where $\mu, \mu_c \neq 1, \mu_c \neq (1-\mu)/\gamma$.

Hence, $w_{lv} > 1$ when:

$$\mu_c > \frac{1-\mu}{2(1-\mu)-\gamma} = \mu_c^*$$

where $\gamma \neq 2(1-\mu)$. Wages cannot be lower than 1 because in this case the traditional good would be produced in region v and not in region r.

On the contrary, when $0 < \mu_c \leqslant \mu_c^*$, the wages of unskilled workers in the core v must be equal to 1 if the traditional good is produced in the periphery r.

Therefore we may have the two following cases.

When $0 < \mu_c \leqslant \mu_c^*$, agglomeration of manufacturing firms in region v is an equilibrium if the ratio Q_{mir}/Q_{mir}^* from (41) is smaller than 1:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{\left(\tau^{2(\sigma-1)}-1\right)(1-\mu_c\gamma-\mu)}{2}+1\right) < 1 \tag{51}$$

Otherwise, when $1 > \mu_c > \mu_c^*$, agglomeration of the manufacturing sector in region v is an equilibrium when:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{(1-\mu)(1-\mu_c)}{(1-\mu-\gamma)\mu_c}\right)^{-\sigma(1-\gamma-\mu)} \left[\left(\tau^{2(\sigma-1)}-1\right)(1-\mu)(1-\mu_c)+1\right] < 1 \tag{52}$$

Appendix B.

To prove that profits in a neighborhood of a long run equilibrium can be written as a function of the number of firms n

$$\pi_i = u(n) \tag{53}$$

it is necessary to determine the short run equilibrium, which is defined as a set of solutions to equations (54)-(58) below, once n_n and n_s are given. We express them in matrix form. To this end, variables without suffix r define vectors, variables with superscript$\tilde{}$ are 2x2 diagonal matrix with the i-th element of the corresponding vector in position (i,i) and zeros off the diagonal. Matrix T is: $T = \begin{bmatrix} 1 & \tau^{1-\sigma} \\ \tau^{1-\sigma} & 1 \end{bmatrix}$.

Substituting manufacturing prices p_r from (13), expenditures on the manufacturing good E_{mr} from (4), (5) and (10), manufacturing quantities Q_{mir} from (15), and production cost TC_{mir} from (9), into (12), (3), (20), (18), (19) and (21), for $r = n, s$, and given the normalizations of α and β, we obtain a system of 10 equations. The solutions of the system (54)-(58) is given by the set of the 10 "fast" variables (π_{in}, π_{is}, p_{mn}, p_{ms}, w_{hn}, w_{hs}, w_{ls}, $w_{\ln}$, H_n, H_s) for given values of the slow variables n_n and n_s.

In matrix form, the equilibrium is obtained by solving the following system:

- two manufacturing good market short run equilibrium conditions:

$$a + \sigma\tilde{a}\tilde{p}_m^{-\mu}\tilde{w}_l^{-1+\mu+\gamma}\tilde{w}_h^{-\gamma}\pi_i = \tag{54}$$

$$= \mu_c\tilde{a}^{\sigma}\tilde{p}_m^{-\sigma\mu}\tilde{w}_l^{-\sigma(1-\mu-\gamma)}\tilde{w}_h^{-\sigma\gamma}T\tilde{p}_m^{\sigma-1}\left(\tilde{L}w_l + \tilde{H}w_h + \tilde{n}\pi_i\right) +$$
$$+\mu\tilde{a}^{\sigma}\tilde{p}_m^{-\sigma\mu}\tilde{w}_l^{-\sigma(1-\mu-\gamma)}\tilde{w}_h^{-\sigma\gamma}T\tilde{p}_m^{\sigma-1}\tilde{n}\left[(\sigma-1)\pi_i + \tilde{w}_l^{(1-\mu-\gamma)}\tilde{p}_m^{\mu}w_h^{\gamma}\right]$$

- two composite good price indices:

$$0 = p_m^{1-\sigma} - T\tilde{a}^{(-1+\sigma)}\tilde{p}_m^{\mu(1-\sigma)}\tilde{w}_h^{\gamma(1-\sigma)}\tilde{w}_l^{(1-\gamma-\mu)(1-\sigma)}n \tag{55}$$

- two functions that express total wages of skilled workers in the two regions:

$$0 = \tilde{w}_h H - \gamma\tilde{n}\left[(\sigma-1)\pi_i + \tilde{p}_m^{\mu}\tilde{w}_l^{1-\gamma-\mu}w_h^{\gamma}\right] \tag{56}$$

- skilled labor market equilibrium condition together with the condition of equal real wages for the two regions:

$$\begin{bmatrix} \frac{w_{hn}}{p_{mn}^{\mu_c}} \\ H_n + H_s \end{bmatrix} = \begin{bmatrix} \frac{w_{hs}}{p_{ms}^{\mu_c}} \\ \bar{H} \end{bmatrix} \tag{57}$$

- two unskilled labor market conditions:

$$\tilde{w}_l \bar{L} = (1-\gamma-\mu)\,\tilde{n}\left[(\sigma-1)\,\pi_i + \tilde{p}_m^{\mu}\tilde{w}_h^{\gamma}w_l^{1-\gamma-\mu}\right] + \tag{58}$$

$$+\,(1-\mu_c)\,\Lambda(\tilde{w}_h H + \tilde{w}_l \bar{L} + \tilde{n}\pi_i)$$

where $\Lambda = \begin{bmatrix} \lambda_r & 1-\lambda_v \\ 1-\lambda_r & \lambda_v \end{bmatrix}$

Let:

1. $x = (p_{mn}, p_{ms}, w_{hn}, w_{hs}, w_{ls}, w_{\ln}, H_n, H_s)'$, a column vector of eight fast variables;

2. $y = (\pi_i', x')'$ the column vector of the ten fast variables;

3. G_k be a function from R^{12} to R with continuous derivative in the neighborhood of a long run equilibrium (LRE), such that $G_k(y,\ n) = 0$, with $k = 1, 2, ...10$ are the ten equations (54)-(58);

4. $G(y, n) \equiv (G_k(y, n))$.

If the $det = \left[\dfrac{\partial G_k}{\partial y_\iota}\right]_* \neq 0$, where y_ι is a generic element of y and $*$ means that the derivatives are evaluated at a LRE, equation $G(y, n) = 0$ allows us to define in a neighborhood of such LRE function u from R^2 to R^2 with continuous derivative such that

$$\pi_i = u(n)$$

The Jacobian matrix of u in a LRE is denoted by $\dfrac{\partial u}{\partial n}(n^*)$.

Finally, it should be noted that at the long run equilibrium values, that is, at long run equilibrium values of all fast and slow (n_n and n_s) variables, profits should be equal to zero ($\pi_{in} = \pi_{is} = 0$).

Appendix C.

In this appendix we show how we compute the Jacobian matrix evaluated at the long run equilibrium

$$J_1^* = \frac{\partial z}{\partial n}(n^*) = \delta \, M$$

where $M = \frac{\partial u}{\partial n}(n^*)$.

Define:

- the column vector $x = (p_{mn}, p_{ms}, w_{hn}, w_{hs}, w_{ls}, w_{\ln}, H_n, H_s)'$;
- and the two functions f and g, where:

f is defined from R^{12} to R^2 and is derived from the two equations (54) in appendix B, and is such that

$$f(\pi_i, n, x) = 0$$

g is defined from R^{12} to R^8 and is derived from the eight equations (55)-(58), and is such that

$$g(\pi_i, n, x) = 0$$

Total differentials of f and g are respectively given by (59) and (60):

$$A \; d\pi_i + B \; dn + C \; dx = 0 \tag{59}$$

$$D \; d\pi_i + E \; dn + F \; dx = 0 \tag{60}$$

where matrices A, B, C, D, E and F are evaluated at symmetric equilibrium values, that can be computed from the system of equation (54)-(58) and are given below.

Computing dx from (60),

$$dx = -F^{-1} \left(D \; d\pi_i + E \; dn \right)$$

and substituting it into (59), yields:

$$M = \frac{\partial \pi_i}{\partial n} = -(-CF^{-1}D + A)^{-1}(-C \, F^{-1}E + B)$$

Long run symmetric equilibrium values, which can be obtained only if technological development levels are equal in the two regions, $a_n = a_s = 1$, are:

$w_{lr} = 1; \qquad \pi_{ir} = 0; \qquad H_r = \frac{\bar{H}}{2}; \qquad w_{hr} = \frac{2\gamma\mu_c \bar{L}}{\bar{H}(1-\mu-\gamma\,\mu_c)};$

$n_r = (1+\tau^{1-\sigma})^{-\frac{\mu}{1-\sigma+\mu\sigma}} \left(\frac{\mu_c \bar{L}}{1-\mu_c\gamma-\mu}\right)^{\frac{(1-\sigma)(1-\mu-\gamma)}{1-\sigma+\mu\sigma}} \left(\frac{2\gamma}{\bar{H}}\right)^{\frac{\gamma(\sigma-1)}{1-\sigma+\mu\sigma}};$

$p_{mr} = (1+\tau^{1-\sigma})^{\frac{1}{1-\sigma+\mu\sigma}} \left(\frac{\mu_c \bar{L}}{1-\mu_c\gamma-\mu}\right)^{\frac{1-\gamma\sigma}{1-\sigma+\mu\sigma}} \left(\frac{2\gamma}{\bar{H}}\right)^{-\frac{\gamma\sigma}{1-\sigma+\mu\sigma}}$

where $r = n,\ s$.

Solutions are positive for $\mu_c < \frac{1-\mu}{\gamma}$.