Appendix

Here we prove equibrium existence, as well as the properties stated in Proposition 2. Existence and quasi-competitiveness are proved mainly for later convenience – indeed, our model satisfies the conditions studied by Amir and Lambson (2000), who generalize previous work by MacManus (1964) and Roberts and Sonnenschein (1976). Convergence to the competitive limit has been studied by Ruffin (1971) under conditions on demand more restrictive than ours.

We gather here our assumptions:
(1) $C(0) = 0$, $C'(q) \geq 0$ and $C''(q) \geq 0$ for $q > 0$;
(2) $C'_M(q) \geq 0$ for $q > 0$
(3) $y_{\min} \leq C'(0) < y_{\max}$
(4) $f(y) > 0$ for $y \in (y_{\min}, y_{\max})$

Existence
At a symmetric equilibrium, the two (first and second order) conditions are

$$\tfrac{1}{n}(1 - F(p^*, \theta)) - [p^* - C'(\tfrac{1}{n}(1 - F(p^*, \theta))]f(p^*, \theta) = 0 \tag{A.1}$$

$$-f(p^*, \theta)[2 + C''(\cdot)f(p^*, \theta)] - [p^* - C'(\cdot)]\tfrac{\partial f(p^*,\theta)}{\partial p} < 0 \tag{A.2}$$

We show that (i) for any $n \geq 1$, there is a p^* such that (A.1) and (A.2) are satisfied; (ii) profits are non negative, depending (obviously) on K. Of course, (i) and (ii) together make up a short run Cournot equilibrium.

(i) To ease notation, (A.1) can be written as $h(p^*, n, \theta) = 0$, where $h(p, n, \theta) \equiv \frac{1}{n}(1 - F(p, \theta)) - [p - C'(\frac{1}{n}(1 - F(p, \theta))]f(p, \theta)$. For any given finite $n > 0$ and θ, $h(y_{\min}, n, \theta) = \frac{1}{n} - [y_{\min} - C'\left(\frac{1}{n}\right)]f(y_{\min}, \theta) > 0$: trivially, if $f(y_{\min}, \theta) = 0$; but also if $f(y_{\min}, \theta) > 0$, since $C''(\cdot) \geq 0$ implies $C'(1/n) \geq C'(0) \geq y_{\min}$ by Assumption 3. On other hand, there is a $\widehat{p} \leq y_{\max}$ such that $h(\widehat{p}, n, \theta) < 0$: $\widehat{p} = y_{\max}$ if $f(y_{\max}) > 0$, as $-[y_{\max} - C'(0)]f(y_{\max}, \theta) < 0$; while if $f(y_{\max}) = 0$ we note that $\frac{\partial h(y_{\max},n,\theta)}{\partial p} = -[y_{\max} - C'(0)]\frac{\partial f(y_{\max},\theta)}{\partial p} > 0$ by Assumption 4 (implying $\frac{\partial f(y_{\max},\theta)}{\partial p} < 0$), so that $h(\widehat{p}, n, \theta) < 0$ for any $\widehat{p}$ close enough to $y_{\max}$. By continuity, there exists a $p^* \in (y_{\min}, y_{\max})$ such that $h(p^*, n, \theta) = 0$. To prove (A.2), note that h crosses the 0-axis at least once from above; let p^* be one such crossing point, where clearly $\frac{\partial h(p^*,n,\theta)}{\partial p} < 0$: we have $0 < -\frac{\partial h(p^*,n,\theta)}{\partial p} = \frac{\partial f(p^*,\theta)}{\partial p}[p^* - C'(\cdot)] + [1 + \frac{1}{n}C''(\cdot)f(p^*, \theta)]f(p^*, \theta) + \frac{f(p^*,\theta)}{n}$ $\leq \frac{\partial f(p^*,\theta)}{\partial p}[p^* - C'(\cdot)] + f(p^*, \theta)[2 + C''(\cdot)f(p^*, \theta)]$. The last inequality holds for any $n \geq 1$.

(ii) There has to be a value $\overline{K} > 0$, such that for all $K \leq \overline{K}$ equilibrium profits are non negative. This is directly implied by Assumption 2, as (for any finite n) $p^* > C_M(\frac{1}{n}(1 - F(p^*, \theta))$: hence, for $K = 0$ profits are strictly positive at such n, and there follows that such $\overline{K}$ exists.

Quasi-competitiveness
Since p^* satisfies $h(p^*, n, \theta) = 0$, clearly $dp^*/dn = -\frac{\partial h}{\partial n}/\frac{\partial h}{\partial p}$. We know from the above that at p^*, $\frac{\partial h}{\partial p} < 0$. On the other hand, for given p^*, $\frac{\partial h}{\partial n} = -\frac{1}{n^2}[(1 - F(p^*, \theta)) + C''(\cdot)f(p^*, \theta)] < 0$. Hence p^* decreases as n increases.

Monotonic convergence
In order to prove that $\lim_{K \to 0} p^*(n(K, \theta), \theta) = C'(0)$, we first note that $\lim_{K \to 0} n(K, \theta) = \infty$: by Assumption 2, for any finite n profit is positive at $K = 0$, since $p^* > C_M$. Now observe that $\lim_{n \to \infty} p^*(n, \theta) = C'(0)$. To this end, suppose to the contrary that

$$\lim_{n \to \infty} p^*(n, \theta) = \widehat{p} \neq C'(0)$$

Then we would have

$$\lim_{n \to \infty} h(p^*, n, \theta) = 0 - [\widehat{p} - C'(0)]f(\widehat{p}, \theta) \neq 0$$

which cannot be, since it violates (A.1).

As to monotonicity, it is easily checked that the long run equilibrium number of firms $n^*(K, \theta)$ verifies $\partial n^*/\partial K < 0$. Indeed, by the zero profit condition $\widehat{\pi}(p^*(n), \theta) - K = 0$ (where $\widehat{\pi}(p^*(n), \theta)$ is gross profits), and using quasi-competitiveness, one can see that $d\widehat{\pi}/dn < 0$ and hence $\partial n^*/\partial K < 0$. Monotonicity of p^* then follows.