1. Introduction

In recent years, a reform of the supervision authorities has been under discussion in every European country. The basic issue is whether it is appropriate for the central bank, already in charge of monetary policy¹, to be made responsible for supervision² as well. In some countries banking regulation is carried out by the central bank, in others it is the duty of different authorities, possibly in collaboration with the central bank, or lastly it is the task of a single authority separate from the central bank. Among the European countries, the latter is true of the United Kingdom³, Austria⁴, Belgium, Denmark, Germany, Sweden⁵, Hungary⁶, Malta, Estonia and Latvia. On a world level, of Japan and South Korea. Ireland on the other hand is an anomaly. The only authority responsible for bank supervision there is the central bank⁷. What is therefore emerging are two models: one with a single authority and the other with multiple authorities⁸.

¹ On the factors determining the decision to delegate responsibility for monetary policy to the central bank, a valid answer is given by Alesina - Tabellini (2003).
² According to Lannoo(1999), the rethinking of the job of banking supervision can be explained with the growing development of banks and the loss of clarity about the services offered.
³ The FSA in the United Kingdom placed banking supervision outside the central bank, but since one of the functions of regulation is to protect from systemic risk, a close, transparent relationship needs to be established with the central bank and the Ministry of Finance. In 1997 a Memorandum of Understanding was signed between the FSA, the Treasury, and the Bank of England. This Memorandum explains how the three bodies work together to achieve financial stability. But “The Authorities of the United Kingdom underline that models of regulation should reflect the structural and cultural characteristics of local financial services market. What is right for the United Kingdom, is not necessarily right for all”. Sykes (FSA 2004)
⁴ Austria adopted the FMA single authority supervision model in 2002. This body carries out supervision on the banking, insurance, pension fund, and stock market, cooperating with the Austrian central bank. Grunbichler – Darlap (FMA 2004)
⁵ In Sweden the central bank (Riksbank) answers directly to Parliament and has no supervisory role. Its role is to promote a safe and efficient payment system and lender of last resort. Since 1971 supervision has been entrusted to a single authority (Finansinspektionen) which watches over the banking, stock and insurance markets with the purpose of achieving stability and efficiency in the financial system and defending the consumer. Strom (FSA 2004)
⁶ In Hungary the single financial authority was set up in April 2000, with responsibility for the whole financial sector. Decisions taken by the supervising body are final. Appeals can only be made through the courts of justice. Even before the institution of the single authority, the Hungarian central bank never had a banking supervision role. Balogh (FSA 2004)
⁷ Holland is moving in the same direction. Assigning the role of prudential supervision to the central bank involves advantages and disadvantages. Among the pros, we find systemic stability, stability in the system of payments, cost efficiency, good central bank reputation, which can however become a disadvantage if the reputation is bad. There may be other disadvantages in terms of price stability, monetary policy dictated by the banks, over-concentration of power in the hands of the central bank. Prast – Lelyveld (De Nederlandsche Bank 2004)
⁸ In a recent work Masciandaro (2005) uses an empirical model to analyse the possible factors determining the process of reunification in the various European countries. For an examination of the pros and cons of attributing the responsibility for monetary policy and supervision to the central bank, cfr. Peek – Rosengren – Tootell (1999, 2001, 2003); Masciandaro (1993), Eijffinger (2001) and for further
The policymaker, whose main aim is to be re-elected, is the one who decides the institutional setup\(^9\). The political economy choices will be, therefore, influenced by the wish to get the favour of the electors. Theoretical analysis\(^10\) suggests that the politician prefers an expansive monetary policy before the elections and a tight monetary policy, to low inflation, in the post electoral period. The possibility for the government to handle monetary policy depends on institutional design, chosen in every country. Independent central banks or dependent central banks with more conservative central banker limit the possibility for the policymaker to carry out twisted political economics\(^11\). Such setup is not therefore desirable for a politician in the pre-electoral period, especially in that countries characterized by an higher rate of unemployment. The hypothesis is correct if there is agreement, inside the same political class, on the monetary policy choices\(^12\). How is it possible now to connect the different politician’s setup choice, according to the period he is in, with the possibility that the responsibility of supervision policy is entrusted to central banker by the same policymaker? In fact the possibility that the central banker is able or less to have the assignment to pursue the financial stability complicates the picture. From the point of view of the policymaker, the conduct of supervision policy, has some implications in terms of cost-benefits analysis. We can think that in general the politician prefers, especially in electoral period, attitudes of financial accommodation from the supervisor. He should carry out a policy to avoid bankruptcies, because it is politically more advantageous (you think about the diffused benefits for depositors and to the specific advantages for banking and bankers). This, however, does not mean that it is always politically convenient that the central bank has the powers both of monetary policy and of supervision. In fact, if the central banker is inclinable to an accommodating monetary policy, to

---

\(^9\) The first studies on the electoral cycle are those of Nordhaus (1975), Hibbs (1977), then they were taken back by Alesina (1987), Alesina - Sachs (1988).

\(^{10}\) See Kydland - Prescott(1977) and Barro - Gordon (1983a,b) for models on the economic political cycle with rational expectations (time inconsistency).


possess the powers of supervision would mean to be able to develop easier a laxist monetary policy; and this serves the interest of the politician. Otherwise, if the central banker is conservative, he would pay attention to his not accommodating reputation. In this case, he would like to strengthen such reputation, with the same behaviour, also when he carries out the supervision policy.

The present paper intends to analyze theoretically this problem list\textsuperscript{13}.

The paper is structured as follows: after the introduction, in the second section we present the general model. In the third and fourth sections we examine the principal-and-two-agents model in the electoral and non-electoral periods. In the fifth and seventh sections the single-agent contract is analysed, in the electoral and non-electoral periods. The sixth and eighth sections give a comparison between the two contracts. In the ninth section remarks and possible developments are discussed. Finally, our conclusions.

\section*{2. The general model}

This paper\textsuperscript{14} analyses the advantage to be gained in entrusting the tasks of “banking supervision” and “monetary policy” to two agents, Banking Authority (BA) and Central Bank (CB), or to a single agent, CB. In this analysis two periods are examined: electoral and non-electoral. The model is that of a principal with two agents, where the principal is the political group in power, while the agents are, as we have said, BA and CB.

The politician has his own utility function \( U \), which will take on four different values according to whether four different events take place. These events are:

\begin{itemize}
  \item \( Bs = \text{stability in the banking system} \)
  \item \( Ps = \text{price stability} \)
  \item \( -Bs = \text{banking instability} \)
  \item \( -Ps = \text{price instability} \).
\end{itemize}

The utility function \( U \) is defined thus:

\textsuperscript{13}In the literature there are no theoretical models linking the problem of the choice of institutional setup of supervisory bodies, with the electoral cycle.

\textsuperscript{14}The approach adopted is that proposed by Franck - Krausz (2004)
\[
U = \begin{cases} 
    u_1 & \text{if } E_1 = Bs \cap Ps \\
    u_2 & \text{if } E_2 = Bs \cap -Ps \\
    u_3 & \text{if } E_3 = -Bs \cap Ps \\
    u_4 & \text{if } E_4 = -Bs \cap -Ps
\end{cases}
\]

A stable banking system means lack of banking crises. The banking system stability is promoted by a tight regulation policy, while the banking system instability is pursued by a loose regulation policy. We suppose that a low frequency of banking crises is the consequence of a strict regulation. Unless the politician has a blanket preference for stability in the banking system rather than instability regardless of whether or not it is an election period, he will attain greater utility if there is price instability in the election period and stability in the non-election period. This situation, in fact, gives the politician more probability of pursuing his objective, i.e. being re-elected. In the short term, price instability and therefore an increase in inflation - as long as it is not perceived by wage-earners - determines a reduction in real wages and a consequent drop in unemployment, and this may mean that the politician receives more support from electors, thus increasing his chance of being re-elected. In the long term, however, as Friedman asserts\textsuperscript{15}, there is no trade off between inflation and unemployment.

The “inflation surprise” effect is annulled in the long run, when wage-earners re-adjust their expectations in view of the inflation level actually chosen by the authorities. The preference for banking system stability is justified by the fact that a crisis in the system would lead to a loss of confidence among depositors, obviously not desirable for a politician whose immediate goal is to be re-elected. Most of all, in the non-election period it will be to the politician’s advantage to have a stable banking system, because this makes it easier to achieve the objective of price stability. In fact, the presence of failed banks calls for the intervention of the central bank which, as lender of last resort, injects liquidity into the system to cope with the crisis, at the same time altering the equilibrium of the money market.

\textsuperscript{15} Friedman (1968)
In the election period it will therefore be

\[ u_2 > u_1; \ u_4 > u_3 \] (i.e. also \( u_2 > u_4 \) and \( u_1 > u_3 \)).

As established by principal-agent models, the principal offers a contract to the agents giving them the incentive to act in the exclusive interests of the principal. This contract envisages a payment to be made to the agents, appropriate to the results they have attained. To be precise, BA will receive \( t_b \) if there is \( -B_s \), and \( T_b \) if there is \( B_s \) (obviously with \( t_b < T_b \)), and CB will receive \( t_p \) if there is \( P_s \), and \( T_p \) if there is \( -P_s \) (with \( t_p < T_p \)).

To give \( t \) to the agent, the politician will spend \( u(t) \), that is, a higher sum than will actually be paid to the agent; this is also due to the transaction costs he has to sustain.

The politician’s expected net utility will therefore be:

\[
E(U-u) = [u_1 - u(T_b) - u(t_p)] Pr(B_s \cap P_s) + [u_2 - u(T_b) - u(T_p)] Pr(B_s \cap -P_s) + [u_3 - u(t_b) - u(t_p)] Pr(-B_s \cap P_s) + [u_4 - u(t_b) - u(T_p)] Pr(-B_s \cap -P_s)
\]

To obtain a certain outcome each agent must make an “effort”. We will call \( e_b \) the effort of BA and \( e_p \) that of CB. Let us suppose, for the sake of simplicity, that both variables can assume only two values: \( e_b = 0 \) or \( e_b = 1 \) and similarly for \( e_p \). For this reason, the possible couples \((e_b, e_p)\) are four.

To simplify the writing, we put:

\[
e_{00} = (e_b = 0) \cap (e_p = 0) \quad \text{- no effort made in either function -}
\]

\[
e_{01} = (e_b = 0) \cap (e_p = 1) \quad \text{- effort made only for price stability (restrictive monetary policy) -}
\]

\[
e_{10} = (e_b = 1) \cap (e_p = 0) \quad \text{- effort made only for banking stability -}
\]

\[
e_{11} = (e_b = 1) \cap (e_p = 1) \quad \text{- effort made in both functions -}
\]

and \( p_{00}, p_{01}, p_{10}, p_{11} \) the corresponding probabilities.

The probability of achieving a stable banking system is affected by the agents’ behavior in line with political decisions. We therefore introduce the following probabilities:

\[
P_{0b} = Pr(B_s \mid e_{01}) \quad P_{1b} = Pr(B_s \mid e_{10})
\]

\[
P_{2b} = Pr(B_s \mid e_{11}) \quad P_{3b} = Pr(B_s \mid e_{00})
\]

Given the meaning of these probabilities, we expect that:
(2') \[ P_{1b} \geq P_{2b} ; P_{3b} \geq P_{0b} \]

Under equal effort by the banking authorities, the probability of achieving a stable banking system is higher when monetary policy is expansionistic\(^{16}\). 

As for price stability, we insert the following probabilities:

(3) \[ P_{0p} = \Pr(Ps | e_p = 0 \cap -Bs) \quad P_{1p} = \Pr(Ps | e_p = 1 \cap Bs) \]
\[ P_{2p} = \Pr(Ps | e_p = 1 \cap -Bs) \quad P_{3p} = \Pr(Ps | e_p = 0 \cap Bs) \]

The probability of achieving a stable price level depends on whether the monetary authority makes an effort, and on the degree of stability of the banking system, since this is the channel conveying monetary policy. In particular, there is a higher probability of stable prices when the banking system is stable\(^{17}\).

We therefore expect:

(3') \[ P_{0p} \leq P_{2p} ; P_{3p} \leq P_{1p} . \]

The introduction of these conditional probabilities becomes necessary when we look at the agents’ utility. This is a problem of constrained maximization. The politician has his own utility function and to maximize it he has to minimize costs. He will therefore have to identify what size incentives can maximize his expected utility, within certain constraints determined by the contract with the agents. These are incentive and participation constraints.

If we want to express the probabilities present in (1), through the probabilities introduced with (3), keeping in mind the definition of conditional probability\(^{18}\) and the resulting properties, we obtain:

(4) \[ \Pr(Bs \cap Ps) = P_{3p} [P_{3b} p_{00} + P_{1b} p_{10}] + P_{1p} [P_{0b} p_{01} + P_{2b} p_{11}] \]
(5) \[ \Pr(Bs \cap -Ps) = (1- P_{3p}) [P_{3b} p_{00} + P_{1b} p_{10}] + (1 - P_{1p}) [P_{0b} p_{01} + P_{2b} p_{11}] \]
(6) \[ \Pr(-Bs \cap Ps) = P_{0p} [(1 - P_{3b}) p_{00} + (1 - P_{1b}) p_{10}] + P_{2p} [(1 - P_{0b}) p_{01} + (1 - P_{2b}) p_{11}] \]
(7) \[ \Pr(-Bs \cap -Ps) = (1 - P_{0p}) [(1 - P_{3b}) p_{00} + (1 - P_{1b}) p_{10}] + (1 - P_{2p}) [(1 - P_{0b}) p_{01} + (1 - P_{2b}) p_{11}] \]

\(^{16}\) An inflationistic monetary policy is not desirable if one considers the goal of price stability. This conflict of interests is one of the factors in support of giving the roles to different authorities.

\(^{17}\) “Stability of the financial sector is important for monetary authorities, as monetary and financial sector stability are closely connected. History provides many examples where problems in the financial sector led to monetary instability. The Great Depression in the U.S. is probably the best known example where bank failures, combined with an inadequate response by the monetary authorities, resulted in a prolonged economic crises…” (Eijffinger 2001)

\(^{18}\) \( \Pr (A|B) = \frac{\Pr (A \cap B)}{\Pr (B)} \)
We prove the first one (4):

$$\Pr(B_s \cap P_s) = \Pr(B_s \cap P_s \cap (e_p = 0 \cup e_p = 1)) =$$

$$= \Pr(B_s \cap P_s \cap e_p = 0) + \Pr(B_s \cap P_s \cap e_p = 1) =$$

$$= \Pr(P_s | B_s \cap e_p = 0) \Pr(B_s \cap e_p = 0) + \Pr(P_s | B_s \cap e_p = 1) \Pr(B_s \cap e_p = 1) =$$

for (3)

$$= P_3p \Pr(B_s \cap e_p = 0) + P_{1p} \Pr(B_s \cap e_p = 1) =$$

$$= P_3p [\Pr(B_s \cap (e_{00} \cup e_{10}))] + P_{1p} [\Pr(B_s \cap (e_{01} \cup e_{11}))] =$$

$$= P_3p [\Pr(B_s \cap e_{00}) + \Pr(B_s \cap e_{10})] + P_{1p} [\Pr(B_s \cap e_{01}) + \Pr(B_s \cap e_{11})] =$$

$$= P_3p [\Pr(B_s | e_{00}) \Pr(e_{00}) + \Pr(B_s | e_{10}) \Pr(e_{10})] + P_{1p} [\Pr(B_s | e_{01}) \Pr(e_{01}) + \Pr(B_s | e_{11}) \Pr(e_{11})] =$$

$$= P_3p [P_{3b} p_{00} + P_{1b} p_{10}] + P_{1p} [P_{0b} p_{01} + P_{2b} p_{11}].$$

These probabilities (4, 5, 6, 7) will assume different values according to the considered period: electoral or non electoral period.

The politician is sure that given adequate incentives, the authorities will act in his interests. Consequently, in the election period, the monetary authority will recieve incentives to make no effort to maintain stable prices. As we have already seen, in fact, price instability is preferable for the politician in the electoral period, while he always wants a stable banking system. The politician therefore supposes that there will almost certainly be \((e_b = 1 \cap e_p = 0) = e_{10}\), and the probability of this eventuating will be \(p_{10} = 1\). All the other \(p_{ij}\) probabilities will be null. This result will be not the same if we consider the non electoral period. Therefore, in the **electoral period** we can re-write the probabilities (4, 5, 6, 7), in this way:

(4)' \quad \Pr(B_s \cap P_s | e_{10}) = P_{1b} P_3p

(5)' \quad \Pr(B_s \cap -P_s | e_{10}) = P_{1b} (1 - P_3p)

(6)' \quad \Pr(-B_s \cap P_s | e_{10}) = (1 - P_{1b}) P_{0p}

(7)' \quad \Pr(-B_s \cap -P_s | e_{10}) = (1 - P_{1b}) (1 - P_{0p}).

On the other hand, in the **non-electoral period**, since the politician will prefer a stable banking system and price stability, he will give the agents adequate incentives to achieve this goal. To be
precise, he will offer BA $t_b$ if there is $-B_s$, and $T_b$ if there is $B_s$ (obviously with $t_b < T_b$), and he will give CB $t_p$ if there is $-P_s$, and $T_p$ if there is $P_s$ (with $t_p < T_p$). Following the same line of thinking as before, the politician is convinced that both the authorities will make an effort to reach stability, one for prices, the other for the banking system. So for the politician there will be $e_{11}$, and the corresponding probability will be $p_{11} = 1$. All the other probabilities will be null. From (4)-(7) it therefore follows that:

(4)** $\Pr(B_s \cap P_s \mid e_{11}) = P_{2b} P_{1p}$

(5)** $\Pr(B_s \cap -P_s \mid e_{11}) = P_{2b} (1 - P_{1p})$

(6)** $\Pr(-B_s \cap P_s \mid e_{11}) = (1 - P_{2b}) P_{2p}$

(7)** $\Pr(-B_s \cap -P_s \mid e_{11}) = (1 - P_{2b}) (1 - P_{2p})$.\textsuperscript{19}

Consequently, the politician’s expected net utility will assume two different expressions, according to whether one considers the electoral period:

(8) $E(U-u \mid e_{10}) = [u_1 - u(T_b) - u(t_p)] P_{1b} P_{3p} + [u_2 - u(T_b) - u(T_p)] P_{1b} (1 - P_{3p}) + [u_3 - u(t_b) - u(t_p)] (1 - P_{1b}) P_{0p} + [u_4 - u(t_b) - u(T_p)] (1 - P_{1b}) (1 - P_{0p})$

or the non-electoral period:

(9) $E(U-u \mid e_{11}) = [u_1 - u(T_b) - u(T_p)] P_{2b} P_{1p} + [u_2 - u(T_b) - u(t_p)] P_{2b} (1 - P_{1p}) + [u_3 - u(t_b) - u(T_p)] (1 - P_{2b}) P_{2p} + [u_4 - u(t_b) - u(t_p)] (1 - P_{2b}) (1 - P_{2p})$

\textsuperscript{19} Since they will be useful later, let us complete the calculation of the other conditional probabilities, as follows:

(4)**$' $ $\Pr(B_s \cap P_s \mid e_{01}) = P_{0b} P_{1p}$

(5)**$' $ $\Pr(B_s \cap -P_s \mid e_{01}) = P_{0b} (1 - P_{1p})$

(6)**$' $ $\Pr(-B_s \cap P_s \mid e_{01}) = (1 - P_{0b}) P_{2p}$

(7)**$' $ $\Pr(-B_s \cap -P_s \mid e_{01}) = (1 - P_{0b}) (1 - P_{2p})$

(4)**$'' $ $\Pr(B_s \cap P_s \mid e_{00}) = P_{3b} P_{3p}$

(5)**$'' $ $\Pr(B_s \cap -P_s \mid e_{00}) = P_{3b} (1 - P_{3p})$

(6)**$'' $ $\Pr(-B_s \cap P_s \mid e_{00}) = (1 - P_{3b}) P_{0p}$

(7)**$'' $ $\Pr(-B_s \cap -P_s \mid e_{00}) = (1 - P_{3b}) (1 - P_{0p})$.\textsuperscript{19}
where however, this time, \( u_1 > u_2 \) and \( u_3 > u_4 \), i.e. the politician’s utility is higher with a restrictive monetary policy. To make the relations between the different values of \( u_i \) clearer, let us say in the **electoral period**:

\[
\begin{align*}
  u_1 &= G; \\
  u_2 &= G (1 + R); \\
  u_3 &= g; \\
  u_4 &= g (1 + r)
\end{align*}
\]

\( G \): measures the preferences for stability in the banking system;  
\( R \): represents the higher value that the politician obtains if there is price instability at the same time as stability in the banking system;  
\( g \): indicates the preferences for instability in the banking system;  
\( r \): is the higher gains obtained by the politician if there is price instability, given the instability in the banking system; with \( G > g \) and \( r < R \).

Substituting these values in (8), we obtain:

\[
(8)' \quad E(U-u | e^{10}) = [G - u(T_b) - u(T_p)] P_{1b} P_{3p} + [G (1 + R) - u(T_b) - u(T_p)] P_{1b} (1 - P_{3p}) + \\
+ [g - u(T_b) - u(T_p)] (1 - P_{1b}) P_{0p} + [g (1 + r) - u(T_b) - u(T_p)] (1 - P_{1b}) (1 - P_{0p}).
\]

In the **non-electoral period** let us say:

\[
\begin{align*}
  u_1 &= G (1 + R); \\
  u_2 &= G; \\
  u_3 &= g (1 + r); \\
  u_4 &= g
\end{align*}
\]

and from (9) we obtain:

\[
(9)' \quad E(U-u | e^{11}) = [G (1 + R) - u(T_b) - u(T_p)] P_{2b} P_{1p} + [G - u(T_b) - u(T_p)] P_{2b} (1 - P_{1p}) + \\
+ [g (1 + r) - u(T_b) - u(T_p)] (1 - P_{2b}) P_{2p} + [g - u(T_b) - u(T_p)] (1 - P_{2b}) (1 - P_{2p}).
\]

As for the choice of cost function \( u(t) \), we use the function

\[
(10) \quad u(t) = \begin{cases} 
  t^2/2 & \text{if } t \geq 0 \\
  -t^2/2 & \text{if } t < 0
\end{cases}
\]

Thus \( u'(t) = \|t\| \) and \( u \) is increasing.
3. Contract with two agents in the electoral period

The two agents will be encouraged by the politician’s incentives to make an effort to achieve the economic scenario that increases the politician’s chances of being re-elected. Making an effort often involves costs. Let $C_b$ be the cost of BA’s effort and $C_p$ that of CB. These costs will obviously exist only if $e_b = 1$ and, respectively, $e_p = 1$. Therefore $C_b > 0$ if $e_b = 1$ and $C_b = 0$ if $e_b = 0$. Similarly with $C_p$. We consider $C_b$ and $C_p$ constant.

The incentive expected by BA will therefore be

$$E(I_b | e_b) = t_b Pr(-Bs | e_b) + T_b Pr(Bs | e_b) - C_b(e_b)$$

And that of CB will be:

$$E(I_p | e_p) = T_p Pr(-Ps | e_p) + t_p Pr(Ps | e_p) - C_p(e_p).$$

The incentive constraint for BA is that:

$$E(I_b | e_b = 1) \geq E(I_b | e_b = 0)$$

In other words this authority expects a higher incentive when it makes an effort to achieve stability in the banking system, compared to when no effort is made.

Clearly the essential condition for the agent to agree with the contract is that if he makes an effort, he will receive a non negative amount. Therefore the participation constraint is:

$$E(I_b | e_b = 1) \geq 0.$$

In other words, for BA to have the incentive to make an effort it is necessary that conditions (13) and (14) are verified.

Similarly, the incentive and participation constraints for CB will be:

$$E(I_p | e_p = 0) \geq E(I_p | e_p = 1)$$

$$E(I_p | e_p = 0) \geq 0.$$

As it is an election period, we have seen that the politician gains greater utility if there is price instability. It will therefore be to his advantage to make a contract that induces the central bank not to pursue price stability, but which instead fosters the development of a degree of inflation.

Consequently, he will offer a contract that pushes the agent to pursue an expansionist monetary
policy. This means no effort on the part of the agent \((e_p = 0)\). For his part, the agent expects higher remuneration if he pursues instability and – the minimum condition for him to agree to the contract – the remuneration must be positive. In other words, (15) and (16) must hold.

To find the values of \(t_b, T_b, t_p, T_p\) that maximize \(E(U-u)\) under constraints (13)-(16), we need to solve the problem of constrained optimization (see appendix A).

From an analysis of the cases that solve this problem (see appendix A), we obtain that the acceptable solutions are\(^{20}\):

i) \(t_b = -T_b\) \(T_b = C_b / (2P_{1b} - 1)\) if \(P_{3b} \leq 1/2 < P_{1b}\)

ii) \(t_b = -C_b P_{3b} / (P_{1b} - P_{3b})\), \(T_b = C_b (1 - P_{3b}) / (P_{1b} - P_{3b})\) with \(P_{3b} \leq 1/2 \) and \(P_{1b} > P_{3b}\)

iii) \(t_p = -T_p\) with any positive value for \(T_p\), if \(A = 1/2\) with \(A = \Pr(Ps | e_{10})\) (see (A12)).

Keeping in mind that the politician’s expected utility function is:

\[
E(U-u | e_{10}) = [G - u(T_b) - u(t_b)] P_{1b} P_{3p} + [G (1 + R) - u(T_b) - u(T_p)] P_{1b} (1 - P_{3p}) + \\
+ [g - u(t_b) - u(t_p)] (1 - P_{1b}) P_{0p} + [g (1 + r) - u(t_b) - u(T_p)] (1 - P_{1b}) (1 - P_{0p}).
\]

we can rewrite this as:

\[
(17) E(U-u | e_{10}) = H - u(T_b) P_{1b} - u(t_b) (1 - P_{1b}) - u(T_p) (1 - A) - u(t_p) A = \\
= H - [T_b^2 P_{1b} - t_b^2 (1 - P_{1b}) + T_p^2 (1 - A) - t_p^2 A ] / 2
\]

where,

\[
(18) H = G P_{1b} P_{3p} + G (1 + R) P_{1b} (1 - P_{3p}) + g (1 - P_{1b}) P_{0p} + g (1 + r) (1 - P_{1b}) (1 - P_{0p}) = \\
= G P_{1b} [1 + R (1 - P_{3p})] + g (1 - P_{1b}) [1 + r (1 - P_{0p})]
\]

If we substitute the value of incentive payment identified [i); iii)], in (17), we obtain:

\[
(19) E(U-u | e_{10}) = H - [C_b^2 / (2 (2 P_{1b} - 1))]
\]

If we substitute incentive payments [ii); iii)], we obtain:

\[
(20) E(U-u | e_{10}) = H - H_0
\]

where, \(H_0 = C_b^2 [ (1 - P_{3b})^2 P_{1b} - P_{3b}^2 (1 - P_{1b}) ] / (2 (P_{1b} - P_{3b})^2).\)

\(^{20}\) Solutions i) and iii), derive from cases I) and V), in appendix A, which say that when the two agents agree to the politician’s requests, they expect a benefit at least large enough to cover the costs incurred in achieving the goal. If, on the other hand, they act in a way considered “not correct” by the politician, they will be punished by him, for the bad result achieved. In this case they obtain a negative expected benefit (see appendix A). Solution ii) which, it might be objected, may seem less rational from an economic point of view, leads however to the same type of results.
We can prove (see (E1) in appendix) that
\[-1 / (2 P_{1b} - 1) \leq -[(1 - P_{3b})^2 P_{1b} - P_{3b}^2 (1 - P_{1b})] / (P_{1b} - P_{3b})^2\]
and that \([(1 - P_{3b})^2 P_{1b} - P_{3b}^2 (1 - P_{1b})]\) can only be positive (see (E2) in appendix).

So (19) \(\leq\) (20).

Therefore the maximum value of \(E(U-u)\), with two agents in electoral period, is (20).


As we have already said, in the non-electoral period the politician will prefer stability in the banking system and price stability. Therefore, in this period the politician’s utility expected net value will be:

\[
E(U-u) = [G (1 + R) - u(T_b) - u(T_p)] P_{2b} P_{1p} + [G - u(T_b) - u(t_p)] P_{2b} (1 - P_{1p}) + \\
+ [g (1 + r) - u(t_b) - u(T_p)] (1 - P_{2b}) P_{2p} + [g - u(t_b) - u(t_p)] (1 - P_{2b}) (1 - P_{2p}) = \\
H' - u(T_b) P_{2b} - u(t_b) (1 - P_{2b}) - u(T_p) B - u(t_p) (1 - B)
\]

where

\[
H' = G P_{2b} (1 + R P_{1p}) + g (1 - P_{2b}) (1 + r P_{2p})
\]

BA’s incentive and participation constraint is that they are

\[
E(I_b | e_b = 1) \geq E(I_b | e_b = 0)
\]

\[
E(I_b | e_b = 1) \geq 0.
\]

Likewise the incentive and participation constraints for CB will be

\[
E(I_p | e_p = 1) \geq E(I_p | e_p = 0)
\]

\[
E(I_p | e_p = 1) \geq 0.21
\]

As in the previous section, the various cases that can eventuate need to be analysed. This analysis reveals that the possible solutions to the problem of optimization are the following22:

\[
\text{iii) } t_b = T_b = 0 \quad \text{if } P_{0b} = P_{2b} \\
\text{iv) } t_b = -C_b P_{0b} / (P_{2b} - P_{0b}), \quad T_b = C_b (1 - P_{0b}) / (P_{2b} - P_{0b}) \quad \text{if } P_{0b} \leq 1/2 \\
\text{v) } t_p = -A C_p / (B - A), \quad T_p = (1 - A) C_p / (B - A) \quad \text{if } A \leq 1/2
\]

\[\text{21 See appendix B.) for the solution to the problem of constrained optimization.}\\
\text{22 Other solutions, which result by the first-order conditions, are the following :}
\]

13
i) \( t_b = -T_b \) with \( T_b = C_b/(2 \, P_{2b} - 1) \) if \( P_{0b} \leq 1/2 < P_{2b} \)  

\[(27)\]

ii) \( t_p = -C_p/(2 \, B - 1) \), \( T_p = -t_p \) if \( A \leq 1/2 < B \)

These solutions derive from the hypothesis made that when the two agents make an effort, they expect a benefit large enough to cover the cost incurred to achieve the goal. If on the other hand they make no effort they will be punished by the politician. The punishment is the politician’s reward to the agents when they reach a bad performance. In this case, therefore, the agents obtain a negative expected benefit.

Keeping in mind that the politician’s expected utility function in the non-electoral period is:

\[(28)\] \( E(U - u) = H' - u(T_b) \, P_{2b} - u(t_b) \, (1 - P_{2b}) - u(T_p) \, B - u(t_p) \, (1 - B) \)

If we substitute the incentive payments in the politician’s utility function \( E(U - u) \), as we did in the previous case, we obtain

\[(29)\] \( E(U - u) = H' - (1/2) \{[C_b^2 / (2 \, P_{2b} - 1)] + [C_p^2 / (2 \, B - 1)]\} \)

5. **Contract with a single agent in the electoral period.**

Let us now consider the case where both the functions are delegated to a single authority, the central bank. To see whether this situation is advantageous for the politician, we have to calculate the expected utility and compare it with the expected utility with two separate authorities.

We suppose that the politician will offer four different incentive payments to the CB, according to whether four different events take place. In particular, the politician will give to the agent

\[
\begin{align*}
T_{10} & \quad \text{if} \quad E_1 = B_S \cap -P_S \\
T_{11} & \quad \text{“} \quad E_2 = B_S \cap P_S \\
T_{01} & \quad \text{“} \quad E_3 = -B_S \cap P_S \\
T_{00} & \quad \text{“} \quad E_4 = -B_S \cap -P_S
\end{align*}
\]

However, these solutions, which are not very interesting from an economic point of view, will be disregarded; only i) will be considered. This does not change the results.
with (presumably) $T_{10} \geq T_{11} \geq T_{00} \geq T_{01}$.

We should remember that in the electoral period the politician prefers price instability because an inflationist policy, for the reasons seen in the previous sections, increases his probability of re-election.

The politician’s expected net utility in the electoral period is therefore:

\[
E(U - u | e_{10}) = [G - u(T_{11})] \Pr(E_2 | e_{10}) + [G (1 + R) - u(T_{00})] \Pr(E_1 | e_{10}) + \\
[g - u(T_{01})] \Pr(E_3 | e_{10}) + [g (1 + r) - u(T_{00})] \Pr(E_4 | e_{10}) = \\
= [G - u(T_{11})] P_{1b} P_{3p} + [G (1 + R) - u(T_{00})] P_{1b} (1 - P_{3p}) + \\
+ [g - u(T_{01})] (1 - P_{1b}) P_{0p} + [g (1 + r) - u(T_{00})] (1 - P_{1b}) (1 - P_{0p}) = H - H_0'
\]

with

\[
H_0' = u(T_{11}) P_{1b} P_{3p} + u(T_{10}) P_{1b} (1 - P_{3p}) + u(T_{01}) (1 - P_{1b}) P_{0p} + u(T_{00}) (1 - P_{1b}) (1 - P_{0p}).
\]

The incentive expected by CB will be:

\[
E(I_p | e_{bp}) = T_{10} \Pr(E_1 | e_{bp}) + T_{11} \Pr(E_2 | e_{bp}) + T_{01} \Pr(E_3 | e_{bp}) + T_{00} \Pr(E_4 | e_{bp}) - C_{bp}(e_{bp})
\]

where $C_{bp}$ is the cost of the authority when he makes efforts $e_{bp}$.

The central banker expects to gain more if he makes an effort to achieve banking stability, if we have the same conditions for the monetary stability. Moreover he expects better payment if he makes no effort to achieve price stability. This is justified by the awareness that in the electoral period, the politician prefers a degree of instability in order to reach his goal, i.e. re-election.

The incentive and participation constraints, therefore, become:

\[
g_1 = E(I_p | e_{10}) - E(I_p | e_{00}) \geq 0
\]

\[
g_2 = E(I_p | e_{10}) - E(I_p | e_{11}) \geq 0
\]

\[
g_3 = E(I_p | e_{10}) - E(I_p | e_{01}) \geq 0
\]

\[
g_4 = E(I_p | e_{10}) \geq 0
\]

or
\[ g_1 = g_4 - E(I_p \mid e_{00}) \geq 0 \]
\[ g_2 = g_4 - E(I_p \mid e_{11}) \geq 0 \]
\[ g_3 = g_4 - E(I_p \mid e_{01}) \geq 0 \]
\[ g_4 = E(I_p \mid e_{10}) \geq 0 \]

As for the costs \( C_{bp} \) of effort, we define\(^{23}\):

\[
C_{bp}(e_{bp}) = \begin{cases} 
0 & \text{if } e_{00} \\
C_{bp} & \text{if } e_{11} \\
C_{bp} - C_p & \text{if } e_{10} \\
C_{bp} - C_b & \text{if } e_{01}.
\end{cases}
\]

Also here \( C_{bp} \) is constant, as \( C_b \) and \( C_p \).

6. Comparison of the two contracts in the electoral period.

We have seen that in the election period, if the politician appoints two agents, his expected utility is:
\[ E(U-u \mid e_{10}) = H - H_0. \]

On the other hand, the utility expected by the politician, when he entrusts the task to a single agent is

\[ (1)' \quad E(U-u \mid e_{10}) = H - H_0' \]

To prove that in the electoral period it is to the politician’s advantage to give the two tasks to a single agent - for an appropriate allocation of the size of incentive payments \( T_{ij} \), compatible with the constraints (C1)(see appendix C.) \( g_k \geq 0 \) for \( k = 1,2,3,4 \) - one needs to show that it is possible to have
\[ H_0' \leq H_0. \]

\(^{23}\) For the problem of constrained optimization see appendix C.).
Let us suppose that the politician rewards the central banker, by paying him the same amount set for the banking authority under conditions of banking stability\(^{24}\), provided he makes an effort to achieve both banking stability and price instability. He will be punished, however, in all other cases.

So if we put:

\[ T_{10} = T_b = C_b \frac{(1 - P_{3b})}{(P_{1b} - P_{3b})} \] and

\[ T_{11} = T_{01} = T_{00} = t_b = -C_b \frac{P_{3b}}{(P_{1b} - P_{3b})}, \]

we get

\[ 2 H_0 = (T_b^2 + t_b^2) P_{1b} - t_b^2, \]

while

\[ 2 H_0' = (T_b^2 + t_b^2) P_{1b} (1 - P_{3p}) - t_b^2 \]

and the result is therefore \( H_0' < H_0 \), as was foreseen.

For this result to be valid, the allocation considered needs to be compatible with the constraints \( g_k \geq 0 \) given by (C1)(see in appendix C.). Let us check it.

With the incentive payments considered, the result is:

\[ g_1 \geq 0 \text{ equivalent to } C_b (1 - P_{3b}) + C_p \geq C_{bp} \]

\[ g_2 \geq 0 \text{ equivalent to } C_b \left[ P_{1b} (1 - P_{3p}) - P_{2b} (1 - P_{1p}) \right]/(P_{1b} - P_{3b}) + C_p \geq 0. \]

From conditions (2)' and (3)' it follows that the expression in square brackets is positive and, therefore, the previous inequality is always true.

\[ g_3 \geq 0 \text{ equivalent to } C_b \left[ P_{1b} (1 - P_{3p}) - P_{3b} \right] \geq (C_{bp} - C_p) (P_{1b} - P_{3b}) \]

For the same reasons as before, the expression in square brackets is proved to be positive and, therefore, if \( C_b \leq C_p \) the third constraint is also shown to be valid.

\[ g_4 \geq 0 \text{ equivalent to } C_b \left[ P_{1b} (1 - P_{3p}) - P_{3b} \right] \geq (C_{bp} - C_p) (P_{1b} - P_{3b}) \]

which can also be written

\[ C_{bp} \leq C_p + C_b \frac{1 - P_{1b} P_{3p}}{(P_{1b} - P_{3b})} \]

and is likely to be \( P_{1b} P_{3p} / (P_{1b} - P_{3b}) \leq 1 \), if \( P_{3b} \) is small enough compared to \( P_{1b} \).

\(^{24}\) This hypothesis fits in with the idea that there are economies of scope (\( c_{bp} < c_b + c_p \)), which emerges from the model.
We can therefore conclude that the choice of incentive payments is compatible with the constraints if:

\[ C_{bp} \leq C_p + C_b \]

i.e. if there are economies of scope;

\[ C_b \leq C_p \]

if the task of the central bank is more demanding (this is clear if one thinks of the fact that the central bank is also responsible for banking stability as lender of last resort);

\[ P_{3b} < P_{1b} \]

it is natural to expect this given the definition of these probabilities (in other words it is normal to expect that it is easier to achieve banking stability if the authority responsible makes an effort in this direction).

In these cases, in the election period it is to the politician’s advantage to appoint a single agent for the two roles.


In the post-electoral period, government authorities will want to contain the negative effects, in terms of inflation, deriving from the non-socially beneficial decisions taken, during the election period, to maximize the probability of re-election.

For this reason in the non-electoral period the politician will prefer price stability and stability in the banking system. He will therefore offer the CB the following payments

\[ T_{11} \]

if \( E_1 = B_s \cap P_s \)

\[ T_{10} \]

“ \( E_2 = B_s \cap -P_s \)

\[ T_{01} \]

“ \( E_3 = -B_s \cap P_s \)

\[ T_{00} \]

“ \( E_4 = -B_s \cap -P_s \)

with (presumably)

\[ (33) \] \( T_{11} \geq T_{10} \), \( T_{01} \geq T_{00} \).

The politician’s expected net utility in the non-electoral period will be:

\[ E(U-u | e_{11}) = H’ - K’ \]
where
\[ H' = G P_{2b} (1 + P_{1p} R) + g (1 - P_{2b}) (1 + r P_{2p}) \]

and
\[ K' = P_{2b} [u(T_{11}) P_{1p} + u(T_{10}) (1 - P_{1p})] + (1 - P_{2b}) [u(T_{01}) P_{2p} + u(T_{00}) (1 - P_{2p})] \]
The first one (H') is the benefit expected by the politician, while (K') is the cost for the politician.

The incentive expected by CB will be:
\[ E(I_p | e_{bp}) = T_{11} Pr(E_1 | e_{bp}) + T_{10} Pr(E_2 | e_{bp}) + T_{01} Pr(E_3 | e_{bp}) + T_{00} Pr(E_4 | e_{bp}) - C_{bp}(e_{bp}) \]
The incentive and participation constraints become:
\[ g_1 = E(I_p | e_{11}) - E(I_p | e_{10}) \geq 0 \]
\[ g_2 = E(I_p | e_{11}) - E(I_p | e_{01}) \geq 0 \]
\[ g_3 = E(I_p | e_{11}) - E(I_p | e_{00}) \geq 0 \]
\[ g_4 = E(I_p | e_{11}) \geq 0 \]
or
\[ g_1 = g_4 - E(I_p | e_{10}) \geq 0 \]
\[ g_2 = g_4 - E(I_p | e_{01}) \geq 0 \]
\[ g_3 = g_4 - E(I_p | e_{00}) \geq 0 \]
\[ g_4 = E(I_p | e_{11}) \geq 0 \]
The costs of effort will assume the same values already seen in (32), that is\(^{25}\):
\[ C_{bp}(e_{bp}) = \begin{cases} 0 & \text{if } e_{00} \\ C_{bp} & \text{if } e_{11} \\ C_{bp} - C_{p} & \text{if } e_{10} \\ C_{bp} - C_{b} & \text{if } e_{01} \end{cases} \]

\(^{25}\) For the problem of constrained optimization, see appendix D. We will not examine any particular case, but we will go straight on to compare the two contracts: the two-agent and the single-agent contract.
8. Comparison of the two contracts in the non-electoral period.

The politician’s expected net utility in the non-electoral period is

\[ E(U - u | e_{11}) = H' - K'' \]

with \( K'' = \frac{1}{2} \left[ \frac{C_b^2}{2P_{2b} - 1} \right] + \frac{C_p^2}{2B - 1} \}

in the case of two agents, and

\[ E(U - u | e_{11}) = H' - K' \]

with \( K' = P_{2b} [u(T_{11}) P_{1p} + u(T_{10}) (1 - P_{1p})] + (1 - P_{2b}) [u(T_{01}) P_{2p} + u(T_{00}) (1 - P_{2p})] \)

in the case of a single agent.

The politician will prefer to appoint a single agent also in the non-electoral period if

\[ \min K' \leq \min K''. \]

To prove this, it will be sufficient to show that, for an appropriate allocation of incentive payments \( T_{ij} \), a value of \( K' \) that is \( \leq \min K'' \) can be obtained.

Hypothesizing that

\[ C_b = C_p = C \text{ and } P_{2b} = B \]

where \( B = \Pr (Ps | e_{11}) \) (see (A13)) and \( P_{2b} = \Pr (Bs | e_{11}) \),

and remembering the value of the payments identified:

i) \( t_b = -T_b \) with \( T_b = \frac{C_b}{2P_{2b} - 1} \) if \( P_{0b} \leq 1/2 < P_{2b} \)

ii) \( t_p = -T_p \) with \( T_p = \frac{C_p}{2B - 1} \), if \( A \leq 1/2 < B \)

then in the two-agent contract in the non-electoral period, one would have

\[ T_b + T_p = [2C/(2P_{2b} - 1)] \]

\[ T_b + t_p = t_b + T_p = 0 \]

\[ t_b + t_p = [-2C/(2P_{2b} - 1)] \]

This suggests for the single-agent contract, an incentive payment of the type:

\[ T_{11} = T > 0 \]

\[ T_{10} = T_{01} = 0 \]

\[ T_{00} = -T \]

With these payments, the politician’s expected net utility would be:
\[ E(U-u | e_{11}) = H' - P_{2b} \left[ u(T_{11}) P_{1p} + u(T_{10}) (1 - P_{1p}) \right] + (1 - P_{2b}) \left[ u(T_{01}) P_{2p} + u(T_{00}) (1 - P_{2p}) \right] = \]
\[ = H' - \left( \frac{1}{2} \right) T^2 \left[ P_{2b} P_{1p} - (1 - P_{2b}) (1 - P_{2p}) \right] = H' - \left( \frac{1}{2} \right) T^2 \left( B - 1 + P_{2b} \right) \]

This attribution is valid, however, only if it satisfies the constraints. Testing shows:

\[ g_1 \geq 0 \quad T [B - A - (P_{1b} - P_{2b})] \geq C_p \]
\[ g_2 \geq 0 \quad T (P_{2b} - P_{0b}) (P_{1p} - P_{2p} + 1) \geq C_b \]
\[ g_3 \geq 0 \quad T (B - D + P_{2b} - P_{3b}) \geq C_{bp} \]
\[ g_4 \geq 0 \quad T (B + P_{2b} - 1) \geq C_{bp} \]

We see that the coefficient of \( T \) in \( g_2 \geq 0 \) is positive if \( P_{2b} \geq P_{0b} \) and, therefore, the constraint \( g_2 \) can be satisfied (for \( T > C_b / [(P_{2b} - P_{0b}) (P_{1p} - P_{2p} + 1)] \)).

In the hypothesis that \( P_{2b} = B \) the constraints become:

\[ g_1 \geq 0 \quad T [2 P_{2b} - (A + P_{1b})] \geq C_p \]
\[ g_2 \geq 0 \quad T (P_{2b} - P_{0b}) (P_{1p} - P_{2p} + 1) \geq C_b \]
\[ g_3 \geq 0 \quad T [2 P_{2b} - (P_{3b} + D)] \geq C_{bp} \]
\[ g_4 \geq 0 \quad T (2 P_{2b} - 1) \geq C_{bp} \]

We see that \( 2 P_{2b} - 1 \geq 0 \) by hypothesis, so

\[ g_4 \geq 0 \quad \iff T \geq C_{bp} / [(2 P_{2b} - 1)]. \]

Since \( D \leq B = P_{2b} \), then \( 2 P_{2b} - (P_{3b} + D) > P_{2b} - P_{3b} \). Moreover if we suppose that \( P_{3b} < P_{2b} \) (or \( P_{3b} = \Pr(Bs | e_{00}) < P_{2b} = \Pr(Bs | e_{11}) \)), then the coefficient of \( T \) in \( g_3 \geq 0 \) is also positive and the result is that \( g_3 \geq 0 \) equivalent to

\[ g_3 \geq 0 \quad \iff T \geq C_{bp} / [(2 P_{2b} - P_{3b} - D)] \]

\[ g_2 \geq 0 \quad \iff \text{if } P_{1p} = P_{2p} \quad T (P_{2b} - P_{0b}) (P_{1p} - P_{2p} + 1) \geq C_b \]

There is increasing compatibility between choice of payments and the constraints

- the lower \( P_{0b} \) and \( P_{3b} \): the probability of banking system stability when there is no effort in this direction by the agent (consistent with the hypotheses made);
- the higher \( P_{1p} \): the probability of price stability when there is effort in this direction by the agent, in the presence of banking stability.
It must be remembered that we put \( B = P_{2b} \), i.e. \( \Pr(P_s|e11) = \Pr(B_s|e11) \).

The tightest condition becomes \( g_4 \). We therefore choose this as payment and say:

\[
T = C_{bp} / [(2 P_{2b} - 1)].
\]

Under these conditions, therefore, the politician’s expected net utility is:

\[
E(U-u | e_{11}) = H' - (1/2) T^2 (B - 1 + P_{2b}) = H' - (1/2) T^2 (2 P_{2b} - 1) =
\]

\[
H' - (1/2) \{C_{bp}^2 / [(2 P_{2b} - 1)^2]\}[(2 P_{2b} - 1)] = H' - (1/2) \{C_{bp}^2 / [(2 P_{2b} - 1)]\}
\]

Consequently, the politician will prefer to entrust the appointment to a single agent also in the non-electoral period if:

\[
(1/2) \{C_{bp}^2 / [(2 P_{2b} - 1)]\} \leq (1/2) \{[C_b^2 / (2 P_{2b} - 1)] + [C_p^2 / (2 B -1)]\}
\]

But since we hypothesized that \( C_p = C_b = C e P_{2b} = B \), then we have:

\[
\{C_{bp}^2 / [(2 P_{2b} - 1)]\} \leq 2C^2 / (2 P_{2b} - 1)
\]

Therefore the politician will prefer to entrust the appointment to a single agent also in the non-electoral period if\(^\text{26}\):

\[
C_{bp}^2 \leq 2C^2 = C_b^2 + C_p^2.
\]

The politician’s choice of single or multiple authorities is therefore not tied to electoral factors.

What emerges from this model, on the basis of the hypotheses made and the constraints imposed, is that the politician always prefers the single authority if the following conditions come about:

- \(C_{bp} \leq C_p + C_b\) i.e. if there are economies of scope;

- \(C_b \leq C_p\) if the task of the central bank is more demanding (and this is clear if we consider that the central bank is also responsible for banking stability as lender of last resort);

\(^\text{26}\) It is, however, very easy to give an example in which the previous conditions do not exist, which precludes the use of the previous procedure to prove the advantageousness of using a single agent. In contrast, it is not at all easy to give an example to show, at least for some values of the parameters, that it is more advantageous to entrust the roles to separate agents. This is due to the fact that it is difficult to solve the problem of optimization with a single agent and consequently the minimum of \( K \) is unknown. We can however conjecture that it is always more advantageous to entrust the functions to a single agent for a simple reason that is discussed in the next section.
• $P_{3b} < P_{1b}$ it is natural to expect this given the definition of these probabilities (in other words it is normal to expect banking stability to be easier to achieve if there is effort in this direction on the part of the authority responsible);

• the lower $P_{0b}$ and $P_{3b}$, or the higher the risk of banking crises;

• the higher $P_{1p}$ is, the probability of price stability when the monetary agent makes an effort to attain this, in the presence of stability in the banking system.

9. Remarks and possible developments

The advantage of entrusting the roles to a single agent can however be conjectured using a simple thought process.

The minimum cost that the politician has to bear for incentives must at least cover the costs of effort and must therefore equal $C_{bp}$, if there is a single agent, and $C_p + C_b$ if there are two. If $C_{bp} < C_p + C_b$, then it is clear that it is always more convenient to appoint a single agent.

In the election period, the politician’s spending on incentives is even lower, in that as he does not want the central bank to be over-zealous, it will only equal $C_b$ for two agents and $C_{bp} - C_p$ for one agent. Therefore the minimum spending for the politician must be in the electoral period.

If however we introduce a reputation cost, $R$, in cases in which although the central bank wants to act, it refrains from doing so in order to please the politician, then in the electoral period if two agents are appointed, the politician’s cost will be equivalent to $C_b + (R/2)$ if there are two agents and to $C_{bp} - C_p + R$ if there is one agent.

In the non-electoral period, the politician’s costs will be equivalent to $C_b + C_p$ for two agents and to $C_{bp}$ for one agent.
Comparing these costs shows that, if \( C_{bp} < C_p + C_b \) then, in the non-electoral period, it will certainly be more convenient to appoint a single agent. In the election period, it will all depend on the difference

\[
C_{bp} - C_p + R - [C_b + (R/2)] = C_{bp} - (C_b + C_p) + (R/2)
\]

If this difference is zero, it will not matter at all whether one or two agents are appointed. If the value is >0, then one agent costs more than two agents and therefore it is better to appoint two agents; if on the other hand \( C_{bp} - (C_b + C_p) + (R/2) < 0 \), it will be more suitable to appoint a single agent. The cost difference will rise in proportion to \( R \), the reputation cost, assessed by the central governor.

Firstly, it would be interesting to identify proxies to measure the governor’s reputation effect.

Secondly, an empirical analysis could be made of whether or not these indicators of reputation are connected to the outcome of monetary and supervision policy. It should be remembered that the problem of measurability also exists for the second function mentioned.

10. Conclusions

This paper analyses the advantage to be gained in entrusting the jobs of “banking supervision” and “monetary policy” to two agents, Banking Authority (BA) and Central Bank (CB), or to a single agent, CB. In examining the policy maker’s choice between single or multiple authorities, the role of the political cycles was appraised. For this purpose, two periods were examined: electoral and non-electoral. The model is that of a principal with two agents, where the principal is the political group in power, while the agents are, as we have said, BA and CB.

The reached conclusion is that the political chooses the institutional design of regulatory authorities without being influenced by the electoral cycle. What is interesting is the importance of the costs of "capture", related to the different institutional hypotheses. The political will have convenience to choose a centralized setup, in the pre-electoral period, when the value that the head of the CB (governor) gives to his reputation is rather low. This will occur with higher probability when the
central banker is little conservative. A governor with a lower inflation aversion will leave "to involve" himself from the political in a more expansionary monetary policy. In this case, monetary policy and supervision policy, go to the same direction. The probability, in fact, to have a stable banking system is higher when monetary policy is more “easy-going”. Therefore the presence of an a little conservative central banker means choice of a design of monetary and supervision policy both centralized in the hands of the central bank.

Otherwise if the governor of the central bank is very adverse to the inflation, hardly the government will "capture" the governor. In such case, it will be advantageous for the political to assign the responsibility of supervision policy to an authority distinguished from the central bank.
A. Contract with two agents in the electoral period.

Problem of constrained optimization

The Lagrangian for this problem is

\[(A1) \quad L(\theta, \lambda) = E(U - u) + \lambda_1 g_1 + \lambda_2 g_2 + \lambda_3 g_3 + \lambda_4 g_4\]

assuming:

\[(A2) \quad g_1 = E(I_b | e_b = 1) - E(I_b | e_b = 0) = (T_b - t_b) [Pr(Bs | e_b = 1) - Pr(Bs | e_b = 0) - C_b]\]

\[(A3) \quad g_2 = E(I_b | e_b = 1) = t_b [1 - Pr(Bs | e_b = 1)] + T_b Pr(Bs | e_b = 1) - C_b\]

\[(A4) \quad g_3 = E(I_p | e_p = 0) - E(I_p | e_p = 1) = (T_p - t_p) [Pr(Ps | e_p = 1) - Pr(Ps | e_p = 0)] + C_p\]

\[(A5) \quad g_4 = E(I_p | e_p = 0) = t_p Pr(Ps | e_p = 0) + T_p [1 - Pr(Ps | e_p = 0)]\]

The first-order conditions are given by

\[(A3) \quad \partial L/\partial t_b = 0, \quad \partial L/\partial T_b = 0, \quad \partial L/\partial t_p = 0, \quad \partial L/\partial T_p = 0\]

\[(A4) \quad \lambda_1 g_1 = 0\]

\[(A5) \quad \lambda_2 g_2 = 0\]

\[(A6) \quad \lambda_3 g_3 = 0\]

\[(A7) \quad \lambda_4 g_4 = 0\]

\[(A7) \quad g_1 \geq 0, \quad g_2 \geq 0, \quad g_3 \geq 0, \quad g_4 \geq 0\]

\[(A8) \quad \lambda_i \geq 0\]

We express constraints $g_i$ through the probabilities introduced with (3). For this purpose, we see that, as in $E(U-u)$, in making his assessments each agent can be expected to think that the other agent is almost sure to make the choice that is most advantageous for himself. For instance, BA will think that, as it is an election period, CB will make no effort, while CB will be convinced that BA will make an effort. In formulae this mean that conditions (13)-(16) become

\[(A7) \quad E(I_b | e_b = 1 \cap e_p = 0) = E(I_b | e_{10}) \geq E(I_b | e_b = 0 \cap e_p = 0) = E(I_b | e_{00})\]

\[(A8) \quad E(I_b | e_{10}) \geq 0\]

\[(A9) \quad E(I_p | e_b = 1 \cap e_p = 0) = E(I_p | e_{10}) \geq E(I_p | e_b = 1 \cap e_p = 1) = E(I_p | e_{11})\]

\[(A10) \quad E(I_p | e_{10}) \geq 0\]

and the $g_i$ will become
\[ g_1 = (T_b - t_b) (P_{1b} - P_{3b}) - C_b \]
\[ g_2 = t_b (1 - P_{1b}) + T_b P_{1b} - C_b \]
\[ g_3 = (T_p - t_p) [B - A] + C_p \]
\[ g_4 = t_p A + T_p [1 - A] \]

(A11)

To express (A9) and (A10) we calculate the conditional probabilities of \( P_s \) with respect to \( e_{ij} \):

\[ A = Pr(P_s | e_{10}) = Pr(P_s \cap Bs | e_{10}) + Pr(P_s \cap -Bs | e_{10}) = \]
\[ = Pr(Bs | e_{10}) Pr(P_s | Bs \cap e_{10}) + Pr(-Bs | e_{10}) Pr(P_s | - Bs \cap e_{10}) = \]
\[ = P_{1b} P_{3p} + (1 - P_{1b}) P_{0p}. \]

Likewise:

\[ B = Pr(P_s | e_{11}) = Pr(P_s \cap Bs | e_{11}) + Pr(P_s \cap -Bs | e_{11}) = \]
\[ = Pr(Bs | e_{11}) Pr(P_s | Bs \cap e_{11}) + Pr(-Bs | e_{11}) Pr(P_s | - Bs \cap e_{11}) = \]
\[ = P_{2b} P_{1p} + (1 - P_{2b}) P_{2p}. \]

(A12)

\[ C = Pr(P_s | e_{01}) = P_{0b} P_{1p} + (1 - P_{0b}) P_{2p} \]

(A13)

\[ D = Pr(P_s | e_{00}) = P_{3b} P_{1p} + (1 - P_{3b}) P_{0p} \]

It should be noticed that if \( P_{0p} \leq P_{3p} \) then \( A \leq P_{3p} \) and if \( P_{1p} > P_{2p} \) then \( B > P_{2p} \).

With equal effort being made by the banking authority, prices have more probability of being stable if there is an effort in this direction on the part of the agent of monetary policy. We therefore expect \( A \leq B \) and \( D \leq C \). Moreover, with equal effort being made by the authority in charge of monetary policy, the probability of stable prices is greater if the banking system is stable (see (3')) and therefore we expect \( D \leq C \leq B; A \leq B \).

This results in:

\[ g_1 = (T_b - t_b) (P_{1b} - P_{3b}) - C_b \]
\[ g_2 = t_b (1 - P_{1b}) + T_b P_{1b} - C_b \]
\[ g_3 = (T_p - t_p) [B - A] + C_p \]
\[ g_4 = t_p A + T_p [1 - A] \]

(A16)

The first-order conditions (A3) translate into:

\[ -u'(t_b) [P_{0p} (1 - P_{1b}) + (1 - P_{0p}) (1 - P_{1b})] - \lambda_1 (P_{1b} - P_{3b}) + \lambda_2 (1 - P_{1b}) = 0 \]
\[ -u'(T_b) [P_{3p} P_{1b} + (1 - P_{3p}) P_{1b}] + \lambda_3 (P_{1b} - P_{3b}) + \lambda_2 P_{1b} = 0 \]
\[ -u'(t_p) A - \lambda_3 (B - A) + \lambda_4 A = 0 \]
\[ -u'(T_p) (1 - A) + \lambda_3 (B - A) + \lambda_4 (1 - A) = 0 \]

or, if \( u'(t) \) is substituted with the value of the derivative of the utility function considered in point (10), we have:
\[
|t_b| = \left[ -\lambda_1 (P_{1b} - P_{3b}) + \lambda_2 (1 - P_{1b}) \right] / (1 - P_{1b})
\]
\[
|T_b| = \left[ \lambda_1 (P_{1b} - P_{3b}) + \lambda_2 P_{1b} \right] / P_{1b}
\]
\[
|t_p| = -\lambda_3 (B - A) / A + \lambda_4
\]
\[
|T_p| = \lambda_3 (B - A) / (1 - A) + \lambda_4
\]

Therefore, provided \(0 < P_{1b} < 1\) and \(0 < A < 1\)
\[
|t_b| = \lambda_2 - \lambda_1 (P_{1b} - P_{3b}) / (1 - P_{1b})
\]
\[
|T_b| = \lambda_2 + \lambda_1 (P_{1b} - P_{3b}) / P_{1b}
\]
\[
|t_p| = \lambda_4 - \lambda_3 (B - A) / A
\]
\[
|T_p| = \lambda_4 + \lambda_3 (B - A) / (1 - A)
\]

Conditions (A4)-(A6) lead to the examination of various cases, simplified by the fact that the first two of (A4) are related to \(t_b\) and \(T_b\), while the other two are related to \(t_p\) and \(T_p\). As we want to find solutions that maximize \(E(U-u)\), since \(-u\) is decrescent, the solution to the problem will be the one that makes \(u\) the lowest. Remember that \(u(t)\) is the cost incurred by the politician to pay the agents of the two different authorities. When this cost is lower, the politician’s utility is greater.

**Analysis of the cases that solve the optimization problem with two agents in the electoral period.**

In examining the various cases that can eventuate, we must remember that considering \(\lambda_i = 0\) simply means ignoring the constraint \(g_i \geq 0\).

The cases we should examine to verify conditions (A4-A6) are:

I) \(\lambda_1 = 0, g_2 = 0, g_1 \geq 0\)

II) \(\lambda_2 = 0, g_1 = 0, g_2 \geq 0\)

III) \(\lambda_1 = 0, \lambda_2 = 0\)

IV) \(g_1 = 0, g_2 = 0\)

V) \(\lambda_3 = 0, g_4 = 0, g_3 \geq 0\)

VI) \(\lambda_4 = 0, g_3 = 0, g_4 \geq 0\)

VII) \(\lambda_3 = 0, \lambda_4 = 0\)

VIII) \(g_3 = 0, g_4 = 0\)

For I) \(\lambda_1 = 0, g_2 = 0, g_1 \geq 0\) we have:
\[
|t_b| = \lambda_2
\]
\[
|T_b| = \lambda_2
\]
\[ t_b (1 - P_{1b}) + T_b P_{1b} - C_b = 0 \]
\[ (T_b - t_b) (P_{1b} - P_{3b}) - C_b \geq 0 \]

It follows that, if \( t_b = T_b \), then for the fourth equation the result is \(-C_b \geq 0\), and therefore it can only be \( C_b = 0 \) and \( t_b = T_b = 0 \).

If, however, \( t_b = -T_b \) then, for the third equation, we have
\[ (A19) \quad -t_b = T_b = C_b / (2 P_{1b} - 1) \]

and it must be
\[ (A20) \quad P_{3b} \leq 1/2 < P_{1b} \]

The first inequality derives from the fourth equation.

For II) \( \lambda_2 = 0, g_1 = 0 \) we have:
\[ |t_b| = -\lambda_1 (P_{1b} - P_{3b}) / (1 - P_{1b}) \]
\[ |T_b| = \lambda_1 (P_{1b} - P_{3b}) / P_{1b} \]
\[ (T_b - t_b) (P_{1b} - P_{3b}) - C_b = 0 \]
\[ t_b (1 - P_{1b}) + T_b P_{1b} - C_b \geq 0. \]

The first equation can be verified only if \( \lambda_1 = 0 \) or \( P_{1b} = P_{3b} \), but in both cases there would be \( t_b = T_b = 0 \) and then, for the third and fourth, there would be \( C_b \leq 0 \) and therefore \( C_b = 0 \).

III) \( \lambda_1 = 0, \lambda_2 = 0 \) would give, for (A18)
\[ t_b = T_b = 0 \]
as in the previous case, provided \( C_b = 0 \).

Lastly, IV) \( g_1 = 0, g_2 = 0 \) gives the system:
\[ (T_b - t_b) (P_{1b} - P_{3b}) - C_b = 0 \]
\[ t_b (1 - P_{1b}) + T_b P_{1b} - C_b = 0 \]
whose solution is:
\[ (A21) \quad t_b = -C_b P_{3b} / (P_{1b} - P_{3b}) \]
\[ T_b = C_b (1 - P_{3b}) / (P_{1b} - P_{3b}) \]

This solution is obtained from (A18) by saying:
\[ \lambda_1 = C_b P_{1b} (1 - P_{1b}) (1 - 2 P_{3b}) / (P_{1b} - P_{3b})^2 \]
\[ \lambda_2 = C_b (P_{1b} + P_{3b} - 2 P_{1b} P_{3b}) / (P_{1b} - P_{3b}) \).

For it to be \( \lambda_1 \geq 0 \), it will have to be \( P_{3b} \leq 1/2 \), as well as being \( P_{1b} > P_{3b} \).

Let us examine the case V) \( \lambda_3 = 0, g_4 = 0 \).

From (A18) it is deduced that \( |t_p| = |T_p| \) and, from \( g_4 = 0 \), it follows that either \( t_p = T_p = 0 \), or \( t_p = -T_p \) and, from \( g_4 = 0 \), it follows that \( T_p (1 - 2A) = 0 \). Therefore, either we return to case \( T_p = 0 \) or \( A = 1/2 \). In the latter case, the condition \( g_3 \geq 0 \) translates into \( T_p \geq -C_p/(2(B - A)) \), that is, any non-negative value of \( T_p \) is acceptable.
Case VI) $\lambda_4 = 0, g_3 = 0$ gives, for (A18):

$$|t_p| = -\lambda_3 \frac{(B - A)}{A}$$

$$|T_p| = \lambda_3 \frac{(B - A)}{(1 - A)}$$

$$(T_p - t_p) [B - A] + C_p = 0$$

$$t_p A + T_p [1 - A] \geq 0$$

The first can be satisfied only if $t_p = 0$ and we have this if $\lambda_3 = 0$ or $B = A$. In both cases there would be $T_p = 0$ and from the third $C_p = 0$, against the hypotheses. This case can therefore not be verified.

Case VII) $\lambda_3 = 0, \lambda_4 = 0$ gives

(A22) $$t_p = T_p = 0$$

already seen in case V).

Finally, case VIII) $g_3 = 0, g_4 = 0$ is equivalent to:

$$T_p - t_p = -C_p/ (B - A)$$

$$T_p = A (T_p - t_p)$$

This case is possible only if $C_p = 0$ and if so the solution is

$$T_p = t_p = 0$$

or if $A > B$ and this goes against common sense.

B. Contract with two agents in the non-electoral period

Problem of constrained optimization

The constraints can be expressed in short form, by saying:

$$g_1 = E(I_b \mid e_b = 1) - E(I_b \mid e_b = 0) = (T_b - t_b) [Pr(Bs \mid e_b = 1) - Pr(Bs \mid e_b = 0)] - C_b$$

$$g_2 = E(I_b \mid e_b = 1) = t_b [1 - Pr(Bs \mid e_b = 1)] + T_b Pr(Bs \mid e_b = 1) - C_b$$

$$g_3 = E(I_p \mid e_p = 1) - E(I_p \mid e_p = 0) = (T_p - t_p) [Pr(Ps \mid e_p = 1) - Pr(Ps \mid e_p = 0)] - C_p$$

$$g_4 = E(I_p \mid e_p = 1) = t_p [1 - Pr(Ps \mid e_p = 1)] + T_p Pr(Ps \mid e_p = 1) - C_p$$

and therefore constraints (23)-(26) can be written:

(B2) $$g_1 \geq 0 \quad g_2 \geq 0 \quad g_3 \geq 0 \quad g_4 \geq 0.$$  

Proceeding as in the previous case, conditions of the 1st order are given by (A3)-(A6) and, for the same reasons, we have

$$g_1 = (T_b - t_b) [Pr(Bs \mid e_{b1}) - Pr(Bs \mid e_{b0})] - C_b = (T_b - t_b) (P_{2b} - P_{0b}) - C_b$$

$$g_2 = t_b (1 - P_{2b}) + T_b P_{2b} - C_b$$

$$g_3 = (T_p - t_p) [Pr(Ps \mid e_{p1}) - Pr(Ps \mid e_{p0})] - C_p = (T_p - t_p) [B - A] - C_p$$

$$g_4 = t_p [1 - Pr(Ps \mid e_{p1})] + T_p Pr(Ps \mid e_{p1}) - C_p = t_p [1 - B] + T_p B - C_p$$

30
Conditions (A3) translate into:

\[-u'(t_b) [P_{2p} (1 - P_{2b}) + (1 - P_{2p}) (1 - P_{2b})] - \lambda_1 (P_{2b} - P_{0b}) + \lambda_2 (1 - P_{2b}) = 0\]

\[-u'(T_b) [P_{1p} P_{2b} + (1 - P_{1p}) P_{2b}] + \lambda_1 (P_{2b} - P_{0b}) + \lambda_2 P_{2b} = 0\]

\[-u'(t_p) (1 - B) - \lambda_3 (B - A) + \lambda_4 (1 - B) = 0\]

\[-u'(T_p) B + \lambda_3 (B - A) + \lambda_4 B = 0,\]

which gives:

\[| t_b | = \frac{\lambda_2 - \lambda_1 (P_{2b} - P_{0b})}{(1 - P_{2b})}\]

\[| T_b | = \frac{\lambda_2 + \lambda_1 (P_{2b} - P_{0b})}{P_{2b}}\]

\[| t_p | = \frac{\lambda_4 - \lambda_3 (B - A)}{(1 - B)}\]

\[| T_p | = \frac{\lambda_4 + \lambda_3 (B - A)}{B}\]

(C4)

C. Contract with a single agent in the electoral period.

Problem of constrained optimization

Keeping in mind the conditional probabilities (4)\textsuperscript{i}-(7)\textsuperscript{i} the constraints become

\[g_1 = T_{10} [\Pr(E_1 | e_{10}) - \Pr(E_1 | e_{00})] + T_{11} [\Pr(E_2 | e_{10}) - \Pr(E_2 | e_{00})] + \]
\[+ T_{01} [\Pr(E_3 | e_{10}) - \Pr(E_3 | e_{00})] + T_{00} [\Pr(E_4 | e_{10}) - \Pr(E_4 | e_{00})] - (C_{bp} - C_p) =\]
\[= T_{10} [P_{1b} (1 - P_{3p}) - P_{3b} (1 - P_{3p})] + T_{11} [P_{1b} P_{3p} - P_{3b} P_{3p}] +\]
\[+ T_{01} [(1 - P_{1b}) P_{0p} - (1 - P_{3b}) P_{0p}] + T_{00} [(1 - P_{1b})(1 - P_{0p}) - (1 - P_{3b})(1 - P_{0p})] - (C_{bp} - C_p) =\]
\[= (P_{1b} - P_{3b}) [T_{10} (1 - P_{3p}) + T_{11} P_{3p} - T_{01} P_{0p} - T_{00} (1 - P_{0p})] - (C_{bp} - C_p) \geq 0\]

\[g_2 = T_{10} [\Pr(E_1 | e_{10}) - \Pr(E_1 | e_{11})] + T_{11} [\Pr(E_2 | e_{10}) - \Pr(E_2 | e_{11})] +\]
\[+ T_{01} [\Pr(E_3 | e_{10}) - \Pr(E_3 | e_{11})] + T_{00} [\Pr(E_4 | e_{10}) - \Pr(E_4 | e_{11})] + C_p =\]
\[= T_{10} [P_{1b} (1 - P_{3p}) - P_{2b} (1 - P_{1p})] + T_{11} [P_{1b} P_{3p} - P_{2b} P_{1p}] +\]
\[+ T_{01} [(1 - P_{1b}) P_{0p} - (1 - P_{2b}) P_{2p}] + T_{00} [(1 - P_{1b})(1 - P_{0p}) - (1 - P_{2b})(1 - P_{2p})] + C_p \geq 0\]

(C1)

\[g_3 = T_{10} [\Pr(E_1 | e_{10}) - \Pr(E_1 | e_{01})] + T_{11} [\Pr(E_2 | e_{10}) - \Pr(E_2 | e_{01})] +\]
\[+ T_{01} [\Pr(E_3 | e_{10}) - \Pr(E_3 | e_{01})] + T_{00} [\Pr(E_4 | e_{10}) - \Pr(E_4 | e_{01})] - (C_p - C_b) =\]
\[= T_{10} [P_{1b} (1 - P_{3p}) - P_{0b} (1 - P_{1p})] + T_{11} [P_{1b} P_{3p} - P_{0b} P_{1p}] +\]
\[+ T_{01} [(1 - P_{1b}) P_{0p} - (1 - P_{0b}) P_{2p}] + T_{00} [(1 - P_{1b})(1 - P_{0p}) - (1 - P_{0b})(1 - P_{2p})] +\]
\[+ (C_p - C_b) \geq 0\]
\[
g_4 = T_{10} \Pr(E_1 \mid e_{10}) + T_{11} \Pr(E_2 \mid e_{11}) + T_{01} \Pr(E_3 \mid e_{10}) + T_{00} \Pr(E_4 \mid e_{10}) - (C_{bp} - C_p) = \\
= T_{10} [P_{1b} (1 - P_{3p})] + T_{11} [P_{1b} P_{3p}] + T_{01} [(1 - P_{1b}) P_{0p}] + \\
+ T_{00} [(1 - P_{1b}) (1 - P_{0p})] - (C_{bp} - C_p) \geq 0
\]

Considering the Lagrangian (A1), the conditions of the 1st order are given (A3)-(A6) and in particular (A3) translate into:

-\(u^\prime(T_{10}) (1 - P_{3p}) P_{1b} + \lambda_1 [(P_{1b} - P_{3b})(1 - P_{3p})] + \lambda_2 [P_{1b} (1 - P_{3p}) - P_{2b} (1 - P_{1p})] + \lambda_3 [P_{1b} (1 - P_{3p}) - P_{0b} (1 - P_{1p})] + \lambda_4 [P_{1b} (1 - P_{3p})] = 0\)

-\(u^\prime(T_{11}) P_{3p} P_{1b} + \lambda_1 [P_{1b} P_{3p} - P_{3b} P_{3p}] + \lambda_2 [P_{1b} P_{3p} - P_{2b} P_{1p}] + \lambda_3 [P_{1b} P_{3p} - P_{0b} P_{1p}] + \lambda_4 [P_{1b} P_{3p}] = 0\)

-\(u^\prime(T_{01}) P_{0p} (1 - P_{1b}) + \lambda_1 [(1 - P_{1b}) P_{0p} - (1 - P_{3b}) P_{0p}] + \lambda_2 [(1 - P_{1b}) P_{0p} - (1 - P_{2b}) P_{2p}] + \lambda_3 [(1 - P_{1b}) P_{0p} - (1 - P_{0b}) P_{0p}] + \lambda_4 [(1 - P_{1b}) (1 - P_{0p})] = 0\)

which we can also write:

\[
|T_{10}| = \lambda_1 (1 - P_{3b}) / P_{1b} + \lambda_2 [1 - P_{2b} (1 - P_{1p}) / (P_{1b} (1 - P_{3p}))] + \lambda_3 [1 - P_{0b} (1 - P_{1p}) / (P_{1b} (1 - P_{3p}))] + \lambda_4
\]

\[
|T_{11}| = \lambda_1 (1 - P_{3b}) / P_{1b} + \lambda_2 [1 - P_{2b} P_{1p} / (P_{1b} P_{3p})] + \lambda_3 [1 - P_{0b} P_{1p} / (P_{1b} P_{3p})] + \lambda_4
\]

\(\text{(C2)}\)

\[
|T_{01}| = \lambda_1 [1 - (1 - P_{3b}) / (1 - P_{1b})] + \lambda_2 [1 - (1 - P_{2b}) P_{2p} / ((1 - P_{1b}) P_{0p})] + \lambda_3 [1 - (1 - P_{0b}) P_{2p} / (1 - P_{1b}) P_{0p}] + \lambda_4
\]

\[
|T_{00}| = \lambda_1 [1 - (1 - P_{3b}) / (1 - P_{1b})] + \lambda_2 [1 - (1 - P_{2b}) (1 - P_{2p}) / (1 - P_{1b}) (1 - P_{0p})] + \lambda_3 [1 - (1 - P_{0b}) (1 - P_{2p}) / (1 - P_{1b}) (1 - P_{0p})] + \lambda_4
\]

or in more explicit terms:

\[
|T_{10}| = \hat{\lambda}_1 \left[1 - \frac{\Pr(E_1 \mid e_{10})}{\Pr(E_1 \mid e_{10})}\right] + \hat{\lambda}_2 \left[1 - \frac{\Pr(E_2 \mid e_{10})}{\Pr(E_2 \mid e_{10})}\right] + \hat{\lambda}_3 \left[1 - \frac{\Pr(E_3 \mid e_{10})}{\Pr(E_3 \mid e_{10})}\right] + \hat{\lambda}_4
\]

\[
|T_{10}| = \hat{\lambda}_1 \left[1 - \frac{\Pr(E_2 \mid e_{10})}{\Pr(E_2 \mid e_{10})}\right] + \hat{\lambda}_2 \left[1 - \frac{\Pr(E_3 \mid e_{10})}{\Pr(E_3 \mid e_{10})}\right] + \hat{\lambda}_3 \left[1 - \frac{\Pr(E_4 \mid e_{10})}{\Pr(E_4 \mid e_{10})}\right] + \hat{\lambda}_4
\]

\[
|T_{01}| = \hat{\lambda}_1 \left[1 - \frac{\Pr(E_2 \mid e_{10})}{\Pr(E_2 \mid e_{10})}\right] + \hat{\lambda}_2 \left[1 - \frac{\Pr(E_3 \mid e_{10})}{\Pr(E_3 \mid e_{10})}\right] + \hat{\lambda}_3 \left[1 - \frac{\Pr(E_4 \mid e_{10})}{\Pr(E_4 \mid e_{10})}\right] + \hat{\lambda}_4
\]

\[
|T_{00}| = \hat{\lambda}_1 \left[1 - \frac{\Pr(E_2 \mid e_{10})}{\Pr(E_2 \mid e_{10})}\right] + \hat{\lambda}_2 \left[1 - \frac{\Pr(E_3 \mid e_{10})}{\Pr(E_3 \mid e_{10})}\right] + \hat{\lambda}_3 \left[1 - \frac{\Pr(E_4 \mid e_{10})}{\Pr(E_4 \mid e_{10})}\right] + \hat{\lambda}_4
\]
The solution to this problem is not easy, however it is less important to know the value of incentives that maximize $E (U - u | e_{10})$, than to know if it is more advantageous to appoint two agents or a single agent. To resolve the problem, we have to compare the utility expected by the politician with a single agent, with the expected utility with two separate authorities.

D. Contract with a single agent in the non-electoral period.

Problem of constrained optimization

Keeping conditional probabilities $(4)^{(7)}$ in mind the constraints become

$$g_1 = T_{11} [\Pr(E_1 | e_{11}) - \Pr(E_1 | e_{10})] + T_{10} [\Pr(E_2 | e_{11}) - \Pr(E_2 | e_{10})] +
+ T_{01} [\Pr(E_3 | e_{11}) - \Pr(E_3 | e_{10})] + T_{00} [\Pr(E_4 | e_{11}) - \Pr(E_4 | e_{10})] - \{C_{bp} - (C_{bp} - C_p)\} =
= T_{11} [P_{2b} P_{1p} - P_{1b} P_{3p}] + T_{10} [P_{2b} (1 - P_{1p}) - P_{1b} (1 - P_{3p})] +
+ T_{01} [(1 - P_{2b}) P_{2p} - (1 - P_{1b}) P_{0p}] + T_{00} [(1 - P_{2b}) (1 - P_{2p}) - (1 - P_{1b}) (1 - P_{0p})] - C_p \geq 0$$

$$g_2 = T_{11} [\Pr(E_1 | e_{11}) - \Pr(E_1 | e_{01})] + T_{10} [\Pr(E_2 | e_{11}) - \Pr(E_2 | e_{01})] +
+ T_{01} [\Pr(E_3 | e_{11}) - \Pr(E_3 | e_{01})] + T_{00} [\Pr(E_4 | e_{11}) - \Pr(E_4 | e_{01})] - C_b =
= T_{11} [P_{2b} P_{1p} - P_{0b} P_{1p}] + T_{10} [P_{2b} (1 - P_{1p}) - P_{0b} (1 - P_{1p})] +
+ T_{01} [(1 - P_{2b}) P_{2p} - (1 - P_{0b}) P_{2p}] + T_{00} [(1 - P_{2b}) (1 - P_{2p}) - (1 - P_{0b}) (1 - P_{2p})] - C_b =
= (P_{2b} - P_{0b}) [T_{11} P_{1p} + T_{10} (1 - P_{1p}) - T_{01} P_{2p} - T_{00} (1 - P_{2p})] - C_b \geq 0$$

(D1)

$$g_3 = T_{11} [\Pr(E_1 | e_{11}) - \Pr(E_1 | e_{00})] + T_{10} [\Pr(E_2 | e_{11}) - \Pr(E_2 | e_{00})] +
+ T_{01} [\Pr(E_3 | e_{11}) - \Pr(E_3 | e_{00})] + T_{00} [\Pr(E_4 | e_{11}) - \Pr(E_4 | e_{00})] - C_{bp} =
= T_{11} [P_{2b} P_{1p} - P_{3b} P_{3p}] + T_{10} [P_{2b} (1 - P_{1p}) - P_{3b} (1 - P_{3p})] +
+ T_{01} [(1 - P_{2b}) P_{2p} - (1 - P_{3b}) P_{0p}] + T_{00} [(1 - P_{2b}) (1 - P_{2p}) - (1 - P_{3b}) (1 - P_{0p})] - C_{bp} \geq 0$$

$$g_4 = T_{11} P_{2b} P_{1p} + T_{10} P_{2b} (1 - P_{1p}) + T_{01} (1 - P_{2b}) P_{2p} + T_{00} (1 - P_{2b}) (1 - P_{2p}) - C_{bp} =
= P_{2b} [T_{11} P_{1p} + T_{10} (1 - P_{1p})] + (1 - P_{2b}) [T_{01} P_{2p} + T_{00} (1 - P_{2p})] - C_{bp} \geq 0.$$ 

Considering the Lagrangian (A1), the conditions of the 1st order are given (A3)-(A6) and in particular (A3) translate into :

$$-u'(T_{11}) P_{2b} P_{1p} + \lambda_1 [P_{2b} P_{1p} - P_{1b} P_{3p}] + \lambda_2 [P_{2b} P_{1p} - P_{0b} P_{1p}] + \lambda_3 [P_{2b} P_{1p} - P_{3b} P_{3p}] + \lambda_4 P_{2b} P_{1p} = 0$$

$$-u'(T_{10}) P_{2b} (1 - P_{1p}) + \lambda_1 [P_{2b} (1 - P_{1p}) - P_{1b} (1 - P_{3p})] + \lambda_2 [P_{2b} (1 - P_{1p}) - P_{0b} (1 - P_{1p})] +
+ \lambda_3 [P_{2b} (1 - P_{1p}) - P_{3b} (1 - P_{3p})] + \lambda_4 P_{2b} (1 - P_{1p}) = 0.$$
\[-u'(T_{11}) (1 - P_{2b}) P_{2p} + \lambda_1 [(1 - P_{2b}) P_{2p} - (1 - P_{1b}) P_{0p}] + \lambda_2 [(1 - P_{2b}) P_{2p} - (1 - P_{0b}) P_{2p}] +
\lambda_3 [(1 - P_{2b}) P_{2p} - (1 - P_{3b}) P_{0p}] + \lambda_4 (1 - P_{2b}) P_{2p} = 0\]

\[-u'(T_{00}) (1 - P_{2b}) (1 - P_{2p}) + \lambda_1 [(1 - P_{2b}) (1 - P_{2p}) - (1 - P_{1b}) (1 - P_{0b})] +
\lambda_2 [(1 - P_{2b}) (1 - P_{2p}) - (1 - P_{0b}) (1 - P_{2p})] + \lambda_3 [(1 - P_{2b}) (1 - P_{2p}) - (1 - P_{3b}) (1 - P_{0p})] +
\lambda_4 (1 - P_{2b}) (1 - P_{2p}) = 0\]

which we can also write:

\[|T_{11}| = \lambda_1 [1 - P_{1b} P_{3p} / (P_{2b} P_{1p})] + \lambda_2 [1 - P_{0b} P_{1p} / (P_{2b} P_{1p})] + \lambda_3 [1 - P_{3b} P_{3p} / (P_{2b} P_{1p})] + \lambda_4
\]

\[|T_{10}| = \lambda_1 [1 - P_{1b} (1 - P_{3p}) / (P_{2b} (1 - P_{1p}))] + \lambda_2 [1 - P_{0b} (1 - P_{1p}) / (P_{2b} (1 - P_{1p}))] +
\lambda_3 [1 - (1 - P_{3b}) P_{0p} / ((1 - P_{2b}) P_{2p})] + \lambda_4
\]

\[|T_{01}| = \lambda_1 [1 - (1 - P_{1b}) P_{0p} / ((1 - P_{2b}) P_{2p})] + \lambda_2 [1 - (1 - P_{0b}) P_{2p} / ((1 - P_{2b}) P_{2p})] +
\lambda_3 [1 - (1 - P_{3b}) P_{0p} / ((1 - P_{2b}) P_{2p})] + \lambda_4
\]

\[|T_{00}| = \lambda_1 [1 - (1 - P_{1b}) (1 - P_{0p}) / ((1 - P_{2b}) (1 - P_{2p}))] + \lambda_2 [1 - (1 - P_{0b}) (1 - P_{2p}) / ((1 - P_{2b}) (1 - P_{2p}))]
\]

\[+ \lambda_3 [1 - (1 - P_{3b}) (1 - P_{0p}) / ((1 - P_{2b}) (1 - P_{2p}))] + \lambda_4\]

E. We prove some inequalities

\[(E1) \quad \frac{1}{2x - 1} \geq \frac{(1 - y)^2 x - y^2 (1 - x)}{(x - y)^2}\]

since \(2x - 1 > 0\) and \(0 < x, y < 1\).

(E1) is equivalent to

\[(x - y)^2 \geq (1 - y)^2 x - y^2 (1 - x) / (2x - 1)\]

\[x^2 + y^2 - 2xy \geq [x + 2y^2 x - 2x y - y^2] (2x - 1)\]

\[x^2 + y^2 - 2xy \geq 2x^2 + 4x^2 y^2 - 4 x^2 y - 2xy^2 - x - 2xy^2 + 2xy + y^2\]

\[4x^2 y + 4xy^2 + x \geq x^2 + 4x^2 y^2 + 4xy\]

\[4xy + 4y^3 + 1 \geq x + 4xy + 4y\]

\[4xy (1 - y) + 1 - x \geq 4 y (1 - y)\]

\[1 - x \geq 4 y (1 - y) (1 - x)\]

34
\[ \frac{1}{4} \geq y (1 - y) \]

this last inequality being true, the first is true.

(E2) \((1 - y)^2 x - y^2 (1 - x)\)

can only be positive, if \(0 < y \leq 1/2\) and \(y \leq x\).

\((1 - y)^2 x - y^2 (1 - x) \geq 0\)

equivalent to

\[ \left( \frac{1 - y}{y} \right)^2 \geq \frac{1 - x}{x} ; \]

since if \(0 < y \leq 1/2\) then \((1 - y)/y \geq 1\), we get \((1 - y)/y \geq (1 - y)/y\) and as function

\(f(x) = \frac{(1 - x)}{x}\)

is descending and \(y \leq x\), we will have

\[ \left( \frac{1 - y}{y} \right)^2 \geq (1 - y)/y \geq (1 - x)/x \]

the equality is valid only if

\(x = y\).

(E3) \(y/ [2(x - y)] \leq [(1 - y)^2 x - y^2 (1 - x)]/[2(x - y)^2] \leq 1 / [2(x - y)]\)

if \(0 < y \leq 1/2\) and \(y \leq x\) for the first inequality and \(0 < y \leq 1/2 \leq x\) for the second.

The first inequality is equivalent to

\(y(x - y) \leq (1 - y)^2 x - y^2 (1 - x) \Leftrightarrow yx \leq (1 - y)^2 x + y^2 x\)

which means:

\(yx(1 - y) \leq (1 - y)^2 x \Leftrightarrow y \leq (1 - y) \Leftrightarrow y \leq 1/2.\)

The second inequality is equivalent to

\((1 - y)^2 x - y^2 (1 - x) \leq x - y \Leftrightarrow (1 + y^2 - 2y)x - y^2 + y^2 x \leq x - y\)

which means:

\(2 y^2 x + y \leq y (2x + y) \Leftrightarrow 2yx + 1 \leq 2x + y \Leftrightarrow 1 - y \leq 2x (1 - y)\)

That is, \(x \geq 1/2.\)
References


18) Franck R. – Krausz M. (2004), “Political business cycles under separation of monetary policy from banking supervision”, *Bar Ilan University, Department of Economics*


21) Greenspan A. (1994), “Testimony on banking regulation, before the United States Senate Committee on Banking, Housing and Urban Affairs”, 2 March


