{array}$$

b)

$$\begin{array}{ccc} E \times_M T\,M & \xrightarrow{\tilde{H} \times id_{TM}} & J'E \times_M T\,M \\ & H \searrow \quad \swarrow C & \\ & T\,E & \end{array}$$

Henceforth we will write $\mathcal{J}$, Γ and H for $\tilde{\mathcal{J}}$, $\tilde{\Gamma}$ and $\tilde{H}$.

5 PROPOSITION.

Let $c : R \to E$ be a curve. The following conditions are equivalent.

a) $H \circ \sigma^{01} \circ j'c \equiv H \circ c = j'c$

a') $H \circ h \circ d\,c \equiv H \circ (c, d(p \circ c)) = d\,c$

b) $\Gamma \circ j'c = 0$

b') $\Gamma \circ d\,c = 0$.

Hence a curve $c : R \to E$ is HORIZONTAL if the previous conditions hold.

6 PROPOSITION.

Let η be a vector bundle. Let Γ be a connection.
The following conditions are equivalent.

a) $\Gamma : J'E \to \overline{J'E}$ is a vector bundle morphism on M.

a') $\Gamma : TE \to \bar{\nu}TE$ is a vector bundle morphism on TM.

b) $H : E \to J'E$ is a vector bundle morphism on TM.

b') $H : hTE \to TE$ is a vector bundle morphism on TM.

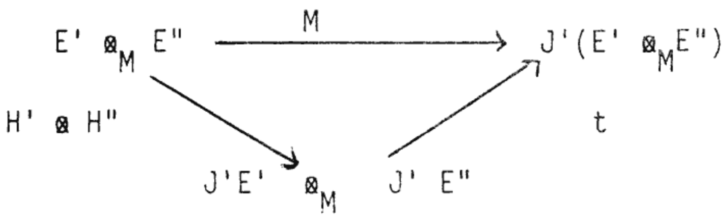
Hence a connection (horizontal section) is LINEAR if the previous conditions hold.

7 PROPOSITION.

Let Γ' and Γ'' be two linear connections of η' and η'', respectively
The tensor product of Γ' and Γ'' is the connection Γ on $\eta' \otimes \eta''$
associated with the horizontal section

$$H = t \circ (H' \otimes H'') \quad .$$

Hence the following diagram is commutative



8 PROPOSITION.

Let Γ be a linear connection on η .

The dual connection of Γ is the linear connection Γ^* on η^* associated with the unique horizontal section H^*, which makes commutative the following diagram

$$\begin{array}{ccc} J'E \times_M J'E^* & \xrightarrow{<,>} & T^*M \\ \uparrow{\scriptstyle (H,H^*)} & & \uparrow{\scriptstyle 0} \\ E \times_M E^* & \longrightarrow & M \end{array}$$ $\dot{-}$

9 PROPOSITION.

Let Γ be a linear connection on η . Let $v : M \to E$ be a section .
We get $\nabla v = (id_{T^*M} \otimes \perp\!\!\perp_E) \circ \Gamma \circ j^1 v$ $\dot{-}$

Hence the following diagram is commutative

$$\begin{array}{ccc} J'E & \xrightarrow{\Gamma} & T^*M \otimes_E \nu TE \\ {\scriptstyle j^1 v}\uparrow & & \downarrow{\scriptstyle id_{T^*M} \otimes \perp\!\!\perp_E} \\ M & \xrightarrow[\nabla v]{} & T^*M \otimes_E E \end{array}$$

10 PROPOSITION.

Let $\eta \equiv \tau M$ and let $g : TM \times_M TM \to R$ be a non degenerate symmetrical bilinear map.

The Riemannian connection Γ induced by g is associated with the unique linear section

$$H : TM \to J'TM$$

such that

a) the following diagram is commutative

$$\begin{array}{ccc} J'TM \times_M J'TM & \xrightarrow{g} & T^*M \\ \uparrow{\scriptstyle (H,H)} & & \uparrow{\scriptstyle 0} \\ TM & \longrightarrow & M \end{array}$$

b) the torsion $\Theta = 0$ $\dot{-}$

R E F E R E N C E S

1 R. PALAIS, Foundations of global non-linear Analysis, Benjamin, New York, 1968.

2 R. HERMANN, Vector bundles in Mathematical Physics, Vol. I, Benjamin, New York, 1970.

3 P. LIBERMANN, Connexions d'ordre supérieur et tenseur de structure. Atti Conv. Int. Geom. Diff., Bologna, 1967.

5 R. OUZILOU, Expression symplectique des problèmes variationelles. Symposia Mathematica, Vol. WIV, Acc. Press, 1974.

6 S. E. SALVIOLI, On the theory of geometric objects. J. Diff. Geom. 7 (1972) 257-278.

7 M.MODUGNO-G.STEFANI, Some results on second tangent and cotangent spaces, to appear.