defined over the support \([0, 1]\). By aggregating the individual demand curves, we obtain market demand functions linear in \(\theta\) of the type:

\[
X_{i1}^d = \left( \frac{p_i}{q} \right)^{-\sigma} \frac{1}{q} (\alpha_1 - \beta_1 \theta) \\
X_{i2}^d = \left( \frac{p_i}{q} \right)^{-\sigma} \frac{1}{q} (\alpha_2 + \beta_2 \theta)
\]

where \(\alpha_j\) and \(\beta_j\) \((j = 1, 2)\) are positive numbers. As expected, \(X_{i1}^d\) is decreasing \((X_{i2}^d\) is increasing) in \(\theta\).

We now apply the above demand framework in the analysis of market equilibrium.

### 3 Pricing and market equilibrium: the case with exogenous mark-up

Following the standard Dixit-Stiglitz approach, we assume that each firm faces the following cost function

\[
C(x_i) = a + cx_i
\]

If each firm maximizes its own profits under the demand constraint (3) and taking \(q\) as given, the symmetric short run equilibrium price is

\[
p_i = p = c \frac{\sigma}{\sigma - 1}
\]

and since \(q = pm^{1-\sigma}\),

\[
X_i = X = \frac{S(pm^{1-\sigma}, \theta)}{pn}
\]

According to equation (4), the equilibrium mark-up is fully determined by the exogenous cost and demand parameters, and is therefore independent

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2 This density, a mixture of a uniform and a quadratic beta distribution, is unimodal and symmetric. It is easy to check that the parameter \(\theta \in [0, 1]\) is a mean preserving spread, so that an increase in \(\theta\) increases income dispersion.

3 In particular we have \(\alpha_1 \approx 0.4426\), \(\beta_1 \approx 0.0047\), \(\alpha_2 \approx 0.1443\), \(\beta_2 \approx 0.0074\).
of the properties of income distribution. Indeed, this result is due to two main assumptions, both implicit in the Dixit-Stiglitz approach. The first is the CES formulation for the composite differentiated good; the second is the so-called negligibility hypothesis, according to which each firm is assumed to neglect the effect of its own price decisions on the aggregate price \( q \). We shall relax this latter assumption in the next section.

Equation (5) shows that the distribution parameter affects the short run optimal production decision: at the given price the changes in the demand size induced by distributional shocks are met through quantity adjustments.

Let us now consider the long run market equilibrium. By using (4) and (5) in the zero profit condition \((p - c) X = a\), we get

\[
\frac{S(p_n, \theta)}{p_n} = \frac{a}{c \left( \frac{1}{\sigma - 1} \right)}
\]

which makes it clear that the distributional parameter \( \theta \) affects, through the above size-effect on demand, the equilibrium number of firms and therefore the degree of product differentiation. The role of \( \theta \) is synthetized in the following proposition.

**Proposition 2** If the intersectoral elasticity of demand for the composite differentiated good \( y \), \( \eta_{yq} = - (q/y) (\partial y/\partial q) \), is lower than the intrasectoral elasticity \( \sigma \), then an increase in income dispersion reduces (increases) the equilibrium number of varieties of a necessary (luxury) differentiated good.

**Proof.** By implicit differentiation of (6), we get

\[
\frac{dn}{d\theta} = - \frac{s}{pm^2} \left( \frac{\partial S}{\partial q} \frac{\partial q}{\partial n} \frac{\partial q}{\partial n} - 1 \right)
\]

Given that \( (q/S) (\partial S/\partial q) = 1 - \eta_{yq} \), and that \( (n/q) (\partial q/\partial n) = 1/(1 - \sigma) \), the denominator is negative if \( \eta_{yq} < \sigma \). This condition ensures uniqueness of equilibrium (Dixit and Stiglitz, 1977,p.300), and if it is verified, \( \text{sign} \left( \frac{dn}{d\theta} \right) = \text{sign} \left( \frac{\partial S}{\partial \theta} \right) \). Making use of Proposition 1, Proposition 2 then follows straightforwardly. \( \Box \)

\(^4\)In a Chamberlinian framework, it amounts to imposing that the \( dd \) curve be more elastic than the \( DD \) curve.
The simple analysis of this section shows clearly that in the standard Dixit-Stiglitz framework of product differentiation, income dispersion plays indeed a role in the long run configuration of market structure, in that it affects the number of available varieties; but it seems to be irrelevant as far as the competitiveness of market is concerned: the price over cost ratio and each firm long run equilibrium output are independent of the distributive parameter.

It may be noticed that, in a love-for-variety framework, this effect on the degree of product differentiation is a non negligible, side welfare effect of distributonal shocks, which affects all consumers utility in the same direction, independently of their displacement in the scale of incomes.

4 Pricing and market equilibrium: the case with endogenous mark-up

In the simple model of the previous section income dispersion exerts only a size effect on demand, the direction of which depends on the concavity or convexity of the Engel’s curve. However, there exists in principle an additional transmission mechanism of distributive shocks to the goods market, namely that of market demand elasticity. This issue has already been dealt with in a homogenous product setup: in that framework an increase in income dispersion reduces for a relevant price range both the size and the elasticity of market demand (e.g., Benassi, Chirco and Scrimitore, 2002).

Within the love-for-variety models of product differentiation, the CES formulation of preferences is consistent with an endogenous mark-up only if the above mentioned negligibility hypothesis is abandoned and the so-called price index effect is taken into account. The idea that firms do not neglect the change in the price index \( q \) induced by their own pricing decisions has been introduced by Yang and Heijdra (1993) and generates an endogenous mark-up behaviour in the macroeconomic analyses by Bratsiotis and Martin (1999), Wu and Zhang (2000), Linnemann (2001), Benassi, Chirco and Colombo (2002).

If the perceived elasticity of the price index with respect to the individual price is not nil, \( \nu_{q_i} = (p_i/q) \frac{\partial q}{\partial p_i} \neq 0 \), then the own price elasticity (in