

We can order the a_i in such a way that $a_1, \dots, a_p \in \langle f(x) \rangle$ and $a_{p+1}, \dots, a_n \in \langle f(x) \rangle - \langle g(x) \rangle$. Therefore it is $x \in \bar{A}_1^f \cap \dots \cap \bar{A}_p^f$, and so $W(x) \cap A_i^f \neq \emptyset$, $\forall i = 1, \dots, p$. Hence there are in $W(x)$ p points x_1, \dots, x_p such that $x_i \in A_i^f$, $\forall i = 1, \dots, p$. Then it is $x \in W(x_1) \cap \dots \cap W(x_p) \subseteq V(A_1^f) \cap \dots \cap V(A_p^f)$. Moreover it is $x \in \bar{A}_{p+1}^g \cap \dots \cap \bar{A}_n^g$, and by *ii*) it follows $x \in V(A_{p+1}^g) \cap \dots \cap V(A_n^g)$. Hence we obtain the contradiction $x \in V(A_1^f) \cap \dots \cap V(A_n^g)$.

Now if we consider any o -pattern h of g , we obtain the sought function. In fact we have:

i') $h: S \rightarrow G$ is completely o -regular (see [5], Proposition 7).

ii') h is weakly p -constant by the definition of o -pattern of a quasi-constant function.

iii') h is completely o -homotopic to f . Since the homotopy F is completely quasi-regular by *iii*), there exists an o -pattern E of F (which is completely o -regular by [5], Proposition 7). Moreover we can choose E such that $E(x, 0) = f(x)$, $E(x, 1) = h(x)$, $\forall x \in S$, since f and h are completely o -regular i.e. $f(x) \in H(\langle f(x) \rangle) = H(\langle F(x, 0) \rangle)$ and $h(x) \in H(\langle g(x) \rangle) = H(\langle F(x, 1) \rangle)$, $\forall x \in S$. Then h is completely o -homotopic to f by E . \square

REMARK 1. If W is a closed set, we can give the function g , by choosing as constant image of $X_j \in P$ any vertex of $H(\{f(\bar{X}_j)\})$.

REMARK 2. - If S is a compact metric space, we can determine a real positive number r and choose partitions P with mesh $< r$. In fact, we have just to calculate $enl(A_1, \dots, A_n)$, $\forall n$ -tuple a_1, \dots, a_n non-headed; so the real number r is given by $\frac{1}{2} \inf(enl(A_1, \dots, A_n))$.

REMARK 3. - If G is an undirected graph, the function g can be chosen quasi-constant. Moreover if S is a compact metric space, by Remark to Definition 2, we have just to consider the couples of non-adjacent vertices a_h, a_k and then to find the distances $d(A_h, A_k)$ rather than the enlargabilities $enl(A_h, A_k)$. Consequently, if we put $r' = \inf(d(A_h, A_k))$ and $r = \frac{1}{2} \inf(enl(A_h, A_k))$, since by Remark 3 to Definition 3 it follows $r' \leq 4r$, we can choose a covering $P = \{X_j\}$, $j \in J$, with mesh $< \frac{r'}{4}$. So we obtain again Property 7 of [8].

3) The third normalization theorem.

By comparing the second normalization theorem for directed and

