Appendix

Consider the dispersed solution $a^{'} = b^{'} = (\gamma - 2)(t-2)/4\gamma t$. We have already verified that it satisfies the SOCs for all non-negative values of γ, and the feasibility requirements for $t \geq t_{\min} = 2(2-\gamma)/(\gamma+2)$. Now we have to check under which conditions it is also consistent and deviation-proof. Since the locations corresponding to (6) respectively maximize (3) and (4), which assume duopolistic interaction over the entire market, consistency requires that from those locations both firms deliver positive quantities at all sites. Given that the site in which each firm delivers the lowest quantity is the opposite endpoint of the market segment, this requirement is met if, e.g., the quantity delivered by firm 1, $q_1^* = [2 - \gamma + \gamma t\,|1-b-x| - 2t\,|a-x|]/(2+\gamma)(2-\gamma)$ is positive in $x = 1$, when evaluated at (6). This is actually verified for $t < t^{cover} = 2(3\gamma+2)(\gamma-2)/\left(\gamma^2 - 8\gamma - 4\right)$, with $t^{cover} > t_{\min}$ for $\gamma \in (0,1]$.[4]

Therefore, in the case of substitutability, there is an interval of values of t for which the dispersed solution is consistent. For it to be a SPNE, however, we have to prove that for these values of t there is no incentive for one of the two firms to deviate, thus generating a different pattern of market coverage (with monopolistic areas for one or both firms). For this purpose we first notice that, if firm 2 chooses $b^{'} = (\gamma-2)(t-2)/4\gamma t$, for $t \in \left[t_{\min}, 2\left(\gamma - \gamma^2 + 2\right)/(3\gamma+2)\right]$, the decisions of firm 1 are such that:

(i) if $a \in \left](10t + 6\gamma - 12 - \gamma t)/8t, (3\gamma t + 2t + 2\gamma - 4)/4\gamma t\right]$, the entire market is covered by both firms and profits of firm 1 are

$$\Pi_1(a,\gamma,t) = \int_0^1 \left(\pi_1^D(a, b^*(\gamma,t), \gamma, t, x)\right) dx \tag{A1}$$

(ii)if $a \in \left]\frac{1}{t\gamma}\left((2-t)\frac{\gamma-2}{2\gamma} - 2 + \gamma + 2t\right), (10t + 6\gamma - 12 - \gamma t)/8t\right]$, firm 2 (but not firm 1) is monopolist in a segment external to its location and at the extreme of its market side; profits of firm 1 are therefore

$$\Pi_1(a,\gamma,t) = \int_0^{\rho_1} \left(\pi_1^D(a, b^*(\gamma,t), \gamma, t, x)\right) dx \tag{A2}$$

(iii) if $a \in \left]\left(2t + 3\gamma t - 4 - 2\gamma + 2\gamma^2\right)/4\gamma t, \frac{1}{t\gamma}\left((2-t)\frac{\gamma-2}{2\gamma} - 2 + \gamma + 2t\right)\right]$, both firms are monopolist in a segment external to their location and at the extreme of their market sides; in this case we have that

$$\Pi_1(a,\gamma,t) = \int_0^{\rho_2} \left(\pi_1^M(a,\gamma,t)\right) dx + \int_{\rho_2}^{\rho_1} \left(\pi_1^D(a, b^*(\gamma,t), \gamma, t, x)\right) dx \tag{A3}$$

(iv) if $a \in \left[0, \left(2t + 3\gamma t - 4 - 2\gamma + 2\gamma^2\right)/4\gamma t\right]$, both firms are monopolist in a segment in their market side which includes their locations, so that

$$\Pi_1(a,\gamma,t) = \int_0^{\rho_3} \pi_1^M(a,t,x)\, dx + \int_{\rho_3}^{\rho_4} \left(\pi_1^D(a, b^*(\gamma,t), \gamma, t, x)\right) dx \tag{A4}$$

[4]We would have obtained the same condition by setting $q_2^* > 0$ in $x = 0$.

where the options (iii) and (iv) are viable only if $t > 2\left(\gamma-\gamma^2+2\right)/(3\gamma+2)$, a threshold value which belongs to $[t_{\min}, t^{cover}]$,[5] and

$$
\begin{aligned}
\rho_1 &= \frac{12-6\gamma-3\gamma t-2t+8at}{4(2-\gamma)t}, \rho_2 = \frac{2-\gamma-2t+(t-2)\frac{\gamma-2}{2\gamma}+\gamma ta}{(\gamma-2)t} \\
\rho_3 &= \frac{\gamma-2+2t-(t-2)\frac{\gamma-2}{2\gamma}+\gamma ta}{(\gamma+2)t}, \rho_4 = \frac{2-\gamma+\gamma t-\frac{1}{4}(t-2)(\gamma-2)+2ta}{(\gamma+2)t}
\end{aligned}
$$

are the relevant boundary points between the sections of the segment with different coverage configurations, Clearly, $\pi_1^M(a,t,x) = (1-t(a-x)/2)^2$ is the monopoly profit at a location x.

By substituting for $\pi_1^D(a, b^*(\gamma,t), \gamma, t, x)$ and $\pi_1^M(a,t,x)$ in the above expressions, it is easy to check that for both intervals of t the profit function is continuous overall. Moreover, tedious but straightforward calculations show that the profit functions (A2), (A3) and (A4) have no local maxima in their domain. This implies that the maximum $a^{'} = (\gamma-2)(t-2)/4\gamma t$ of the profit function (A1) is not only consistent (i.e. belongs to the domain of that function), but is also a global maximum.

A similar procedure can be applied also to the agglomerated equilibrium.

[5] Notice that for $t = 2\left(\gamma-\gamma^2+2\right)/(3\gamma+2)$, the lower bound of the interval in (ii) is equal to zero.

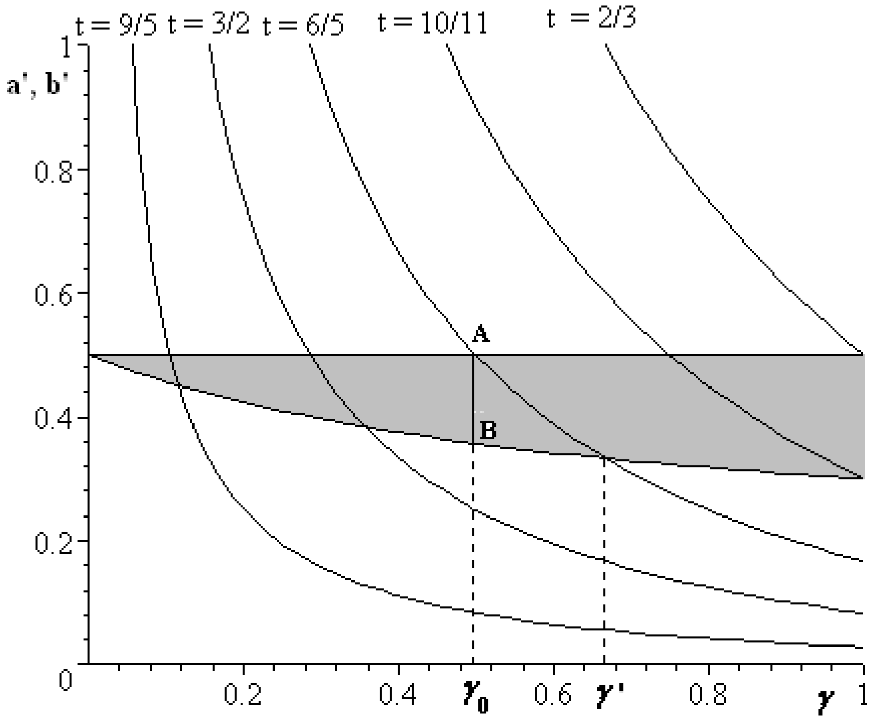

Figure 1. The pattern of the dispersed equilibrium