

0) Background.  
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Let  $X$  be a non-empty subset of a finite directed graph  $G$ . A vertex of  $X$  is called a *head* of  $X$  in  $G$  if it is a predecessor of all the other vertices of  $X$ . We denote by  $H_G(X)$  the set of the heads of  $X$  in  $G$ .  $X$  is called *headed* if  $H(X) \neq \emptyset$  and *totally headed* if all the non-empty subsets of  $X$  are headed.

Given a function  $f: S \rightarrow G$ , where  $S$  is a topological space, we denote by capital letter  $V$  the set of all the  $f$ -counterimages of  $v \in G$ , and, if we want to emphasize the function  $f$ , we write  $V^f = f^{-1}(v)$ .

We call *image-envelope* of a point  $x \in S$  by  $f$ , and we denote by  $\langle f(x) \rangle$ , the set of vertices, such that the closure of their  $f$ -counterimages include the point i.e.  $v \in \langle f(x) \rangle \Leftrightarrow x \in \bar{V}^f$ .

A function  $f: S \rightarrow G$  is called *o-regular*, if, for all different  $v, w \in G$ , such that  $v$  is not a predecessor of  $w$ , it is  $V \cap \bar{W} = \emptyset$ . We proved that  $f$  is o-regular iff:

- i)  $\langle f(x) \rangle$  is headed,  $\forall x \in S$ ;
- ii)  $f(x) \in H(\langle f(x) \rangle)$ ,  $\forall x \in S$ . (See [5], Proposition 2).

So it is natural to define a more restrictive class of functions by saying that a function  $f: S \rightarrow G$  is *completely o-regular* (or simply *c.o-regular*) if

- i')  $\langle f(x) \rangle$  is totally headed,  $\forall x \in S$ ;
- ii')  $f(x) \in H(\langle f(x) \rangle)$ ,  $\forall x \in S$ .

Afterwards we also consider functions satisfying only condition i', which we call *completely quasi regular* functions. In [5] we proved that a completely quasi regular function can be replaced by a c.o-regular one by constructing the o-patterns of the function (where an *o-pattern* of a function  $f: S \rightarrow G$  is a function  $g: S \rightarrow G$  such that  $g(x) \in H(\langle f(x) \rangle)$ ,  $\forall x \in S$ ). In the case of pairs of topological spaces  $S, S'$  and of pairs of graphs  $G, G'$  in [5] in order to introduce the o-patterns, we gave the definition of *balanced* function i.e. of a function  $f: S, S' \rightarrow G, G'$  such that  $\langle f(x') \rangle = \langle f'(x') \rangle$ ,  $\forall x' \in S'$ . With reference to this we remember that if the subspace  $S'$  is open in  $S$ , all the functions are balanced.

II) Enlargability of sets in a uniform space.  
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DEFINITION 1. - Let  $(S, \mathcal{W})$  be a uniform space, where the filter  $\mathcal{W}$  is the uniformity of  $S$ . Given a vicinity  $W \in \mathcal{W}$ , we put  $W(x) = \{y \in S / (x, y) \in W\}$ ,  $\forall x \in S$ , and  $W(X) = \bigcup_{x \in X} W(x)$ ,  $\forall X \subset S$ .

' REMARK. - If  $(S, d)$  is a metric space the subsets  $W^\epsilon = \{(n, a) \in S \times S /$