Appendix

A. Contract with two agents in the electoral period.

Problem of constrained optimization

The Lagrangian for this problem is

(A1) $L(t, \lambda) = E(U - u) + \lambda_1 g_1 + \lambda_2 g_2 + \lambda_3 g_3 + \lambda_4 g_4$

assuming:

$g_1 = E(I_b \mid e_b = 1) - E(I_b \mid e_b = 0) = (T_b - t_b)\,[Pr(Bs \mid e_b = 1) - Pr(Bs \mid e_b = 0) - C_b$

(A2) $g_2 = E(I_b \mid e_b = 1) = t_b\,[1 - Pr(Bs \mid e_b = 1)] + T_b\,Pr(Bs \mid e_b = 1) - C_b$

$g_3 = E(I_p \mid e_p = 0) - E(I_p \mid e_p = 1) = (T_p - t_p)\,[Pr(Ps \mid e_p = 1) - Pr(Ps \mid e_p = 0)] + C_p$

$g_4 = E(I_p \mid e_p = 0) = t_p\,Pr(Ps \mid e_p = 0) + T_p\,[1 - Pr(Ps \mid e_p = 0)]$

The first-order conditions are given by

(A3) $\partial L/\partial t_b = 0$, $\partial L/\partial T_b = 0$, $\partial L/\partial t_p = 0$, $\partial L/\partial T_p = 0$

$\lambda_1 g_1 = 0$

(A4) $\lambda_2 g_2 = 0$

$\lambda_3 g_3 = 0$

$\lambda_4 g_4 = 0$

(A5) $g_1 \geq 0$ $g_2 \geq 0$ $g_3 \geq 0$ $g_4 \geq 0$

(A6) $\lambda_i \geq 0$

We express constraints g_i through the probabilities introduced with (3). For this purpose, we see that, as in E(U-u), in making his assessments each agent can be expected to think that the other agent is almost sure to make the choice that is most advantageous for himself. For instance, BA will think that, as it is an election period, CB will make no effort, while CB will be convinced that BA will make an effort. In formulae this mean that conditions (13)-(16) become

(A7) $E(I_b \mid e_b = 1 \cap e_p = 0) = E(I_b \mid e_{10}) \geq E(I_b \mid e_b = 0 \cap e_p = 0) = E(I_b \mid e_{00})$

(A8) $E(I_b \mid e_{10}) \geq 0$

(A9) $E(I_p \mid e_b = 1 \cap e_p = 0) = E(I_p \mid e_{10}) \geq E(I_p \mid e_b = 1 \cap e_p = 1) = E(I_p \mid e_{11})$

(A10) $E(I_p \mid e_{10}) \geq 0$

and the g_i will become

$$
\text{(A11)}\quad
\begin{aligned}
g_1 &= (T_b - t_b)[\Pr(Bs \mid e_{10}) - \Pr(Bs \mid e_{00})] - C_b = (T_b - t_b)(P_{1b} - P_{3b}) - C_b \\
g_2 &= t_b [1 - \Pr(Bs \mid e_{10})] + T_b \Pr(Bs \mid e_{10}) - C_b = t_b (1 - P_{1b}) + T_b P_{1b} - C_b \\
g_3 &= (T_p - t_p)[\Pr(Ps \mid e_{11}) - \Pr(Ps \mid e_{10})] + C_p \\
g_4 &= t_p \Pr(Ps \mid e_{10}) + T_p [1 - \Pr(Ps \mid e_{10})]
\end{aligned}
$$

To express (A9) and (A10) we calculate the conditional probabilies of Ps with respect to e_{ij}:

$$
\text{(A12)}\quad
\begin{aligned}
A = \Pr(Ps \mid e_{10}) &= \Pr(Ps \cap Bs \mid e_{10}) + \Pr(Ps \cap - Bs \mid e_{10}) = \\
&= \Pr(Bs \mid e_{10}) \Pr(Ps \mid Bs \cap e_{10}) + \Pr(-Bs \mid e_{10}) \Pr(Ps \mid - Bs \cap e_{10}) = \\
&= P_{1b} P_{3p} + (1 - P_{1b}) P_{0p}\,.
\end{aligned}
$$

Likewise:

$$
\text{(A13)}\quad
\begin{aligned}
B = \Pr(Ps \mid e_{11}) &= \Pr(Ps \cap Bs \mid e_{11}) + \Pr(Ps \cap - Bs \mid e_{11}) = \\
&= \Pr(Bs \mid e_{11}) \Pr(Ps \mid Bs \cap e_{11}) + \Pr(-Bs \mid e_{11}) \Pr(Ps \mid - Bs \cap e_{11}) = \\
&= P_{2b} P_{1p} + (1 - P_{2b}) P_{2p}.
\end{aligned}
$$

$$
\text{(A14)}\quad C = \Pr(Ps \mid e_{01}) = P_{0b} P_{1p} + (1 - P_{0b}) P_{2p}
$$

$$
\text{(A15)}\quad D = \Pr(Ps \mid e_{00}) = P_{3b} P_{3p} + (1 - P_{3b}) P_{0p}
$$

It should be noticed that if $P_{0p} \leq P_{3p}$ then $A \leq P_{3p}$ and if $P_{1p} > P_{2p}$ then $B > P_{2p}$.

With equal effort being made by the banking authority, prices have more probability of being stable if there is an effort in this direction on the part of the agent of monetary policy. We therefore expect $A \leq B$ and $D \leq C$. Moreover, with equal effort being made by the authority in charge of monetary policy, the probability of stable prices is greater if the banking system is stable (see (3')) and therefore we expect

$D \leq C \leq B$; $A \leq B$.

This results in:

$$
\text{(A16)}\quad
\begin{aligned}
g_1 &= (T_b - t_b)(P_{1b} - P_{3b}) - C_b \\
g_2 &= t_b (1 - P_{1b}) + T_b P_{1b} - C_b \\
g_3 &= (T_p - t_p)[B - A] + C_p \\
g_4 &= t_p A + T_p [1 - A]
\end{aligned}
$$

The first-order conditions (A3) translate into:

$$
\begin{aligned}
&-u'(t_b) [P_{0p} (1 - P_{1b}) + (1 - P_{0p}) (1 - P_{1b})] - \lambda_1 (P_{1b} - P_{3b}) + \lambda_2 (1 - P_{1b}) = 0 \\
&-u'(T_b) [P_{3p} P_{1b} + (1 - P_{3p}) P_{1b}] + \lambda_1 (P_{1b} - P_{3b}) + \lambda_2 P_{1b} = 0 \\
&-u'(t_p) A - \lambda_3 (B - A) + \lambda_4 A = 0 \\
&-u'(T_p) (1 - A) + \lambda_3 (B - A) + \lambda_4 (1 - A) = 0
\end{aligned}
$$

or, if u'(t) is substituted with the value of the derivative of the utiltity function considered in point (10), we have:

$$|t_b| = [-\lambda_1 (P_{1b} - P_{3b}) + \lambda_2 (1 - P_{1b})] / (1 - P_{1b})$$

(A17)
$$|T_b| = [\lambda_1 (P_{1b} - P_{3b}) + \lambda_2 P_{1b}] / P_{1b}$$

$$|t_p| = -\lambda_3 (B - A) / A + \lambda_4$$

$$|T_p| = \lambda_3 (B - A) / (1 - A) + \lambda_4$$

Therefore, provided $0 < P_{1b} < 1$ and $0 < A < 1$

$$|t_b| = \lambda_2 - \lambda_1 (P_{1b} - P_{3b}) / (1 - P_{1b})$$

(A18)
$$|T_b| = \lambda_2 + \lambda_1 (P_{1b} - P_{3b}) / P_{1b}$$

$$|t_p| = \lambda_4 - \lambda_3 (B - A) / A$$

$$|T_p| = \lambda_4 + \lambda_3 (B - A) / (1 - A)$$

Conditions (A4)-(A6) lead to the examination of various cases, simplified by the fact that the first two of (A4) are related to t_b and T_b, while the other two are related to t_p and T_p. As we want to find solutions that maximize E(U-u), since –u is decrescent, the solution to the problem will be the one that makes u the lowest. Remember that u(t) is the cost incurred by the politician to pay the agents of the two different authorities. When this cost is lower, the politician's utility is greater.

Analysis of the cases that solve the optimization problem with two agents in the electoral period.

In examining the various cases that can eventuate, we must remember that considering $\lambda_i = 0$ simply means ignoring the constraint $g_i \geq 0$.

The cases we should examine to verify conditions (A4-A6) are:

I) $\lambda_1 = 0, g_2 = 0, g_1 \geq 0$

II) $\lambda_2 = 0, g_1 = 0, g_2 \geq 0$

III) $\lambda_1 = 0, \lambda_2 = 0$

IV) $g_1 = 0, g_2 = 0$

V) $\lambda_3 = 0, g_4 = 0, g_3 \geq 0$

VI) $\lambda_4 = 0, g_3 = 0, g_4 \geq 0$

VII) $\lambda_3 = 0, \lambda_4 = 0$

VIII) $g_3 = 0, g_4 = 0$

For I) $\boldsymbol{\lambda_1 = 0, g_2 = 0}$, $g_1 \geq 0$ we have:

$$|t_b| = \lambda_2$$

$$|T_b| = \lambda_2$$

$t_b (1 - P_{1b}) + T_b P_{1b} - C_b = 0$

$(T_b - t_b) (P_{1b} - P_{3b}) - C_b \geq 0$

It follows that, if $t_b = T_b$, then for the fourth equation the result is $-C_b \geq 0$, and therefore it can only be $C_b = 0$ and $t_b = T_b = 0$.

If, however, $t_b = - T_b$ then, for the third equation, we have

(A19) $- t_b = T_b = C_b / (2 P_{1b} - 1)$

and it must be

(A20) $P_{3b} \leq 1/2 < P_{1b}$.

The first inequality derives from the fourth eqation.

For II) $\boldsymbol{\lambda_2 = 0, g_1 = 0}$ we have:

$| t_b | = - \lambda_1 (P_{1b} - P_{3b}) / (1 - P_{1b})$

$| T_b | = \lambda_1 (P_{1b} - P_{3b}) / P_{1b}$

$(T_b - t_b) (P_{1b} - P_{3b}) - C_b = 0$

$t_b (1 - P_{1b}) + T_b P_{1b} - C_b \geq 0$.

The first equation can be verified only if $\lambda_1 = 0$ or $P_{1b} = P_{3b}$, but in both cases there would be $t_b = T_b = 0$ and then, for the third and fourth, there would be $C_b \leq 0$ and therefore $C_b = 0$.

III) $\boldsymbol{\lambda_1 = 0, \lambda_2 = 0}$ would give, for (A18)

$t_b = T_b = 0$

as in the previous case, provided $C_b = 0$.

Lastly, IV) $\boldsymbol{g_1 = 0, g_2 = 0}$ gives the system:

$(T_b - t_b) (P_{1b} - P_{3b}) - C_b = 0$

$t_b (1 - P_{1b}) + T_b P_{1b} - C_b = 0$

whose solution is:

(A21) $t_b = - C_b P_{3b} / (P_{1b} - P_{3b})$

$T_b = C_b (1 - P_{3b}) / (P_{1b} - P_{3b})$

This solution is obtained from (A18) by saying:

$\lambda_1 = C_b P_{1b} (1 - P_{1b}) (1 - 2 P_{3b}) / (P_{1b} - P_{3b})^2$

$\lambda_2 = C_b (P_{1b} + P_{3b} - 2 P_{1b} P_{3b}) / (P_{1b} - P_{3b})$.

For it to be $\lambda_1 \geq 0$, it will have to be $P_{3b} \leq 1/2$, as well as being $P_{1b} > P_{3b}$.

Let us examine the case V) $\boldsymbol{\lambda_3 = 0, g_4 = 0}$.

From (A18) it is deduced that $|t_p| = |T_p|$ and, from $g_4 = 0$, it follows that either $t_p=T_p= 0$, or $t_p = - T_p$ and, from $g_4 = 0$, it follows that $T_p (1 - 2A) = 0$. Therefore, either we return to case $T_p = 0$ or $A=1/2$. In the latter case, the condition $g_3 \geq 0$ translates into $T_p \geq - C_p/(2(B - A))$, that is, any non-negative value of T_p is acceptable.

Case VI) $\boldsymbol{\lambda_4 = 0, g_3 = 0}$ gives, for (A18):

$| t_p | = -\lambda_3 (B - A) / A$

$| T_p | = \lambda_3 (B - A) / (1 - A)$

$(T_p - t_p) [B - A] + C_p = 0$

$t_p A + T_p [1 - A] \geq 0$

The first can be satisfied only if $t_p = 0$ and we have this if $\lambda_3 = 0$ or B = A. In both cases there would be $T_p = 0$ and from the third $C_p = 0$, against the hypotheses. This case can therefore not be verified.

Case VII) $\boldsymbol{\lambda_3 = 0, \lambda_4 = 0}$ gives

(A22) $t_p = T_p = 0$

already seen in case V).

Finally, case VIII) $\boldsymbol{g_3 = 0, g_4 = 0}$ is equivalent to:

$T_p - t_p = - C_p / (B - A)$

$T_p = A (T_p - t_p)$

This case is possible only if $C_p = 0$ and if so the solution is

$T_p = t_p = 0$

or if A > B and this goes against common sense.

B. Contract with two agents in the non-electoral period

Problem of constrained optimization

The constraints can be expressed in short form, by saying:

(B1)
$$\begin{aligned}
g_1 &= E(I_b \mid e_b = 1) - E(I_b \mid e_b = 0) = (T_b - t_b) [Pr(Bs \mid e_b = 1) - Pr(Bs \mid e_b = 0) - C_b \\
g_2 &= E(I_b \mid e_b = 1) = t_b [1 - Pr(Bs \mid e_b = 1)] + T_b Pr(Bs \mid e_b = 1) - C_b \\
g_3 &= E(I_p \mid e_p = 1) - E(I_p \mid e_p = 0) = (T_p - t_p) [Pr(Ps \mid e_p = 1) - Pr(Ps \mid e_p = 0)] - C_p \\
g_4 &= E(I_p \mid e_p = 1) = t_p [1 - Pr(Ps \mid e_p = 1)] + T_p Pr(Ps \mid e_p = 1) - C_p
\end{aligned}$$

and therefore constraints (23)-(26) can be written:

(B2) $g_1 \geq 0$ $g_2 \geq 0$ $g_3 \geq 0$ $g_4 \geq 0$.

Proceeding as in the previous case, conditions of the 1st order are given by (A3)-(A6) and, for the same reasons, we have

(B3)
$$\begin{aligned}
g_1 &= (T_b - t_b) [Pr(Bs \mid e_{11}) - Pr(Bs \mid e_{01})] - C_b = (T_b - t_b) (P_{2b} - P_{0b}) - C_b \\
g_2 &= t_b (1 - P_{2b}) + T_b P_{2b} - C_b \\
g_3 &= (T_p - t_p) [Pr(Ps \mid e_{11}) - Pr(Ps \mid e_{10})] - C_p = (T_p - t_p) [B - A] - C_p \\
g_4 &= t_p [1 - Pr(Ps \mid e_{11})] + T_p Pr(Ps \mid e_{11}) - C_p = t_p [1 - B] + T_p B - C_p
\end{aligned}$$

Conditions (A3) translate into :

$$-u'(t_b)\,[P_{2p}(1 - P_{2b}) + (1 - P_{2p})(1 - P_{2b})] - \lambda_1 (P_{2b} - P_{0b}) + \lambda_2 (1 - P_{2b}) = 0$$

$$-u'(T_b)\,[P_{1p} P_{2b} + (1 - P_{1p}) P_{2b}] + \lambda_1 (P_{2b} - P_{0b}) + \lambda_2 P_{2b} = 0$$

$$-u'(t_p)\,(1 - B) - \lambda_3 (B - A) + \lambda_4 (1 - B) = 0$$

$$-u'(T_p)\,B + \lambda_3 (B - A) + \lambda_4 B = 0,$$

which gives:

$$\text{(B4)}\quad \begin{aligned} |\,t_b\,| &= \lambda_2 - \lambda_1 (P_{2b} - P_{0b}) / (1 - P_{2b}) \\ |\,T_b\,| &= \lambda_2 + \lambda_1 (P_{2b} - P_{0b}) / P_{2b} \\ |\,t_p\,| &= \lambda_4 - \lambda_3 (B - A) / (1 - B) \\ |\,T_p\,| &= \lambda_4 + \lambda_3 (B - A) / B \end{aligned}$$

C. Contract with a single agent in the elctoral period.

Problem of constrained optimization

Keeping in mind the conditional probabilities $(4)^i$-$(7)^i$ the constraints become

$$\begin{aligned} g_1 &= T_{10}\,[\Pr(E_1 \mid e_{10}) - \Pr(E_1 \mid e_{00})] + T_{11}\,[\Pr(E_2 \mid e_{10}) - \Pr(E_2 \mid e_{00})] + \\ &\quad + T_{01}\,[\Pr(E_3 \mid e_{10}) - \Pr(E_3 \mid e_{00})] + T_{00}\,[\Pr(E_4 \mid e_{10}) - \Pr(E_4 \mid e_{00})] - (C_{bp} - C_p) = \\ &= T_{10}\,[P_{1b}(1 - P_{3p}) - P_{3b}(1 - P_{3p})] + T_{11}\,[P_{1b} P_{3p} - P_{3b} P_{3p}] + \\ &\quad + T_{01}\,[(1 - P_{1b}) P_{0p} - (1 - P_{3b}) P_{0p}] + T_{00}\,[(1 - P_{1b})(1 - P_{0p}) - (1 - P_{3b})(1 - P_{0p})] - (C_{bp} - C_p) = \\ &= (P_{1b} - P_{3b})\,[T_{10}(1 - P_{3p}) + T_{11} P_{3p} - T_{01} P_{0p} - T_{00}(1 - P_{0p})] - (C_{bp} - C_p) \geq 0 \end{aligned}$$

$$\begin{aligned} g_2 &= T_{10}\,[\Pr(E_1 \mid e_{10}) - \Pr(E_1 \mid e_{11})] + T_{11}\,[\Pr(E_2 \mid e_{10}) - \Pr(E_2 \mid e_{11})] + \\ &\quad + T_{01}\,[\Pr(E_3 \mid e_{10}) - \Pr(E_3 \mid e_{11})] + T_{00}\,[\Pr(E_4 \mid e_{10}) - \Pr(E_4 \mid e_{11})] + C_p = \\ &= T_{10}\,[P_{1b}(1 - P_{3p}) - P_{2b}(1 - P_{1p})] + T_{11}\,[P_{1b} P_{3p} - P_{2b} P_{1p}] + \\ &\quad + T_{01}\,[(1 - P_{1b}) P_{0p} - (1 - P_{2b}) P_{2p}] + T_{00}\,[(1 - P_{1b})(1 - P_{0p}) - (1 - P_{2b})(1 - P_{2p})] + C_p \geq 0 \end{aligned}$$

(C1)

$$\begin{aligned} g_3 &= T_{10}\,[\Pr(E_1 \mid e_{10}) - \Pr(E_1 \mid e_{01})] + T_{11}\,[\Pr(E_2 \mid e_{10}) - \Pr(E_2 \mid e_{01})] + \\ &\quad + T_{01}\,[\Pr(E_3 \mid e_{10}) - \Pr(E_3 \mid e_{01})] + T_{00}\,[\Pr(E_4 \mid e_{10}) - \Pr(E_4 \mid e_{01})] + (C_p - C_b) = \\ &= T_{10}\,[P_{1b}(1 - P_{3p}) - P_{0b}(1 - P_{1p})] + T_{11}\,[P_{1b} P_{3p} - P_{0b} P_{1p}] + \\ &\quad + T_{01}\,[(1 - P_{1b}) P_{0p} - (1 - P_{0b}) P_{2p}] + T_{00}\,[(1 - P_{1b})(1 - P_{0p}) - (1 - P_{0b})(1 - P_{2p})] + \\ &\quad + (Cp - Cb) \geq 0 \end{aligned}$$

$g_4 = T_{10}\, Pr(E_1 \mid e_{10}) + T_{11}\, Pr(E_2 \mid e_{10}) + T_{01}\, Pr(E_3 \mid e_{10}) + T_{00}\, Pr(E_4 \mid e_{10}) - (C_{bp} - C_p) =$

$= T_{10}\, [P_{1b}\, (1 - P_{3p})] + T_{11}\, [P_{1b}\, P_{3p}] + T_{01}\, [(1 - P_{1b})\, P_{0p}] +$

$+ T_{00}\, [(1 - P_{1b})\, (1 - P_{0p})] - (C_{bp} - C_p) \geq 0$

Considering the Lagrangian (A1), the conditions of the 1st order are given (A3)-(A6) and in particular (A3) translate into:

$-u'(T_{10})\, (1 - P_{3p})\, P_{1b} + \lambda_1\, [(P_{1b} - P_{3b})\, (1 - P_{3p})] + \lambda_2\, [P_{1b}\, (1 - P_{3p}) - P_{2b}\, (1 - P_{1p})] +$

$+ \lambda_3\, [P_{1b}\, (1 - P_{3p}) - P_{0b}\, (1 - P_{1p})] + \lambda_4 [P_{1b}\, (1 - P_{3p})] = 0$

$-u'(T_{11})\, P_{3p}\, P_{1b} + \lambda_1\, [P_{1b}\, P_{3p} - P_{3b}\, P_{3p}] + \lambda_2\, [P_{1b}\, P_{3p} - P_{2b}\, P_{1p}] + \lambda_3\, [P_{1b}\, P_{3p} - P_{0b}\, P_{1p}] + \lambda_4\, [P_{1b}\, P_{3p}] = 0$

$-u'(T_{01})\, P_{0p}\, (1 - P_{1b}) + \lambda_1 [(1 - P_{1b})\, P_{0p} - (1 - P_{3b})\, P_{0p}] + \lambda_2\, [(1 - P_{1b})\, P_{0p} - (1 - P_{2b})\, P_{2p}] +$

$+ \lambda_3\, [(1 - P_{1b})\, P_{0p} - (1 - P_{0b})\, P_{2p}] + \lambda_4 [(1 - P_{1b})\, P_{0p}] = 0$

$-u'(T_{00})\, (1 - P_{0p})\, (1 - P_{1b}) + \lambda_1\, [(1 - P_{1b})\, (1 - P_{0p}) - (1 - P_{3b})\, (1 - P_{0p})] +$

$+ \lambda_2\, [(1 - P_{1b})\, (1 - P_{0p}) - (1 - P_{2b})\, (1 - P_{2p})] + \lambda_3\, [(1 - P_{1b})\, (1 - P_{0p}) - (1 - P_{0b})\, (1 - P_{2p})] +$

$+ \lambda_4\, [(1 - P_{1b})\, (1 - P_{0p})] = 0$

which we can also write:

$|T_{10}| = \lambda_1\, (1 - P_{3b} / P_{1b}) + \lambda_2\, [1 - P_{2b}\, (1 - P_{1p}) / (P_{1b}\, (1 - P_{3p}))] +$

$+ \lambda_3\, [1 - P_{0b}\, (1 - P_{1p}) / (P_{1b}\, (1 - P_{3p}))] + \lambda_4$

$|T_{11}| = \lambda_1\, (1 - P_{3b} / P_{1b}) + \lambda_2\, [1 - P_{2b} P_{1p} / (P_{1b}\, P_{3p})] + \lambda_3\, [1 - P_{0b}\, P_{1p} / (P_{1b} P_{3p})] + \lambda_4$

(C2)

$|T_{01}| = \lambda_1\, [1 - (1 - P_{3b}) / (1 - P_{1b})] + \lambda_2\, [1 - (1 - P_{2b})\, P_{2p} / ((1 - P_{1b})\, P_{0p})] +$

$+ \lambda_3\, [1 - (1 - P_{0b})\, P_{2p} / (1 - P_{1b})\, P_{0p}] + \lambda_4$

$|T_{00}| = \lambda_1\, [1 - (1 - P_{3b}) / (1 - P_{1b})\,] + \lambda_2\, [1 - (1 - P_{2b})\, (1 - P_{2p}) / (1 - P_{1b})\, (1 - P_{0p})] +$

$+ \lambda_3\, [1 - (1 - P_{0b})\, (1 - P_{2p}) / (1 - P_{1b})\, (1 - P_{0p})] + \lambda_4$

or in more explicit terms:

$$|\ T_{10}\ | = \lambda_1 \left[1 - \frac{\Pr(E_1|e_{00})}{\Pr(E_1|e_{10})}\right] + \lambda_2 \left[1 - \frac{\Pr(E_1|e_{11})}{\Pr(E_1|e_{10})}\right] + \lambda_3 \left[1 - \frac{\Pr(E_1|e_{01})}{\Pr(E_1|e_{10})}\right] + \lambda_4$$

$$|\ T_{10}\ | = \lambda_1 \left[1 - \frac{\Pr(E_2|e_{00})}{\Pr(E_2|e_{10})}\right] + \lambda_2 \left[1 - \frac{\Pr(E_2|e_{11})}{\Pr(E_2|e_{10})}\right] + \lambda_3 \left[1 - \frac{\Pr(E_2|e_{01})}{\Pr(E_2|e_{10})}\right] + \lambda_4$$

$$|\ T_{01}\ | = \lambda_1 \left[1 - \frac{\Pr(E_3|e_{00})}{\Pr(E_3|e_{10})}\right] + \lambda_2 \left[1 - \frac{\Pr(E_3|e_{11})}{\Pr(E_3|e_{10})}\right] + \lambda_3 \left[1 - \frac{\Pr(E_3|e_{01})}{\Pr(E_3|e_{10})}\right] + \lambda_4$$

$$|\ T_{00}\ | = \lambda_1 \left[1 - \frac{\Pr(E_4|e_{00})}{\Pr(E_4|e_{10})}\right] + \lambda_2 \left[1 - \frac{\Pr(E_4|e_{11})}{\Pr(E_4|e_{10})}\right] + \lambda_3 \left[1 - \frac{\Pr(E_4|e_{01})}{\Pr(E_4|e_{10)}}\right] + \lambda_4$$

The solution to this problem is not easy, however it is less important to know the value of incentives that maximize E $(U - u \mid e_{10})$, than to know if it is more advantageous to appoint two agents or a single agent. To resolve the problem, we have to compare the utility expected by the politician with a single a single agent, with the expected utility with two separate authorities.

D. Contract with a single agent in the non-electoral period.

Problem of constrained optimization

Keeping conditional probabilities $(4)^i$-$(7)^i$ in mind the constraints become

$$g_1 = T_{11}[Pr(E_1 \mid e_{11}) - Pr(E_1 \mid e_{10})] + T_{10}[Pr(E_2 \mid e_{11}) - Pr(E_2 \mid e_{10})] +$$
$$+ T_{01}[Pr(E_3 \mid e_{11}) - Pr(E_3 \mid e_{10})] + T_{00}[Pr(E_4 \mid e_{11}) - Pr(E_4 \mid e_{10})] - [C_{bp} - (C_{bp} - C_p)] =$$
$$= T_{11}[P_{2b} P_{1p} - P_{1b} P_{3p}] + T_{10}[P_{2b}(1 - P_{1p}) - P_{1b}(1 - P_{3p})] +$$
$$+ T_{01}[(1 - P_{2b}) P_{2p} - (1 - P_{1b}) P_{0p}] + T_{00}[(1 - P_{2b})(1 - P_{2p}) - (1 - P_{1b})(1 - P_{0p})] - C_p \geq 0$$

$$g_2 = T_{11}[Pr(E_1 \mid e_{11}) - Pr(E_1 \mid e_{01})] + T_{10}[Pr(E_2 \mid e_{11}) - Pr(E_2 \mid e_{01})] +$$
$$+ T_{01}[Pr(E_3 \mid e_{11}) - Pr(E_3 \mid e_{01})] + T_{00}[Pr(E_4 \mid e_{11}) - Pr(E_4 \mid e_{01})] - C_b =$$
$$= T_{11}[P_{2b} P_{1p} - P_{0b} P_{1p}] + T_{10}[P_{2b}(1 - P_{1p}) - P_{0b}(1 - P_{1p})] +$$
$$+ T_{01}[(1 - P_{2b}) P_{2p} - (1 - P_{0b}) P_{2p}] + T_{00}[(1 - P_{2b})(1 - P_{2p}) - (1 - P_{0b})(1 - P_{2p})] - C_b =$$
$$= (P_{2b} - P_{0b})[T_{11} P_{1p} + T_{10}(1 - P_{1p}) - T_{01} P_{2p} - T_{00}(1 - P_{2p})] - C_b \geq 0$$

(D1)

$$g_3 = T_{11}[Pr(E_1 \mid e_{11}) - Pr(E_1 \mid e_{00})] + T_{10}[Pr(E_2 \mid e_{11}) - Pr(E_2 \mid e_{00})] +$$
$$+ T_{01}[Pr(E_3 \mid e_{11}) - Pr(E_3 \mid e_{00})] + T_{00}[Pr(E_4 \mid e_{11}) - Pr(E_4 \mid e_{00})] - C_{bp} =$$
$$= T_{11}[P_{2b} P_{1p} - P_{3b} P_{3p}] + T_{10}[P_{2b}(1 - P_{1p}) - P_{3b}(1 - P_{3p})] +$$
$$+ T_{01}[(1 - P_{2b}) P_{2p} - (1 - P_{3b}) P_{0p}] + T_{00}[(1 - P_{2b})(1 - P_{2p}) - (1 - P_{3b})(1 - P_{0p})] - C_{bp} \geq 0$$

$$g_4 = T_{11} P_{2b} P_{1p} + T_{10} P_{2b}(1 - P_{1p}) + T_{01}(1 - P_{2b}) P_{2p} + T_{00}(1 - P_{2b})(1 - P_{2p}) - C_{bp} =$$
$$= P_{2b}[T_{11} P_{1p} + T_{10}(1 - P_{1p})] + (1 - P_{2b})[T_{01} P_{2p} + T_{00}(1 - P_{2p})] - C_{bp} \geq 0 .$$

Considering the Lagrangian (A1), the conditions of the 1st order are given (A3)-(A6) and in particular (A3) translate into :

$$-u'(T_{11}) P_{2b} P_{1p} + \lambda_1 [P_{2b} P_{1p} - P_{1b} P_{3p}] + \lambda_2 [P_{2b} P_{1p} - P_{0b} P_{1p}] + \lambda_3 [P_{2b} P_{1p} - P_{3b} P_{3p}] + \lambda_4 P_{2b} P_{1p} = 0$$

$$-u'(T_{10}) P_{2b}(1 - P_{1p}) + \lambda_1 [P_{2b}(1 - P_{1p}) - P_{1b}(1 - P_{3p})] + \lambda_2 [P_{2b}(1 - P_{1p}) - P_{0b}(1 - P_{1p})] +$$
$$+ \lambda_3 [P_{2b}(1 - P_{1p}) - P_{3b}(1 - P_{3p})] + \lambda_4 P_{2b}(1 - P_{1p}) = 0$$

$-u'(T_{01})(1 - P_{2b}) P_{2p} + \lambda_1 [(1 - P_{2b}) P_{2p} - (1 - P_{1b}) P_{0p}] + \lambda_2 [(1 - P_{2b}) P_{2p} - (1 - P_{0b}) P_{2p}] +$

$+ \lambda_3 [(1 - P_{2b}) P_{2p} - (1 - P_{3b}) P_{0p}] + \lambda_4 (1 - P_{2b}) P_{2p} = 0$

$- u'(T_{00})(1 - P_{2b})(1 - P_{2p}) + \lambda_1 [(1-P_{2b})(1 - P_{2p}) - (1 - P_{1b})(1 - P_{0p})] +$

$+ \lambda_2 [(1 - P_{2b})(1 - P_{2p}) - (1 - P_{0b})(1 - P_{2p})] + \lambda_3 [(1 - P_{2b})(1 - P_{2p}) - (1 - P_{3b})(1 - P_{0p})] +$

$+ \lambda_4 (1 - P_{2b})(1 - P_{2p}) = 0$

which we can also write:

$|T_{11}| = \lambda_1 [1 - P_{1b} P_{3p} / (P_{2b} P_{1p})] + \lambda_2 [1 - P_{0b} P_{1p} / (P_{2b} P_{1p})] + \lambda_3 [1 - P_{3b} P_{3p} / (P_{2b} P_{1p})] + \lambda_4$

$|T_{10}| = \lambda_1 [1 - P_{1b} (1 - P_{3p}) / (P_{2b} (1 - P_{1p}))] + \lambda_2 [1 - P_{0b} (1 - P_{1p}) / (P_{2b} (1 - P_{1p}))] +$

(D2) $+ \lambda_3 [1 - P_{3b} (1 - P_{3p}) / (P_{2b} (1 - P_{1p}))] + \lambda_4$

$|T_{01}| = \lambda_1 [1 - (1 - P_{1b}) P_{0p} / ((1 - P_{2b}) P_{2p})] + \lambda_2 [1 - (1 - P_{0b}) P_{2p} / ((1 - P_{2b}) P_{2p})] +$

$+ \lambda_3 [1 - (1 - P_{3b}) P_{0p} / ((1 - P_{2b}) P_{2p})] + \lambda_4$

$|T_{00}| = \lambda_1 [1 - (1 - P_{1b})(1 - P_{0p}) / ((1 - P_{2b})(1 - P_{2p}))] + \lambda_2 [1 - (1 - P_{0b})(1 - P_{2p}) /$

$/ ((1 - P_{2b})(1 - P_{2p}))] + \lambda_3 [1 - (1 - P_{3b})(1 - P_{0p}) / ((1 - P_{2b})(1 - P_{2p}))] + \lambda_4$

E. We prove some inequalities

(E1) $\frac{1}{2x-1} \geq \frac{(1-y)^2 x - y^2(1-x)}{(x-y)^2}$

since 2x – 1 > 0 and 0 < x, y < 1.

(E1) is equivalent to

$(x-y)^2 \geq [(1-y)^2 x - y^2(1-x)](2x-1)$

$x^2 + y^2 - 2xy \geq [x + 2y^2x - 2xy - y^2](2x - 1)$

$x^2 + y^2 - 2xy \geq 2x^2 + 4x^2y^2 - 4x^2y - 2xy^2 - x - 2xy^2 + 2xy + y^2$

$4x^2y + 4xy^2 + x \geq x^2 + 4x^2y^2 + 4xy$

$4xy + 4y^2 + 1 \geq x + 4xy^2 + 4y$

$4xy(1 - y) + 1 - x \geq 4y(1 - y)$

$1 - x \geq 4y(1 - y)(1 - x)$

$1/4 \geq y(1-y)$

this last inequality being true, the first is true.

(E2) $(1-y)^2 x - y^2 (1-x)$

can only be positive, if $0 < y \leq 1/2$ and $y \leq x$.

$(1-y)^2 x - y^2 (1-x) \geq 0$

equivalent to

$((1-y)/y)^2 \geq (1-x)/x$;

since if $0 < y \leq 1/2$ then $(1-y)/y \geq 1$, we get $((1-y)/y)^2 \geq (1-y)/y$ and as funzion

$f(x) = (1-x)/x$

is decrescent and $y \leq x$, we will have

$((1-y)/y)^2 \geq (1-y)/y \geq (1-x)/x$

the equality is valid only if

$x = y$.

(E3) $y/[2(x-y)] \leq [(1-y)^2 x - y^2 (1-x)]/[2(x-y)^2] \leq 1/[2(x-y)]$

if $0 < y \leq 1/2$ and $y \leq x$ for the first inequality and $0 < y \leq 1/2 \leq x$ for the second.

The first inequality is equivalent to

$y(x-y) \leq (1-y)^2 x - y^2 (1-x) \Leftrightarrow yx \leq (1-y)^2 x + y^2 x$

which means :

$yx(1-y) \leq (1-y)^2 x \Leftrightarrow y \leq (1-y) \Leftrightarrow y \leq 1/2$.

The second inequality is equivalent to

$(1-y)^2 x - y^2 (1-x) \leq x - y \Leftrightarrow (1 + y^2 - 2y)x - y^2 + y^2 x \leq x - y$

which means :

$2y^2 x + y \leq y(2x + y) \Leftrightarrow 2yx + 1 \leq 2x + y \Leftrightarrow 1 - y \leq 2x(1-y)$

That is, $x \geq 1/2$.