8. Comparison of the two contracts in the non-electoral period.

The politician's expected net utility in the non-electoral period is

$E(U - u \mid e_{11}) = H' - K''$

with $K'' = (1/2) \{[C_b^2 / (2 P_{2b} - 1)] + [C_p^2 / (2 B - 1)]\}$

in the case of two agents, and

$E(U - u \mid e_{11}) = H' - K'$

with $K' = P_{2b} [u(T_{11}) P_{1p} + u(T_{10}) (1 - P_{1p})] + (1 - P_{2b}) [u(T_{01}) P_{2p} + u(T_{00}) (1 - P_{2p})]$

in the case of a single agent.

The politician will prefer to appoint a single agent also in the non-electoral period if

(36) $\min K' \leq \min K''$.

To prove this, it will be sufficient to show that, for an appropriate allocation of incentive payments T_{ij}, a value of K' that is $\leq \min K''$ can be obtained.

Hypothesizing that

$C_b = C_p = C$ and $P_{2b} = B$ where $B = Pr(Ps \mid e_{11})$ (see (A13)) and $P_{2b} = Pr(Bs \mid e_{11})$,

and remembering the value of the payments identified:

i) $t_b = -T_b$ with $T_b = C_b / (2 P_{2b} - 1)$ if $P_{0b} \leq 1/2 < P_{2b}$

ii) $t_p = -T_p$ with $T_p = C_p / (2 B - 1)$, if $A \leq 1/2 < B$

then in the two-agent contract in the non-electoral period, one would have

$T_b + T_p = [2 C / (2P_{2b} - 1)]$

$T_b + t_p = t_b + T_p = 0$

$t_b + t_p = -[2 C / (2P_{2b} - 1)]$

This suggests for the single-agent contract, an incentive payment of the type:

$T_{11} = T > 0$

$T_{10} = T_{01} = 0$

$T_{00} = -T$

With these payments, the politician's expected net utility would be:

$E(U\text{-}u \mid e_{11}) = H' - P_{2b}\,[u(T_{11})\,P_{1p} + u(T_{10})\,(1 - P_{1p})] + (1 - P_{2b})\,[u(T_{01})\,P_{2p} + u(T_{00})\,(1 - P_{2p})] =$

$= H' - (1/2)\,T^2\,[P_{2b}\,P_{1p} - (1 - P_{2b})\,(1 - P_{2p})] = H' - (1/2)\,T^2\,(B - 1 + P_{2b})$

This attribution is valid, however, only if it satisfies the constraints. Testing shows:

$g_1 \geq 0 \quad T\,[B - A - (P_{1b} - P_{2b})] \geq C_p$

$g_2 \geq 0 \quad T\,(P_{2b} - P_{0b})\,(P_{1p} - P_{2p} + 1) \geq C_b$

$g_3 \geq 0 \quad T\,(B - D + P_{2b} - P_{3b}) \geq C_{bp}$

$g_4 \geq 0 \quad T\,(B + P_{2b} - 1) \geq C_{bp}$

We see that the coefficient of T in $g_2 \geq 0$ is positive if $P_{2b} \geq P_{0b}$ and, therefore, the constraint g_2 can be satisfied (for $T > C_b / [(P_{2b} - P_{0b})\,(P_{1p} - P_{2p} + 1)]$).

In the hypothesis that $P_{2b} = B$ the constraints become:

$g_1 \geq 0 \quad T\,[2\,P_{2b} - (A + P_{1b})] \geq C_p$

$g_2 \geq 0 \quad T\,(P_{2b} - P_{0b})\,(P_{1p} - P_{2p} + 1) \geq C_b$

$g_3 \geq 0 \quad T\,[2\,P_{2b} - (P_{3b} + D)] \geq C_{bp}$

$g_4 \geq 0 \quad T\,(2\,P_{2b} - 1) \geq C_{bp}$

We see that $2\,P_{2b} - 1 > 0$ by hypothesis, so

$g_4 \geq 0 \quad \leftrightarrow \quad T \geq C_{bp} / [(2P_{2b} - 1)]$.

Since $D \leq B = P_{2b}$, then $2P_{2b} - (P_{3b} + D) > P_{2b} - P_{3b}$. Moreover if we suppose that $P_{3b} < P_{2b}$ (or $P_{3b} = Pr(Bs \mid e_{00}) < P_{2b} = Pr(Bs \mid e_{11})$), then the coefficient of T in $g_3 \geq 0$ is also positive and the result is that $g_3 \geq 0$ equivalent to

$g_3 \geq 0 \quad \leftrightarrow \quad T \geq C_{bp} / [(2\,P_{2b} - P_{3b} - D)]$

$g_2 \geq 0 \quad \leftrightarrow$ if $P_{1p} = P_{2p} \quad T\,(P_{2b} - P_{0b})\,(P_{1p} - P_{2p} + 1) \geq C_b$

There is increasing compatibility between choice of payments and the constraints

- the lower P_{0b} and P_{3b}: the probability of banking system stability when there is no effort in this direction by the agent (consistent with the hypotheses made);
- the higher P_{1p}: the probability of price stability when there is effort in this direction by the agent, in the presence of banking stability.

It must be remembered that we put $B = P_{2b}$, (i.e. $Pr(Ps|e11) = Pr(Bs|e11)$).

The tightest condition becomes g_4. We therefore choose this as payment and say:

$T = C_{bp} / [(2 P_{2b} - 1)]$.

Under these conditions, therefore, the politician's expected net utility is:

$E(U\text{-}u \mid e_{11}) = H' - (1/2) T^2 (B - 1 + P_{2b}) = H' - (1/2) T^2 (2 P_{2b} - 1) =$

$H' - (1/2) \{C_{bp}^2 / [(2 P_{2b} - 1)^2]\}[(2 P_{2b} - 1)] = H' - (1/2) \{C_{bp}^2 / [(2 P_{2b} - 1)]\}$

Consequently, the politician will prefer to entrust the appointment to a single agent also in the non-electoral period if:

$(1/2) \{C_{bp}^2 / [(2 P_{2b} - 1)]\} \leq (1/2) \{[C_b^2 / (2 P_{2b} - 1)] + [C_p^2 / (2 B - 1)]\}$

But since we hypothesized that $C_p = C_b = C$ e $P_{2b} = B$, then we have:

$\{C_{bp}^2 / [(2 P_{2b} - 1)]\} \leq 2C^2 / (2 P_{2b} - 1)$

Therefore the politician will prefer to entrust the appointment to a single agent also in the non-electoral period if[26]:

$C_{bp}^2 \leq 2C^2 = C_b^2 + C_p^2$.

The politician's choice of single or multiple authorities is therefore not tied to electoral factors.

What emerges from this model, on the basis of the hypotheses made and the constraints imposed, is that the politician always prefers the single authority if the following conditions come about:

- $C_{bp} \leq C_p + C_b$ i.e. if there are economies of scope;
- $C_b \leq C_p$ if the task of the central bank is more demanding (and this is clear if we consider that the central bank is also responsible for banking stability as lender of last resort);

[26] It is, however, very easy to give an example in which the previous conditions do not exist, which precludes the use of the previous procedure to prove the advantageousness of using a single agent. In contrast, it is not at all easy to give an example to show, at least for some values of the parameters, that it is more advantageous to entrust the roles to separate agents. This is due to the fact that it is difficult to solve the problem of optimization with a single agent and consequently the minimum of K is unknown. We can however conjecture that it is always more advantageous to entrust the functions to a single agent for a simple reason that is discussed in the next section.

•$P_{3b} < P_{1b}$ it is natural to expect this given the definition of these probabilities (in other words it is normal to expect banking stability to be easier to achieve if there is effort in this direction on the part of the authority responsible);

•the lower P_{0b} and P_{3b}, or the higher the risk of banking crises;

• the higher P_{1p} is, the probability of price stability when the monetary agent makes an effort to attain this, in the presence of stability in the banking system.

9. Remarks and possible developments

The advantage of entrusting the roles to a single agent can however be conjectured using a simple thought process.

The minimum cost that the politician has to bear for incentives must at least cover the costs of effort and must therefore equal C_{bp}, if there is a single agent, and $C_p + C_b$ if there are two. If $C_{bp} < C_p + C_b$, then it is clear that it is always more convenient to appoint a single agent.

In the election period, the politician's spending on incentives is even lower, in that as he does not want the central bank to be over-zealous, it will only equal C_b for two agents and $C_{bp} - C_p$ for one agent. Therefore the minimum spending for the politician must be in the electoral period.

If however we introduce a reputation cost, R, in cases in which although the central bank wants to act, it refrains from doing so in order to please the politician, then in the electoral period if two agents are appointed, the politician's cost will be equivalent to

$C_b + (R/2)$ if there are two agents and to

$C_{bp} - C_p + R$ if there is one agent.

In the non-electoral period**,** the politician's costs will be equivalent to

$C_b + C_p$ for two agents and to

C_{bp} for one agent.