## Contents

Preface		v
Chapter	A. Partitions and Algebraic Structures	1
A1.	Partitions and representations	1
A2.	Ferrers diagrams of partitions	2
A3.	Addition on partitions	6
A4.	Multiplication on partitions	6
A5.	Dominance partial ordering	7
Chapter	B. Generating Functions of Partitions	11
B1.	Basic generating functions of partitions	11
B2.	Classical partitions and the Gauss formula	15
B3.	Partitions into distinct parts and the Euler formula	19
B4.	Partitions and the Gauss $q$ -binomial coefficients	24
B5.	Partitions into distinct parts and finite $q$ -differences	27
Chapte	r C. Durfee Rectangles and Classical Partition Identities	29
C1.	q-Series identities of Cauchy and Kummer: Unification	29
C2.	q-Binomial convolutions and the Jacobi triple product	30
C3.	The finite form of Euler's pentagon number theorem	42
Chapte	r D. The Carlitz Inversions and Rogers-Ramanujan Identities	45
D1.	Combinatorial inversions and series transformations	47
D2.	Finite $q$ -differences and further transformation	51
D3.	Rogers-Ramanujan identities and their finite forms	52
Chapte	r E. Basic Hypergeometric Series	57
E1.	Introduction and notation	57
E2.	The $q$ -Gauss summation formula	60
E3.	Transformations of Heine and Jackson	63
E4.	The $q$ -Pfaff-Saalschütz summation theorem	71
E5.	The terminating $q$ -Dougall-Dixon formula	73
E6.	The Sears balanced transformations	74
E7.	Watson's q-Whipple transformation	79

Chapte	r F. Bilateral Basic Hypergeometric Series	85
F1.	Definition and notation	85
F2.	Ramanujan's bilateral $_1\psi_1$ -series identity	88
F3.	Bailey's bilateral $_6\psi_6$ -series identity	89
F4.	Bilateral q-analogue of Dixon's theorem	93
F5.	Partial fraction decomposition method	97
Chapte	r G. The Lagrange Four Square Theorem	105
G1.	Representations by two square sums	105
G2.	Representations by four square sums	107
G3.	Representations by six square sums	108
G4.	Representations by eight square sums	112
G5.	Jacobi's identity and $q$ -difference equations	114
Chapte	r H. Congruence Properties of Partition Function	119
H1.	Proof of $p(5n+4) \equiv 0 \pmod{5}$	121
H2.	Generating function for $p(5n+4)$	122
H3.	Proof of $p(7n+5) \equiv 0 \pmod{7}$	130
H4.	Generating function for $p(7n+6)$	131
H5.	Proof of $p(11n+6) \equiv 0 \pmod{11}$	139
Append	lix: Tannery's Limiting Theorem	149
Bibliography		151