

We can order the  $a_i$  in such a way that  $a_1, \dots, a_p \in \langle f(x) \rangle$  and  $a_{p+1}, \dots, a_n \in \langle f(x) \rangle - \langle g(x) \rangle$ . Therefore it is  $x \in \bar{A}_1^f \cap \dots \cap \bar{A}_p^f$ , and so  $W(x) \cap A_i^f \neq \emptyset$ ,  $\forall i = 1, \dots, p$ . Hence there are in  $W(x)$   $p$  points  $x_1, \dots, x_p$  such that  $x_i \in A_i^f$ ,  $\forall i = 1, \dots, p$ . Then it is  $x \in W(x_1) \cap \dots \cap W(x_p) \subseteq V(A_1^f) \cap \dots \cap V(A_p^f)$ . Moreover it is  $x \in \bar{A}_{p+1}^g \cap \dots \cap \bar{A}_n^g$ , and by *ii*) it follows  $x \in V(A_{p+1}^g) \cap \dots \cap V(A_n^g)$ . Hence we obtain the contradiction  $x \in V(A_1^f) \cap \dots \cap V(A_n^g)$ .

Now if we consider any  $o$ -pattern  $h$  of  $g$ , we obtain the sought function. In fact we have:

*i'*)  $h: S \rightarrow G$  is completely  $o$ -regular (see [5], Proposition 7).

*ii'*)  $h$  is weakly  $p$ -constant by the definition of  $o$ -pattern of a quasi-constant function.

*iii'*)  $h$  is completely  $o$ -homotopic to  $f$ . Since the homotopy  $F$  is completely quasi-regular by *iii*), there exists an  $o$ -pattern  $E$  of  $F$  (which is completely  $o$ -regular by [5], Proposition 7). Moreover we can choose  $E$  such that  $E(x, 0) = f(x)$ ,  $E(x, 1) = h(x)$ ,  $\forall x \in S$ , since  $f$  and  $h$  are completely  $o$ -regular i.e.  $f(x) \in H(\langle f(x) \rangle) = H(\langle F(x, 0) \rangle)$  and  $h(x) \in H(\langle g(x) \rangle) = H(\langle F(x, 1) \rangle)$ ,  $\forall x \in S$ . Then  $h$  is completely  $o$ -homotopic to  $f$  by  $E$ .  $\square$

REMARK 1. If  $W$  is a closed set, we can give the function  $g$ , by choosing as constant image of  $X_j \in P$  any vertex of  $H(\{f(\bar{X}_j)\})$ .

REMARK 2. - If  $S$  is a compact metric space, we can determine a real positive number  $r$  and choose partitions  $P$  with mesh  $< r$ . In fact, we have just to calculate  $enl(A_1, \dots, A_n)$ ,  $\forall n$ -tuple  $a_1, \dots, a_n$  non-headed; so the real number  $r$  is given by  $\frac{1}{2} \inf(enl(A_1, \dots, A_n))$ .

REMARK 3. - If  $G$  is an undirected graph, the function  $g$  can be chosen quasi-constant. Moreover if  $S$  is a compact metric space, by Remark to Definition 2, we have just to consider the couples of non-adjacent vertices  $a_h, a_k$  and then to find the distances  $d(A_h, A_k)$  rather than the enlargabilities  $enl(A_h, A_k)$ . Consequently, if we put  $r' = \inf(d(A_h, A_k))$  and  $r = \frac{1}{2} \inf(enl(A_h, A_k))$ , since by Remark 3 to Definition 3 it follows  $r' \leq 4r$ , we can choose a covering  $P = \{X_j\}$ ,  $j \in J$ , with mesh  $< \frac{r'}{4}$ . So we obtain again Property 7 of [8].

3) The third normalization theorem.

By comparing the second normalization theorem for directed and







