When skilled workers are employed in both regions, we know from Krugman (1991b) that regional real wages of skilled workers are equal at the long run equilibrium:

$$\frac{w_{hn}}{p_{mn}^{\mu_c}} = \frac{w_{hs}}{p_{ms}^{\mu_c}} \tag{18}$$

Moreover, the full employment of skilled workers requires that:

$$\bar{H} = H_n + H_s \tag{19}$$

Total wages of skilled workers in region r are equal to the share of total production costs of the region:

$$H_r w_{hr} = \gamma TC_{mir} n_r \tag{20}$$

Finally, by equating total unskilled workers' demand from the agricultural and the manufacturing sector to their regional supply, we obtain the market clearing condition for unskilled workers in region r:

$$w_{lr}\bar{L} = (1-\gamma-\mu)TC_{mir}n_r + (1-\mu_c)\,\lambda_r(w_{hr}H_r + w_{lr}\bar{L} + n_r\pi_{ir}) \tag{21}$$

$$+\,(1-\mu_c)\,(1-\lambda_v)(w_{hv}H_v + w_{lv}\bar{L} + n_v\pi_{iv})$$

where λ_r and λ_v are, respectively, the shares of agricultural expenditure devoted to domestic production by residents in region r and v, with $r \neq v$.

3 Technological evolution

In this paper we want to investigate what the interregional distribution of the economic activity becomes if interregional knowledge spill-overs take place only when the initial technological gap is not too wide, and when trade costs, taken as a proxy for the obstacles to interaction between firms of different regions, are sufficiently low. Therefore, the critical force lies in the ability of firms located in the receiving regions to use the flow of additional knowledge.

In fact, Verspagen (1991, p. 362-363) points out that the learning abilities of a lagging region or country "depend both on an intrinsic capability, and on its technological distance from the leading country". Furthermore, he maintains that the intrinsic learning capability "is determined by a mixture of social factors (Abramovitz, 1986), education of the workforce (Baumol et al., 1989), the level of the infrastructure, the level of capitalization (mechanization) of the economy, the correspondence of the sectorial mix of production in the leading and following country (Pasinetti, 1981), and other factors."

However, we argue that learning through knowledge spill-overs processes is also enhanced when firms in the less developed regions have more opportunities to observe and learn how all the different phases of production are conducted by firms active in the more developed regions. We believe that such an observation is more likely to occur when the level of integration is higher because natural and artificial barriers to trade are lower. Thus, the productivity of firms producing in a less developed region may be increased through a process of *learning by interacting* with firms that produce in the more developed region. Since knowledge spill-overs do not take place automatically, we find it reasonable to assume that their chances to occur increase when trade costs are "small", while knowledge spill-overs fail to take place when trade costs are "high". Therefore, low trade costs act as a stabilizing force because they favor knowledge spill-overs.[3]

In fact, recent empirical works, such as that by Coe and Helpman (1995) and Keller (2001a,b), illustrate the importance of trade as a mechanism of international knowledge spill-overs.[4]

Particularly, Keller (2001a, p. 5) finds that, for manufacturing industries in the world's seven major industrialized countries during the years between 1970 and 1995, "the scope for knowledge spillovers is severely limited by distance". Furthermore, Keller finds (2001a, p. 1) that "trade patterns account for the majority of all differences in bilateral spillover flows, whereas foreign

[3] Also Baldwin and Forslid (2000) consider knowledge spill-overs as a stabilizing force, but they assume that their size can be determined by policy makers. In particular, they assume that knowledge spill-overs increase when integration takes place through a lowering of the cost of trading information, and that knowledge spill-overs are independent from trade costs viewed as the cost of trading goods. Hence, while in our model, high trade costs entail null knowledge spill-overs, and in this case act as a destabilizing force, Baldwin and Forslid show that high knowledge spill-overs may stabilize the symmetric equilibrium even if trade costs are high.

[4] The theoretical models on which are built these empirical works are the innovation-driven growth models by Grossman and Helpman (1991), Romer (1990) and Aghion and Howitt (1992). Moreover, Keller (2001a) refers also to the New Economic Geography contributions by Krugman (1991a,b) and Fujita, Krugman and Venables (1999).

direct investments and communications flow differences account for circa 15% each", and that "these three channels together account for almost the entire localization effect that would be otherwise attributed to geographic distance".

In order to illustrate the fact that trade acts as a channel through which knowledge spill-overs take place, we assume that learning capabilities ψ depend upon trade costs. Specifically, we assume that when trade costs are above a certain threshold value $\bar{\tau}$, firms in the lagging region are unable to assimilate any of the potential knowledge spill-overs from the leading region, so that the actual learning capabilities ψ of this region are equal to zero. However, when trade costs are below $\bar{\tau}$, the region's learning capabilities rise as trade costs fall. This leads us to assume that

$$\psi(\tau) = \begin{cases} c(\bar{\tau} - \tau) & \text{if} \quad \tau \leqslant \bar{\tau} \\ 0 & \text{if} \quad \tau > \bar{\tau} \end{cases} \tag{22}$$

where $c > 0$ is a parameter that represents the influence on learning abilities of all other above-mentioned factors. For simplicity, we assume that there are no interregional differences in these other factors so that c is the same across regions.

In order to describe how the learning ability affects the production activity in the lagging regions, we assume that the technological level depends on the learning capabilities and on the technological gap between the two regions through the following dynamic equation:

$$\dot{a}_s = \left[(a_s - 1)^3 + \psi\,(1 - a_s) \right] \tag{23}$$

where $a_s \geq 0$.

This specification describes the fact that the technological advantage of a region tends to increase over time - following cumulative processes, that we consider exogenous in this paper - unless the technological gap between the two regions can be closed thanks to interregional learning capabilities, represented by positive values of ψ.[5] Furthermore, equation (23) takes into account the fact that when learning capabilities are small, even though they are positive, firms in the

[5] See Dosi (1988), for instance, that maintains that the technological advantage of a region tends to increase along a technological trajectory because leading regions tend to growth faster.

lagging region may recover their technological lag only when it is not too wide. In fact, when the technological gap is very wide, the amount of knowledge spill-overs required by firms in the lagging region to catch up is very wide. And if this is not the case, because trade costs are high, the lagging region will definitively fall behind.

For the given normalization (7), the north (south) is the technological leader, while the south (north) is the lagging region, when $a_s < 1$ ($a_s > 1$).

Three equilibrium values for a_s, when $\psi > 0$, are:

$$a_s = 1 \qquad a_s = 1 - \sqrt{\psi} \qquad a_s = 1 + \sqrt{\psi}$$

Thus, one of the possible equilibria for equation (23) is given by the symmetric equilibrium, which is characterized by identical regional levels of technology ($a_s = a_n = 1$).[6] In Figure 1, we plot equation (23) when $\psi = 0.5$.

The intercept of the function (23) with the vertical axis is given by $\psi - 1$. Therefore, we may observe that $a_s = 0$ is a *stable* equilibrium with no firm producing the modern good in the south if learning capabilities of this region are very low ($\psi < 1$). In this case, for a given value of c, trade costs are too high ($\tau > \bar{\tau} - 1/c$) to allow firms in the south to assimilate technology spill-overs from the north. When this is so, the technological advantage of the north continuously increases over time.

The "symmetric" equilibrium characterized by $a_s = a_n = 1$ is *stable* if the slope of (23) is negative in a neighborhood of this point, that is, if $\psi > 0$. In this case, trade costs are low enough ($\tau < \bar{\tau}$) to allow firms in the receiving regions to assimilate technology spill-overs.

Insert Figure 1 about here

When we consider only the dynamic equation (23), the symmetric equilibrium is stable only if learning abilities in both regions are positive, that is, if $\psi > 0$. Moreover, when learning

[6] Equilibrium values can also be considered as *steady state equilibrium* values with positive and equal, exogenous growth rate of the technological level. In fact, function (23) expresses relative technological development since the normalization $a_n = 1$ has been adopted.

capabilities are high enough, namely when $\psi > 1$, the symmetric equilibrium is stable for any initial value of $a_s < 1$. Figure 2 shows this case when $\psi = 1.5$.

Insert Figure 2 about here

However, when learning abilities are positive but not too high because trade costs are not low enough, the lagging region may benefit from interregional knowledge spill-overs provided that its technological lag is not too wide. In fact, when the lagging region is the south ($a_s < 1$), firms in this region may benefit from knowledge spill-overs only when the level of the technology of this region is not too low, namely $a_s > 1 - \sqrt{\psi}$. In other words, firms in the south may recover their lag only if the technological gap $(1 - a_s)$ from the leading region is smaller than $\sqrt{\psi}$. On the contrary, when the technological leading region is the south ($a_s > 1$), firms in the northern region may recover their lag, thanks to knowledge spill-overs from the south, only when the technological lag is not too wide for the given learning abilities, that is, if $a_s - 1 < \sqrt{\psi}$. This two conditions taken together entail that the symmetric equilibrium $a_s = a_n = 1$ is a stable equilibrium for expression (23), when $\psi > 0$, only if for a given initial value of the technology level, a_s^0, we have that:

$$1 - \sqrt{\psi} < a_s^0 < \sqrt{\psi} + 1$$

The width of the recoverable lag increases (decreases) when learning capabilities increase (decrease), namely when the economic integration between the two regions becomes higher (lower). In short, when the south is the lagging region ($a_s < 1$), the following three cases may occur for respectively high, low or intermediate trade costs.

Case 1 $\tau > \bar{\tau}$. *When trade costs are too high, the symmetric equilibrium can never be reached because firms in the lagging region cannot benefit from technology spill-overs from the leading region, given the low level of integration.*

Case 2 $\tau < \bar{\tau} - 1/c$. *When trade costs are low, firms in the lagging region can successfully exploit potential technology spill-overs from the leading region and the symmetric equilibrium is stable.*

Case 3 $\bar{\tau} - 1/c < \tau < \bar{\tau}$. *For intermediate trade costs the process of catching up of the lagging region with the leading region may be completed because trade is sufficiently developed to allow firms in the lagging region to interact with the most productive firms in the leading region. However, this happens only when the technological gap between the two regions is not too wide for the given learning abilities. In other words, when:* $1 - a_s^0 < \sqrt{\psi}$.