10. CONSTRUCTION OF SOME LINEAR SYSTEMS

LEMMA 10.1: Let $|D|$ be a complete, non-special linear system and let $j_0,\dots,j_n$ be the $(|D|,P)$-orders, where $n=\dim|D|$. Then the $(|D+P|,P)$-orders are $0,\ j_0+1,\dots,j_n+1$.

THEOREM 10.2: If $|D|$ is a complete, non-special, classical, linear system and $|D'|$ is a complete, base-point-free, linear system, then $|D+D'|$ is classical.

Let $P\in\mathscr{C}$ and let $j_0,\dots,j_n$ be the $(\mathscr{D},P)$-orders for $\mathscr{D}$ canonical. Then $j_0+1=\alpha_1,\dots,j_{g-1}+1=\alpha_g$ are the <u>Weierstrass gaps</u> at P; that is, there does not exist f in $\bar{K}(\mathscr{C})$, regular outside P, such that $\mathrm{ord}_P(f)=-\alpha_i$.

THEOREM 10.3: Let $P\in\mathscr{C}$ and let $\alpha_1,\dots,\alpha_g$ be the Weierstrass gap sequence at P. If the linear system $\mathscr{D}=|dP|$ for some positive integer d, then the $(\mathscr{D},P)$-orders are $\{0,1,\dots,d\}\setminus\{d-\alpha_i \mid \alpha_i \le d\}$.

THEOREM 10.4: With P and $\alpha_1,\dots,\alpha_g$ as above, let V be a canonical divisor, $s\ge 2$ an integer, and $\mathscr{D}=|V+sP|$. Then the $(\mathscr{D},P)$-orders are

$$j_i = i \qquad \text{for } i=0,1,\dots,s-2,$$

$$j_{i+s-2} = s-1+\alpha_i \quad \text{for } i=1,\dots,g\ .$$

THEOREM 10.5: Let P in $\mathscr{C}$ be an ordinary point for the canonical linear system $|V|$ and assume that $|V|$ is classical. Then, for any n such that $0\le n\le g-1$, the linear system $\mathscr{D}=|V-nP|$ is a classical γ^{g-1-n}_{2g-2-n} without base points, and P is $\mathscr{D}$-ordinary.

An important result an linear series is also worth noting.

THEOREM 10.6: The generic curve of genus g has a γ_d^n if and only if

$$d \geq \frac{n}{n+1} g+n.$$

11. THE ESSENTIAL CONSTRUCTION

Given the curve $\mathscr{C}$ with its linear system of hyperplanes and with N the number of its GF(q)-rational points, consider the set $\mathscr{F} = \{P | P\varphi \subset H_p\}$; compare §4 for the plane. So $P \in \mathscr{F} \iff$

$$\det \begin{bmatrix} f_o^q & \cdots\cdots & f_n^q \\ D_t^{(j_o)} f_o & \cdots & D_t^{(j_o)} f_n \\ \vdots & & \vdots \\ D_t^{(j_{n-1})} f_o & \cdots & D_t^{(j_{n-1})} f_n \end{bmatrix} = 0$$

To give an outline first, take the classical case in which $j_i = i$. So, let

$$W' = \det \begin{bmatrix} f_o^q & \cdots\cdots\cdots & f_n^q \\ f_o & \cdots\cdots\cdots & f_n \\ \vdots & & \vdots \\ D^{(n-1)} f_o \cdots & & D^{(n-1)} f_n \end{bmatrix}$$

If $W' \neq 0$, then W is a function of degree