2 The model

We consider a two-region economy ($r = n, s$) in which there are two types of workers, skilled (h) and unskilled (l), indexed by j ($j = h, l$). The case $r = n$ corresponds to the "north" and $r = s$ to the "south". Skilled and unskilled workers have identical preferences and consume a homogenous (traditional or agricultural) good, and several varieties of a (modern or manufactured) good. Varieties of the modern good are produced under increasing returns and sold in markets characterized by monopolistic competition. The agricultural good is produced under constant returns by using only unskilled workers.

The total number of unskilled workers living in each region is the same and equal to $\bar{L}$. Let $\bar{H}$ be the total number of skilled workers. Skilled workers are employed only in the manufacturing sector, and are perfectly mobile between the two regions. Hence, if they are employed in the two regions, we know from Krugman (1991b) that their regional real wages must be equal. Unskilled workers are interregionally immobile but are intersectorally mobile because they may be employed in both sectors.

The production of any variety requires the use of all varieties of the manufactured good as intermediate inputs. Trading the manufactured good between the two regions is costly, while the traditional good is traded without cost. Finally, the efficiency of the technology available for producing the manufactured good may differ between the two regions. Furthermore, knowledge spill-overs are present across regions.

2.1 Preferences and demand

A consumer in region r maximizes a Cobb-Douglas utility function:

$$U(Q_{mr}, Q_{ar}) = Q_{mr}^{\mu_c} Q_{ar}^{1-\mu_c} \tag{1}$$

where Q_{mr} is the quantity of the composite manufactured good and Q_{ar} is the quantity of the agricultural good he/she consumes. The parameter μ_c is the share of consumers' expenditure on manufactures with $0 < \mu_c < 1$.

The budget constraint is given by:

$$p_{mr}Q_{mr} + p_{ar}Q_{ar} = y_{jr} \tag{2}$$

where y_{jr} $(j = l, h)$ is the income of a worker of type j in region r. While unskilled workers' income in region r is given only by their wage (w_{lr}), skilled workers' income is given by the sum of their wage (w_{hr}) and of a share in the profits earned by manufacturing firms located in their region. This is because the total number of firms has not yet reached a long run equilibrium value that ensures zero profits to all firms in the market. As will be seen below, the number of firms in each region reaches its equilibrium value more slowly than all other variables.

Let p_{ar} be the price of the agricultural good in region r, while p_{mr} is the price index of manufactured good in region r. The composite Q_{mr} consumption good is obtained by aggregating the different varieties of the manufacturing good by means of a constant elasticity of substitution sub-utility function:

$$Q_{mr} = \left(\int_{i=1}^{n_n+n_s} Q_{mir}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution between any pair of varieties, whereas n_n and n_s are, respectively, the mass of firms in the north and in the south.

Because all manufacturing firms in a particular region are homogenous, the price index of the composite good in region r is:

$$p_{mr} = \left[n_r p_r^{1-\sigma} + n_v (p_v \tau)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{3}$$

where $r, v = n, s$ and $v \neq r$.

Maximizing (1) under the budget constraint (2), we obtain the demand schedules for the agricultural good and for the composite manufactured good. Total consumers' expenditure on the manufactured good in region r (E_{mcr}) is given by:

$$E_{mcr} = \mu_c (w_{hr} H_r + w_{lr} \bar{L} + n_r \pi_{ir}) \tag{4}$$

where H_r and $\bar{L}$ are, respectively, the number of mobile skilled workers and the number of immobile workers employed in region r, while π_{ir} are profits realized by each firm in the same region. However, in order to determine the total expenditure on the manufactured good in region r (E_{mr}), we must also consider firms expenditure (E_{mmr}):

$$E_{mr} = E_{mcr} + E_{mmr} \tag{5}$$

2.2 Manufacturing sector

Each firm produces under increasing returns one variety of a differentiated good because consumers have a preference for variety, and because there are no scope economies. A firm's output can be either directed towards local demand, or exported according to an iceberg trade cost. Firms in each region are homogenous and their mass n_r is endogenously determined. Producing the quantity Q_{mir} of each variety i requires both a fixed and a variable amount of a composite productive input I_{mir}, which is defined below. While the fixed amount α is the same in the two regions, the variable amount β/a_r may differ. Specifically, the more productive region is characterized by a higher value of a_r.

The production function of the firm supplying variety i is given by:

$$Q_{mir} = \frac{a_r(I_{mir} - \alpha)}{\beta} \tag{6}$$

where $\alpha, \beta, a_r > 0$. The magnitude a_r reflects the level of development of the technology in region r. For notational simplicity, we normalize the value of a_r in region n to 1:

$$a_n = 1 \tag{7}$$

In the same spirit as Krugman's and Venables' (1995), we define the input I_{mir} as a Cobb-Douglas composite of three production factors: two primary factors (skilled and unskilled workers) and one intermediate factor (the manufactured composite good D_{mir}):

$$I_{mir} = \frac{H_{mir}^{\gamma} L_{mir}^{1-\gamma-\mu} D_{mir}^{\mu}}{(1-\gamma-\mu)^{1-\gamma-\mu}\mu^{\mu}\gamma^{\gamma}} \tag{8}$$

where $0 < \mu < 1$, $0 < \gamma < 1$ and $1 - \gamma - \mu > 0$.

For simplicity, we assume that the elasticity of substitution with which varieties enter consumers' utility function is the same as that with which they enter firms' production function. Therefore, the composite good D_{mir} demanded by each firm i has the same shares of the composite good Q_{mr} demanded by consumers.

Finally, the minimization of the total cost of production under the constraint given by (6) and (8) yields the cost function of the firm supplying variety i:

$$TC_{mir} = p_{mr}^{\mu} w_{hr}^{\gamma} w_{lr}^{1-\gamma-\mu} \left(\frac{\beta}{a_r} Q_{mir} + \alpha \right) \tag{9}$$

Since μ, γ and $1 - \gamma - \mu$ respectively represent the shares of firms' total production costs devoted to the composite good, to skilled and unskilled workers, firms' total expenditure on manufactures in region r (E_{mmr}) is given by:

$$E_{mmr} = \mu n_r TC_{mir} \tag{10}$$

2.3 Equilibrium

The traditional good is produced under constant returns, employing only unskilled workers:

$$Q_{ar} = L_{ar}$$

Given the assumptions of perfect competition and absence of trade costs, the price of this good is equal to the wage of unskilled workers:

$$p_{ar} = w_{lr}$$

Moreover, when the traditional good is produced in both regions, it is sold at the same price, and wages received by unskilled workers in the two regions are equal. The traditional good is chosen as the numéraire: $p_{ar} = 1$.

Consumers' and firms' demand functions in region r for varieties produced in the same region or imported from region v, are obtained by the minimization of the expenditure on manufacturing goods under the constraint given by the aggregation of all varieties in the composite good. Let us consider, for the moment, prices and quantities regardless of the location of production. So, we must solve the following problem:

$$\min_{Q_{mi}} \int_{i=1}^{n} p_i Q_{mi} di \qquad s.t. \qquad Q_m = \left(\int_{i=1}^{n} Q_{mi}^{\rho} di \right)^{\frac{1}{\rho}}$$

where $0 < \rho = \frac{\sigma - 1}{\sigma} < 1$. p_i is the price of variety i, and ρ represents the intensity of the preference for variety in manufactured goods.

The solution of the constrained minimization problem yields the following demand function:

$$Q_{mi} = \frac{p_i^{-\sigma}}{p_m^{1-\sigma}} E_m \tag{11}$$

where the total expenditure on manufacturing goods (E_m) is:

$$E_m = p_m Q_m$$

However, we should notice that the existence of trade costs implies that the price of an imported variety is higher than the one paid in the region in which it is produced. More precisely, in order to import one unit of a variety, $\tau > 1$ units have to be shipped. Therefore, the demand function for the locally produced variety i in region r (Q_{mir}^{rd}) is given by:

$$Q_{mir}^{rd} = \frac{p_{ir}^{-\sigma}}{p_{mr}^{1-\sigma}} E_{mr}$$

Similarly, the demand function for the imported variety i produced in region v (Q_{miv}^{rd}) is given by:

$$Q_{miv}^{rd} = \frac{(\tau p_{iv})^{-\sigma}}{p_{mr}^{1-\sigma}} \tau E_{mr}$$

where $r, v = n, s$ and $v \neq r$. Hence, the aggregate demand (Q_{mir}^{d}) for a firm producing variety i in region r is:

$$Q_{mir}^{d} = Q_{mir}^{rd} + Q_{mir}^{vd} = p_{ir}^{-\sigma} \left(\frac{1}{p_{mr}^{1-\sigma}} E_{mr} + \frac{\tau^{1-\sigma}}{p_{mv}^{1-\sigma}} E_{mv} \right) \tag{12}$$

Each firm producing in region r maximizes profits (π_{ir}) under the aggregate demand function (12). It is readily verified that each firm sets a price with a constant mark-up over the marginal cost MC_{mir}:

$$p_{ir} = \frac{\sigma}{\sigma - 1} MC_{mir} = \left(\frac{\sigma \beta}{(\sigma - 1) a_r} \right) p_{mr}^{\mu} w_{hr}^{\gamma} w_{lr}^{1-\gamma-\mu} \tag{13}$$

Thereafter we will drop the suffix i from the price of each variety produced in region r because all firms in a particular region are assumed to be equal.

Profits of a firm producing in region r are given by:

$$\pi_{ir} = p_{mr}^{\mu} w_{hr}^{\gamma} w_{lr}^{1-\gamma-\mu} \left(\frac{\beta}{(\sigma-1)\, a_r} Q_{mir} - \alpha \right) \tag{14}$$

Following Puga (1999), without loss of generality, the values of α and β can be chosen as follows:

$$\alpha = \frac{1}{\sigma} \quad \text{and} \quad \beta = \frac{\sigma-1}{\sigma}$$

so that profits may be rewritten as follows:

$$\pi_{ir} = p_{mr}^{\mu} w_{hr}^{\gamma} w_{lr}^{1-\gamma-\mu} \frac{1}{\sigma} \left(\frac{1}{a_r} Q_{mir} - 1 \right) \tag{15}$$

The manufacturing sector is characterized by monopolistic competition. When there is free entry and exit, profits are equal to zero at the long run equilibrium. Therefore, the long run equilibrium price of each variety is equal to the average cost of production AC_{mir}:

$$p_r = \frac{TC_{mir}}{Q_{mir}} = AC_{mir} \tag{16}$$

Equation (13) together with equation (16) allows us to compute the long run equilibrium size of each firm in region r:

$$Q_{mir}^{*} = \frac{\alpha a_r (\sigma - 1)}{\beta} = a_r$$

Given the normalization above, profits are equal to zero for northern and southern firms when their production levels are respectively:

$$Q_{\min}^{*} = 1 \qquad \text{and} \qquad Q_{mis}^{*} = a_s \tag{17}$$

The free entry and exit condition implies that the following expression must be satisfied at a long run equilibrium:

$$n_r \pi_{ir} = 0 \qquad \pi_{ir} \leqslant 0 \qquad n_r \geq 0$$

When skilled workers are employed in both regions, we know from Krugman (1991b) that regional real wages of skilled workers are equal at the long run equilibrium:

$$\frac{w_{hn}}{p_{mn}^{\mu_c}} = \frac{w_{hs}}{p_{ms}^{\mu_c}} \tag{18}$$

Moreover, the full employment of skilled workers requires that:

$$\bar{H} = H_n + H_s \tag{19}$$

Total wages of skilled workers in region r are equal to the share of total production costs of the region:

$$H_r w_{hr} = \gamma TC_{mir} n_r \tag{20}$$

Finally, by equating total unskilled workers' demand from the agricultural and the manufacturing sector to their regional supply, we obtain the market clearing condition for unskilled workers in region r:

$$w_{lr}\bar{L} = (1-\gamma-\mu)TC_{mir}n_r + (1-\mu_c)\,\lambda_r(w_{hr}H_r + w_{lr}\bar{L} + n_r\pi_{ir}) \tag{21}$$

$$+\,(1-\mu_c)\,(1-\lambda_v)(w_{hv}H_v + w_{lv}\bar{L} + n_v\pi_{iv})$$

where λ_r and λ_v are, respectively, the shares of agricultural expenditure devoted to domestic production by residents in region r and v, with $r \neq v$.

3 Technological evolution

In this paper we want to investigate what the interregional distribution of the economic activity becomes if interregional knowledge spill-overs take place only when the initial technological gap is not too wide, and when trade costs, taken as a proxy for the obstacles to interaction between firms of different regions, are sufficiently low. Therefore, the critical force lies in the ability of firms located in the receiving regions to use the flow of additional knowledge.

In fact, Verspagen (1991, p. 362-363) points out that the learning abilities of a lagging region or country "depend both on an intrinsic capability, and on its technological distance from the leading