1 Introduction

The "New Economic Geography" literature, which has flourished in the last decade, describes how the interactions of centripetal and centrifugal forces determine the locational decisions of firms and workers between two or more regions involved in trade. The interactions of these forces endogenously determine the size and the productivity of the regional economies. The market outcome is typically affected by the degree of integration among regions.

Increasing returns play a major role in these models that assume decreasing costs of production within each firm. Moreover, pecuniary externalities arise because of the assumptions of increasing returns at the firm level and trade costs in the manufacturing (or modern) sector. Pecuniary externalities induce mobile agents, workers or firms, to move towards regions where the size of the manufacturing sector is bigger. In this way, either consumers (if they demand goods produced in the modern sector) or firms (if they use these goods as production factors), may reduce the share of goods on which trade costs should be paid, if they did not move, and agents had to import them from other regions. However, each manufacturing firm (or consumer) that moves where pecuniary economies are larger, increases the incentive for its customer firms and workers to move in the same direction. These movements, in turn, increase the size of the region of destination and, therefore, the incentive for other firms and consumers to move towards the same region. Hence, concepts such as "backward and forward linkages" (Hirshman, 1958) or "cumulative causation" (Myrdal, 1957) turn out to be fundamental in this body of literature.

Centripetal forces, which favor cumulative causation and, therefore, a spatial concentration of the sector with increasing returns, are generated by three main factors: (1) workers' mobility when the final sector exhibits increasing returns (Krugman, 1991b); (2) backward and forward linkages between firms producing intermediate and final goods, when intermediate goods are produced under increasing returns (Venables, 1996);[1] (3) technological advantage of production in a

[1] Fujita and Thisse (2000a) point out that the assumption of the existence of an imperfectly competitive intermediate sector is sufficient to lead to a core-periphery structure.

particular region (Baldwin and Forslid, 2000). On the contrary, centrifugal forces are generated by: (1) immobile demand sources (such as that generated by immobile workers); (2) stronger competition for limited productive factors, and in good markets for firms that operate in core regions; (3) technological knowledge spill-overs from regions with a more productive modern sector towards less developed regions. Whenever centripetal forces are stronger than centrifugal forces, the modern sector tends to be completely agglomerated in one region, while a uniform distribution of the economic activity emerges when centrifugal forces are stronger.

Moreover, the interest in location and economic growth issues has recently led many economists to investigate how the main conclusions of the New Economic Geography literature can be affected when pecuniary externalities interact with the dynamic and external economies of scale introduced in New Endogenous Growth models by Romer (1990) and Grossman and Helpman (1991). Waltz (1996), Martin and Ottaviano (1999) and Fujita and Thisse (2000b), introduce dynamic economies of scale in New Economic Geography models by means of R&D activities, while Baldwin and Forslid (2000) introduce them by means of capital formation processes.[2] The general result of this kind of model is that the dynamic economies of scale tend to strengthen centripetal forces. Nevertheless, as Fujita and Thisse (2000b) point out "even those that stay put in the periphery are better off than under dispersion provided that the growth effect triggered by the agglomeration is strong enough".

Puga and Venables (1999, p. 292) observe that the "economic development may not be a gradual process of convergence by all countries, but instead involve countries moving sequentially from the group of poor countries to the group of rich countries". They show that an exogenous productivity increase of all primary factors strengthens centripetal forces in developed countries by

[2] Moreover, Martin and Ottaviano (1996) analyze the effects of pecuniary externalities arising among firms producing manufacturing goods under increasing returns that are costly traded, and innovative firms that use manufacturing goods to produce new patents. New patents are then acquired by manufacturing firms with a fixed cost. Hence, Martin and Ottaviano (1996) show that growth is more sustained, and agglomeration is stronger when the market size is wider, the share of the differentiated good demanded by consumers is higher and when labor demand, the elasticity of substitution between any pair of varieties, trade costs, innovation costs and the subjective rate of discount are smaller.

increasing immobile workers' wages. In turn, this may lead some firms to start their production in a less developed country, where wages are lower. Besides, Puga and Venables assume that firms that start their production in newly industrialized countries may adopt the same technology as that used by firms in the leading countries. In other words, they do not focus on technological differences. By contrast, in this paper we want to stress that when there are technological differences, the lagging regions may not always be able to catch up with the leading ones, even though there are "potential" technological knowledge spill-overs. In fact, we will see that some conditions must be satisfied before there can be a process of catching up, while *potential technological knowledge spill-overs do not take place automatically towards firms in a lagging region*. In this respect, we concur with Verspagen (1991, p. 361) when he claims:

> "The basic (implicit) intuition behind the convergence hypothesis seems to be that international knowledge spill-overs take place automatically. In the (economic) literature dealing with the nature of technological change in more detail (e.g. Dosi, 1988) it is argued that this assumption is indeed a heroic one. Since the process of (international) technology spill-over is essentially a process of adoption of new techniques at the microeconomic (firm) level, the capabilities of the "receiving" country (firm) to "assimilate" (foreign) technological knowledge are critical to the success of diffusion. If countries (firms) do not have the relevant capabilities to assimilate new knowledge, spill-overs may not take place at all."

The purpose of this paper is threefold. First, we want to show how the above-mentioned forces can interact when workers have different employment opportunities. Specifically, two types of workers are considered: skilled and unskilled. Skilled workers are interregionally mobile and employed by the manufacturing (also called modern) sector characterized by increasing returns at the firm level; unskilled workers are not interregionally mobile but are intersectorally mobile because they can be employed in both the modern and the traditional sectors. Backward and forward linkages exist between manufacturing firms producing final and intermediate goods.

Second, we want to show that a richer analysis of the interactions of all the above-mentioned forces can be conducted when the regional levels of the technological development of the modern sector may differ and change over time. As a consequence, in our model we allow explicitly for differences in the regional levels of the technology.

Third, we want to account for the fact that the lagging regions are not always able to catch up completely with the leading regions. Here, a complete catch up can be achieved when (i) the technological gap between the two regions is not too wide and (ii) firms in the lagging region have enough opportunities to *learn by interacting* (e.g. watching) the technologies used by firms in the leading region. In particular, this may occur only if the lagging regions' learning capabilities to assimilate potential technological spill-overs are sufficiently large. The chances of benefiting from these spill-overs depends on the opportunity to interact with firms operating in the leading regions. Since this opportunity is higher when regions are more integrated, *our work stresses the relevance of trade costs levels - as a proxy for the difficulty of interacting - in allowing a successful process of catching up.* More precisely, we represent trade costs by iceberg costs that are particularly suitable to describe the cost of the "distance" between any two regions, as well as the cost of all other natural and artificial barriers to trade. Therefore, while knowledge spill-overs may take place from a leading region towards a neighbouring lagging region, they fail to occur if the lagging region is very far, because its firms have less opportunities to interact with firms in the more developed region.

The model is presented in section 2, while section 3 deals with the necessary conditions for a complete process of catching up to occur. In section 4, the equilibria for given levels of the technology in the two regions are analyzed. More precisely, agglomeration equilibria are studied in 4.1, while the symmetric equilibrium is analyzed in 4.2. Using new dynamics based on technological spill-overs, we discuss the different equilibria in section 5, where it is shown that the level of trade costs is critical in determining whether a catching up process can be successfully completed or not. Section 6 gives the conclusions.

2 The model

We consider a two-region economy ($r = n, s$) in which there are two types of workers, skilled (h) and unskilled (l), indexed by j ($j = h, l$). The case $r = n$ corresponds to the "north" and $r = s$ to the "south". Skilled and unskilled workers have identical preferences and consume a homogenous (traditional or agricultural) good, and several varieties of a (modern or manufactured) good. Varieties of the modern good are produced under increasing returns and sold in markets characterized by monopolistic competition. The agricultural good is produced under constant returns by using only unskilled workers.

The total number of unskilled workers living in each region is the same and equal to $\bar{L}$. Let $\bar{H}$ be the total number of skilled workers. Skilled workers are employed only in the manufacturing sector, and are perfectly mobile between the two regions. Hence, if they are employed in the two regions, we know from Krugman (1991b) that their regional real wages must be equal. Unskilled workers are interregionally immobile but are intersectorally mobile because they may be employed in both sectors.

The production of any variety requires the use of all varieties of the manufactured good as intermediate inputs. Trading the manufactured good between the two regions is costly, while the traditional good is traded without cost. Finally, the efficiency of the technology available for producing the manufactured good may differ between the two regions. Furthermore, knowledge spill-overs are present across regions.

2.1 Preferences and demand

A consumer in region r maximizes a Cobb-Douglas utility function:

$$U(Q_{mr}, Q_{ar}) = Q_{mr}^{\mu_c} Q_{ar}^{1-\mu_c} \tag{1}$$

where Q_{mr} is the quantity of the composite manufactured good and Q_{ar} is the quantity of the agricultural good he/she consumes. The parameter μ_c is the share of consumers' expenditure on manufactures with $0 < \mu_c < 1$.

The budget constraint is given by:

$$p_{mr}Q_{mr} + p_{ar}Q_{ar} = y_{jr} \tag{2}$$

where y_{jr} $(j = l, h)$ is the income of a worker of type j in region r. While unskilled workers' income in region r is given only by their wage (w_{lr}), skilled workers' income is given by the sum of their wage (w_{hr}) and of a share in the profits earned by manufacturing firms located in their region. This is because the total number of firms has not yet reached a long run equilibrium value that ensures zero profits to all firms in the market. As will be seen below, the number of firms in each region reaches its equilibrium value more slowly than all other variables.

Let p_{ar} be the price of the agricultural good in region r, while p_{mr} is the price index of manufactured good in region r. The composite Q_{mr} consumption good is obtained by aggregating the different varieties of the manufacturing good by means of a constant elasticity of substitution sub-utility function:

$$Q_{mr} = \left(\int_{i=1}^{n_n+n_s} Q_{mir}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution between any pair of varieties, whereas n_n and n_s are, respectively, the mass of firms in the north and in the south.

Because all manufacturing firms in a particular region are homogenous, the price index of the composite good in region r is:

$$p_{mr} = \left[n_r p_r^{1-\sigma} + n_v (p_v \tau)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{3}$$

where $r, v = n, s$ and $v \neq r$.

Maximizing (1) under the budget constraint (2), we obtain the demand schedules for the agricultural good and for the composite manufactured good. Total consumers' expenditure on the manufactured good in region r (E_{mcr}) is given by:

$$E_{mcr} = \mu_c (w_{hr} H_r + w_{lr} \bar{L} + n_r \pi_{ir}) \tag{4}$$

where H_r and $\bar{L}$ are, respectively, the number of mobile skilled workers and the number of immobile workers employed in region r, while π_{ir} are profits realized by each firm in the same region. However, in order to determine the total expenditure on the manufactured good in region r (E_{mr}), we must also consider firms expenditure (E_{mmr}):

$$E_{mr} = E_{mcr} + E_{mmr} \tag{5}$$

2.2 Manufacturing sector

Each firm produces under increasing returns one variety of a differentiated good because consumers have a preference for variety, and because there are no scope economies. A firm's output can be either directed towards local demand, or exported according to an iceberg trade cost. Firms in each region are homogenous and their mass n_r is endogenously determined. Producing the quantity Q_{mir} of each variety i requires both a fixed and a variable amount of a composite productive input I_{mir}, which is defined below. While the fixed amount α is the same in the two regions, the variable amount β/a_r may differ. Specifically, the more productive region is characterized by a higher value of a_r.

The production function of the firm supplying variety i is given by:

$$Q_{mir} = \frac{a_r(I_{mir} - \alpha)}{\beta} \tag{6}$$

where $\alpha, \beta, a_r > 0$. The magnitude a_r reflects the level of development of the technology in region r. For notational simplicity, we normalize the value of a_r in region n to 1:

$$a_n = 1 \tag{7}$$

In the same spirit as Krugman's and Venables' (1995), we define the input I_{mir} as a Cobb-Douglas composite of three production factors: two primary factors (skilled and unskilled workers) and one intermediate factor (the manufactured composite good D_{mir}):

$$I_{mir} = \frac{H_{mir}^{\gamma} L_{mir}^{1-\gamma-\mu} D_{mir}^{\mu}}{(1-\gamma-\mu)^{1-\gamma-\mu}\mu^{\mu}\gamma^{\gamma}} \tag{8}$$

where $0 < \mu < 1$, $0 < \gamma < 1$ and $1 - \gamma - \mu > 0$.

For simplicity, we assume that the elasticity of substitution with which varieties enter consumers' utility function is the same as that with which they enter firms' production function. Therefore, the composite good D_{mir} demanded by each firm i has the same shares of the composite good Q_{mr} demanded by consumers.

Finally, the minimization of the total cost of production under the constraint given by (6) and (8) yields the cost function of the firm supplying variety i:

$$TC_{mir} = p_{mr}^{\mu} w_{hr}^{\gamma} w_{lr}^{1-\gamma-\mu} \left(\frac{\beta}{a_r} Q_{mir} + \alpha \right) \tag{9}$$

Since μ, γ and $1 - \gamma - \mu$ respectively represent the shares of firms' total production costs devoted to the composite good, to skilled and unskilled workers, firms' total expenditure on manufactures in region r (E_{mmr}) is given by:

$$E_{mmr} = \mu n_r TC_{mir} \tag{10}$$

2.3 Equilibrium

The traditional good is produced under constant returns, employing only unskilled workers:

$$Q_{ar} = L_{ar}$$

Given the assumptions of perfect competition and absence of trade costs, the price of this good is equal to the wage of unskilled workers:

$$p_{ar} = w_{lr}$$

Moreover, when the traditional good is produced in both regions, it is sold at the same price, and wages received by unskilled workers in the two regions are equal. The traditional good is chosen as the numéraire: $p_{ar} = 1$.

Consumers' and firms' demand functions in region r for varieties produced in the same region or imported from region v, are obtained by the minimization of the expenditure on manufacturing goods under the constraint given by the aggregation of all varieties in the composite good. Let us consider, for the moment, prices and quantities regardless of the location of production. So, we must solve the following problem:

$$\min_{Q_{mi}} \int_{i=1}^{n} p_i Q_{mi} di \qquad s.t. \qquad Q_m = \left(\int_{i=1}^{n} Q_{mi}^{\rho} di \right)^{\frac{1}{\rho}}$$

where $0 < \rho = \frac{\sigma - 1}{\sigma} < 1$. p_i is the price of variety i, and ρ represents the intensity of the preference for variety in manufactured goods.

The solution of the constrained minimization problem yields the following demand function:

$$Q_{mi} = \frac{p_i^{-\sigma}}{p_m^{1-\sigma}} E_m \tag{11}$$

where the total expenditure on manufacturing goods (E_m) is:

$$E_m = p_m Q_m$$

However, we should notice that the existence of trade costs implies that the price of an imported variety is higher than the one paid in the region in which it is produced. More precisely, in order to import one unit of a variety, $\tau > 1$ units have to be shipped. Therefore, the demand function for the locally produced variety i in region r (Q_{mir}^{rd}) is given by:

$$Q_{mir}^{rd} = \frac{p_{ir}^{-\sigma}}{p_{mr}^{1-\sigma}} E_{mr}$$

Similarly, the demand function for the imported variety i produced in region v (Q_{miv}^{rd}) is given by:

$$Q_{miv}^{rd} = \frac{(\tau p_{iv})^{-\sigma}}{p_{mr}^{1-\sigma}} \tau E_{mr}$$

where $r, v = n, s$ and $v \neq r$. Hence, the aggregate demand (Q_{mir}^{d}) for a firm producing variety i in region r is:

$$Q_{mir}^{d} = Q_{mir}^{rd} + Q_{mir}^{vd} = p_{ir}^{-\sigma} \left(\frac{1}{p_{mr}^{1-\sigma}} E_{mr} + \frac{\tau^{1-\sigma}}{p_{mv}^{1-\sigma}} E_{mv} \right) \tag{12}$$

Each firm producing in region r maximizes profits (π_{ir}) under the aggregate demand function (12). It is readily verified that each firm sets a price with a constant mark-up over the marginal cost MC_{mir}:

$$p_{ir} = \frac{\sigma}{\sigma - 1} MC_{mir} = \left(\frac{\sigma \beta}{(\sigma - 1) a_r} \right) p_{mr}^{\mu} w_{hr}^{\gamma} w_{lr}^{1-\gamma-\mu} \tag{13}$$

Thereafter we will drop the suffix i from the price of each variety produced in region r because all firms in a particular region are assumed to be equal.

Profits of a firm producing in region r are given by:

$$\pi_{ir} = p_{mr}^{\mu} w_{hr}^{\gamma} w_{lr}^{1-\gamma-\mu} \left(\frac{\beta}{(\sigma-1)\, a_r} Q_{mir} - \alpha \right) \tag{14}$$

Following Puga (1999), without loss of generality, the values of α and β can be chosen as follows:

$$\alpha = \frac{1}{\sigma} \quad \text{and} \quad \beta = \frac{\sigma-1}{\sigma}$$

so that profits may be rewritten as follows:

$$\pi_{ir} = p_{mr}^{\mu} w_{hr}^{\gamma} w_{lr}^{1-\gamma-\mu} \frac{1}{\sigma} \left(\frac{1}{a_r} Q_{mir} - 1 \right) \tag{15}$$

The manufacturing sector is characterized by monopolistic competition. When there is free entry and exit, profits are equal to zero at the long run equilibrium. Therefore, the long run equilibrium price of each variety is equal to the average cost of production AC_{mir}:

$$p_r = \frac{TC_{mir}}{Q_{mir}} = AC_{mir} \tag{16}$$

Equation (13) together with equation (16) allows us to compute the long run equilibrium size of each firm in region r:

$$Q_{mir}^{*} = \frac{\alpha a_r (\sigma - 1)}{\beta} = a_r$$

Given the normalization above, profits are equal to zero for northern and southern firms when their production levels are respectively:

$$Q_{\min}^{*} = 1 \qquad \text{and} \qquad Q_{mis}^{*} = a_s \tag{17}$$

The free entry and exit condition implies that the following expression must be satisfied at a long run equilibrium:

$$n_r \pi_{ir} = 0 \qquad \pi_{ir} \leqslant 0 \qquad n_r \geq 0$$

When skilled workers are employed in both regions, we know from Krugman (1991b) that regional real wages of skilled workers are equal at the long run equilibrium:

$$\frac{w_{hn}}{p_{mn}^{\mu_c}} = \frac{w_{hs}}{p_{ms}^{\mu_c}} \tag{18}$$

Moreover, the full employment of skilled workers requires that:

$$\bar{H} = H_n + H_s \tag{19}$$

Total wages of skilled workers in region r are equal to the share of total production costs of the region:

$$H_r w_{hr} = \gamma T C_{mir} n_r \tag{20}$$

Finally, by equating total unskilled workers' demand from the agricultural and the manufacturing sector to their regional supply, we obtain the market clearing condition for unskilled workers in region r:

$$w_{lr}\bar{L} = (1-\gamma-\mu) T C_{mir} n_r + (1-\mu_c)\,\lambda_r (w_{hr} H_r + w_{lr}\bar{L} + n_r \pi_{ir}) \tag{21}$$

$$+\,(1-\mu_c)\,(1-\lambda_v)(w_{hv} H_v + w_{lv}\bar{L} + n_v \pi_{iv})$$

where λ_r and λ_v are, respectively, the shares of agricultural expenditure devoted to domestic production by residents in region r and v, with $r \neq v$.

3 Technological evolution

In this paper we want to investigate what the interregional distribution of the economic activity becomes if interregional knowledge spill-overs take place only when the initial technological gap is not too wide, and when trade costs, taken as a proxy for the obstacles to interaction between firms of different regions, are sufficiently low. Therefore, the critical force lies in the ability of firms located in the receiving regions to use the flow of additional knowledge.

In fact, Verspagen (1991, p. 362-363) points out that the learning abilities of a lagging region or country "depend both on an intrinsic capability, and on its technological distance from the leading country". Furthermore, he maintains that the intrinsic learning capability "is determined by a mixture of social factors (Abramovitz, 1986), education of the workforce (Baumol et al., 1989), the level of the infrastructure, the level of capitalization (mechanization) of the economy, the correspondence of the sectorial mix of production in the leading and following country (Pasinetti, 1981), and other factors."

However, we argue that learning through knowledge spill-overs processes is also enhanced when firms in the less developed regions have more opportunities to observe and learn how all the different phases of production are conducted by firms active in the more developed regions. We believe that such an observation is more likely to occur when the level of integration is higher because natural and artificial barriers to trade are lower. Thus, the productivity of firms producing in a less developed region may be increased through a process of *learning by interacting* with firms that produce in the more developed region. Since knowledge spill-overs do not take place automatically, we find it reasonable to assume that their chances to occur increase when trade costs are "small", while knowledge spill-overs fail to take place when trade costs are "high". Therefore, low trade costs act as a stabilizing force because they favor knowledge spill-overs.[3]

In fact, recent empirical works, such as that by Coe and Helpman (1995) and Keller (2001a,b), illustrate the importance of trade as a mechanism of international knowledge spill-overs.[4]

Particularly, Keller (2001a, p. 5) finds that, for manufacturing industries in the world's seven major industrialized countries during the years between 1970 and 1995, "the scope for knowledge spillovers is severely limited by distance". Furthermore, Keller finds (2001a, p. 1) that "trade patterns account for the majority of all differences in bilateral spillover flows, whereas foreign

[3] Also Baldwin and Forslid (2000) consider knowledge spill-overs as a stabilizing force, but they assume that their size can be determined by policy makers. In particular, they assume that knowledge spill-overs increase when integration takes place through a lowering of the cost of trading information, and that knowledge spill-overs are independent from trade costs viewed as the cost of trading goods. Hence, while in our model, high trade costs entail null knowledge spill-overs, and in this case act as a destabilizing force, Baldwin and Forslid show that high knowledge spill-overs may stabilize the symmetric equilibrium even if trade costs are high.

[4] The theoretical models on which are built these empirical works are the innovation-driven growth models by Grossman and Helpman (1991), Romer (1990) and Aghion and Howitt (1992). Moreover, Keller (2001a) refers also to the New Economic Geography contributions by Krugman (1991a,b) and Fujita, Krugman and Venables (1999).

direct investments and communications flow differences account for circa 15% each", and that "these three channels together account for almost the entire localization effect that would be otherwise attributed to geographic distance".

In order to illustrate the fact that trade acts as a channel through which knowledge spill-overs take place, we assume that learning capabilities ψ depend upon trade costs. Specifically, we assume that when trade costs are above a certain threshold value $\bar{\tau}$, firms in the lagging region are unable to assimilate any of the potential knowledge spill-overs from the leading region, so that the actual learning capabilities ψ of this region are equal to zero. However, when trade costs are below $\bar{\tau}$, the region's learning capabilities rise as trade costs fall. This leads us to assume that

$$\psi(\tau) = \begin{cases} c(\bar{\tau} - \tau) & \text{if} \quad \tau \leqslant \bar{\tau} \\ 0 & \text{if} \quad \tau > \bar{\tau} \end{cases} \tag{22}$$

where $c > 0$ is a parameter that represents the influence on learning abilities of all other above-mentioned factors. For simplicity, we assume that there are no interregional differences in these other factors so that c is the same across regions.

In order to describe how the learning ability affects the production activity in the lagging regions, we assume that the technological level depends on the learning capabilities and on the technological gap between the two regions through the following dynamic equation:

$$\dot{a}_s = \left[(a_s - 1)^3 + \psi\,(1 - a_s) \right] \tag{23}$$

where $a_s \geq 0$.

This specification describes the fact that the technological advantage of a region tends to increase over time - following cumulative processes, that we consider exogenous in this paper - unless the technological gap between the two regions can be closed thanks to interregional learning capabilities, represented by positive values of ψ.[5] Furthermore, equation (23) takes into account the fact that when learning capabilities are small, even though they are positive, firms in the

[5] See Dosi (1988), for instance, that maintains that the technological advantage of a region tends to increase along a technological trajectory because leading regions tend to growth faster.

lagging region may recover their technological lag only when it is not too wide. In fact, when the technological gap is very wide, the amount of knowledge spill-overs required by firms in the lagging region to catch up is very wide. And if this is not the case, because trade costs are high, the lagging region will definitively fall behind.

For the given normalization (7), the north (south) is the technological leader, while the south (north) is the lagging region, when $a_s < 1$ ($a_s > 1$).

Three equilibrium values for a_s, when $\psi > 0$, are:

$$a_s = 1 \qquad a_s = 1 - \sqrt{\psi} \qquad a_s = 1 + \sqrt{\psi}$$

Thus, one of the possible equilibria for equation (23) is given by the symmetric equilibrium, which is characterized by identical regional levels of technology ($a_s = a_n = 1$).[6] In Figure 1, we plot equation (23) when $\psi = 0.5$.

The intercept of the function (23) with the vertical axis is given by $\psi - 1$. Therefore, we may observe that $a_s = 0$ is a *stable* equilibrium with no firm producing the modern good in the south if learning capabilities of this region are very low ($\psi < 1$). In this case, for a given value of c, trade costs are too high ($\tau > \bar{\tau} - 1/c$) to allow firms in the south to assimilate technology spill-overs from the north. When this is so, the technological advantage of the north continuously increases over time.

The "symmetric" equilibrium characterized by $a_s = a_n = 1$ is *stable* if the slope of (23) is negative in a neighborhood of this point, that is, if $\psi > 0$. In this case, trade costs are low enough ($\tau < \bar{\tau}$) to allow firms in the receiving regions to assimilate technology spill-overs.

Insert Figure 1 about here

When we consider only the dynamic equation (23), the symmetric equilibrium is stable only if learning abilities in both regions are positive, that is, if $\psi > 0$. Moreover, when learning

[6] Equilibrium values can also be considered as *steady state equilibrium* values with positive and equal, exogenous growth rate of the technological level. In fact, function (23) expresses relative technological development since the normalization $a_n = 1$ has been adopted.

capabilities are high enough, namely when $\psi > 1$, the symmetric equilibrium is stable for any initial value of $a_s < 1$. Figure 2 shows this case when $\psi = 1.5$.

Insert Figure 2 about here

However, when learning abilities are positive but not too high because trade costs are not low enough, the lagging region may benefit from interregional knowledge spill-overs provided that its technological lag is not too wide. In fact, when the lagging region is the south ($a_s < 1$), firms in this region may benefit from knowledge spill-overs only when the level of the technology of this region is not too low, namely $a_s > 1-\sqrt{\psi}$. In other words, firms in the south may recover their lag only if the technological gap $(1-a_s)$ from the leading region is smaller than $\sqrt{\psi}$. On the contrary, when the technological leading region is the south ($a_s > 1$), firms in the northern region may recover their lag, thanks to knowledge spill-overs from the south, only when the technological lag is not too wide for the given learning abilities, that is, if $a_s - 1 < \sqrt{\psi}$. This two conditions taken together entail that the symmetric equilibrium $a_s = a_n = 1$ is a stable equilibrium for expression (23), when $\psi > 0$, only if for a given initial value of the technology level, a_s^0, we have that:

$$1 - \sqrt{\psi} < a_s^0 < \sqrt{\psi} + 1$$

The width of the recoverable lag increases (decreases) when learning capabilities increase (decrease), namely when the economic integration between the two regions becomes higher (lower). In short, when the south is the lagging region ($a_s < 1$), the following three cases may occur for respectively high, low or intermediate trade costs.

Case 1 $\tau > \bar{\tau}$. *When trade costs are too high, the symmetric equilibrium can never be reached because firms in the lagging region cannot benefit from technology spill-overs from the leading region, given the low level of integration.*

Case 2 $\tau < \bar{\tau} - 1/c$. *When trade costs are low, firms in the lagging region can successfully exploit potential technology spill-overs from the leading region and the symmetric equilibrium is stable.*

Case 3 $\bar{\tau} - 1/c < \tau < \bar{\tau}$. *For intermediate trade costs the process of catching up of the lagging region with the leading region may be completed because trade is sufficiently developed to allow firms in the lagging region to interact with the most productive firms in the leading region. However, this happens only when the technological gap between the two regions is not too wide for the given learning abilities. In other words, when:* $1 - a_s^0 < \sqrt{\psi}$.

4 The full agglomeration and the symmetric equilibria

In this section, we analyze the full agglomeration and the symmetric equilibria when the regional levels of technology (a_r) are given.[7] In other words, equation (23) is not considered here, but will be introduced in the following section.

4.1 Agglomeration

For given regional levels of the technology (a_r), agglomeration of the modern sector in region v is a *sustainable equilibrium* when no firm finds it profitable to relocate or start its production in region r (where $v, r = s, n$ and $v \neq r$). In other words, full agglomeration of the modern sector in region v is a sustainable equilibrium if, and only if, with all firms located in region v, the sales of a (potential) firm relocating to region r (Q_{mir}) are less than the level required to break even (Q^*_{mir}):

$$Q_{mir} < Q^*_{mir}$$

Following Puga and Venables (1996) and Puga (1999), we compute in appendix A the conditions for the full agglomeration in region v to be a sustainable equilibrium when the regional levels of technology a_v and a_r are given.

Moreover, in appendix A it is shown that two cases may arise according to two different ranges of the parameters of the model. The first one arises when $0 < \mu_c \leqslant \mu^*_c$, and is characterized by the fact that the wages of unskilled workers are the same in the two regions ($w_{lv} = w_{lr} = 1$). The second case arises when $\mu_c > \mu^*_c$, and is characterized by a higher wage for unskilled workers in the region (v) in which the agglomeration of the modern sector takes place. Moreover, while in the first case the traditional sector may be active in both regions, in the second case region r is completely specialized in the production of the traditional good, and region v in the production

[7] There could be also a third type of equilibrium when $0 < \psi < 1$. In fact, there are two asymmetric equilibria characterized by technological level equal to $a_n = 1$ in the north and respectively $a_s = 1 - \sqrt{\psi}$ or $a_s = 1 + \sqrt{\psi}$ in the south. However, because these would never be stable equilibria for equation (23), we do not consider this type of equilibrium.

of the varieties of the modern good. The threshold value μ_c^* is given by:

$$\mu_c^* = \frac{(1-\mu)}{2(1-\mu)-\gamma}$$

4.1.1 Case I. $0 < \mu_c \leqslant \mu_c^*$

First, we consider the case in which the wages of unskilled workers in the core region v are equal to those of unskilled workers in the periphery r (that is, $w_{lv} = w_{lr} = 1$), and in which the traditional good may be produced in both regions. In this case, agglomeration in region v is a sustainable configuration when:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{\left(\tau^{2(\sigma-1)}-1\right)(1-\mu_c\gamma-\mu)}{2} + 1\right) < 1 \tag{24}$$

Expression (24) cannot be "easily turned into a closed-form solution for the range of trade costs for which agglomeration is sustainable".[8] However, following Puga (1999), we notice that the value of Q_{mir}/Q_{mir}^* approaches $\left(\frac{a_v}{a_r}\right)^{1-\sigma}$ when τ tends to 1, and that its derivative is negative for τ close to 1. Moreover, when τ becomes infinitely large so does Q_{mir}/Q_{mir}^*, provided that $\sigma > 1/(1-\mu-\gamma\mu_c)$.

Let us define τ^* as the value of τ, below which agglomeration becomes sustainable because $Q_{mir}/Q_{mir}^* < 1$.

The graphic in Figure 3 plots Q_{mir}/Q_{mir}^* as a function of trade costs, for given values of other parameters such that $0 < \mu_c \leqslant \mu_c^*$.[9] When trade costs are higher than τ^*, the full agglomeration of the manufacturing sector in region v is not a sustainable configuration because a firm may start its production in region r without suffering losses, given that $Q_{mir}/Q_{mir}^* \geq 1$. On the contrary, full agglomeration in region v is a sustainable equilibrium if trade costs are smaller than τ^* because $Q_{mir}/Q_{mir}^* < 1$.

Insert Figure 3 about here

[8] See Puga (1999), p. 318.

[9] The graphic is obtained for the following parameter values: $\gamma = 0.1$, $\mu = 0.3$, $\mu_c = 0.4$, $\sigma = 5$, $a_v = a_r = 1$.

When the level of integration between the two regions is high, that is, when τ tends to 1, we observe that

$$\lim_{\tau \to 1} \frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tag{25}$$

Therefore, when the level of the technology of the core region v is higher than that of the periphery r, that is, when $a_v > a_r$, and the two regions are highly integrated ($\tau \to 1$), full agglomeration of the modern sector in region v is more likely to occur because (25) is:

$$\lim_{\tau \to 1} \frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} < 1 \qquad \forall \sigma > 1$$

4.1.2 Case II. $\mu_c > \mu_c^*$

When $\mu_c > \mu_c^*$, the share of consumers' expenditures on manufacturing goods is high enough to yield full agglomeration of the manufacturing sector in region v as well as full specialization of the two regions. More precisely, in this case manufacturing goods are produced only in the core region v, and the agricultural good only in the periphery r, the wages of unskilled workers being now lower in the periphery than in the core. In fact, in appendix A we show that, while in region r unskilled workers earn 1, in region v their wage is given by:

$$w_{lv} = \frac{(1-\mu-\gamma)\mu_c}{(1-\mu)(1-\mu_c)}$$

where $\mu, \mu_c \neq 1$, $\mu_c \neq (1-\mu)/\gamma$.[10]

The previous expression outlines the fact that the wage w_{lv} of unskilled workers in region v increases when the share $(1-\mu-\gamma)$ of total cost of production of firms devoted to unskilled workers increases, when the share μ of total cost devoted to manufacturing intermediate goods decreases, and when the share μ_c of consumers' expenditures on manufacturing goods increases.

Agglomeration in region v is a sustainable configuration if:

$$\frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{(1-\mu)(1-\mu_c)}{(1-\mu-\gamma)\mu_c}\right)^{-\sigma(1-\gamma-\mu)} \left[\left(\tau^{2(\sigma-1)}-1\right)(1-\mu)(1-\mu_c)+1\right] < 1 \tag{26}$$

[10] In Appendix A we show that when $\mu_c > \mu_c^*$, $w_{lv} = \dfrac{(1-\mu-\gamma)\mu_c}{(1-\mu)(1-\mu_c)} > 1$.

Again, it is worth noting that the value of Q_{mir}/Q^*_{mir} becomes infinitely large when τ tends to ∞, provided that $\sigma > 1/(1-\mu-\gamma\mu_c)$.

However, expression (26) implies that not only high trade costs, but also sufficiently low trade costs may lead some firms to locate their production in the periphery r where the wages of unskilled workers are lower. Indeed, Q_{mir}/Q^*_{mir} may become higher than (or equal to) 1 for low trade costs. In this case, the agglomeration of the modern sector in region v may be unsustainable not only for high, but also for low trade costs.

Let τ^{**} be the value of trade costs at which agglomeration becomes unsustainable for $\tau \leqslant \tau^{**}$ (with $\tau^* < \tau^{**}$). Figure 4 plots Q_{mir}/Q^*_{mir} when parameters are such that $\mu_c > \mu_c^*$ and that τ^{**} exists.[11]

Insert Figure 4 about here

A necessary condition for τ^{**} to exist is that $Q_{mir}/Q^*_{mir} \geq 1$ when $\tau = 1$, that is:

$$\frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \left(\frac{(1-\mu)(1-\mu_c)}{(1-\mu-\gamma)\mu_c}\right)^{-\sigma(1-\gamma-\mu)} \geq 1 \tag{27}$$

Expression (27) is true when the wages of unskilled workers are such that:

$$w_{lv}^{\sigma(1-\gamma-\mu)} = \left(\frac{(1-\mu-\gamma)\mu_c}{(1-\mu)(1-\mu_c)}\right)^{\sigma(1-\gamma-\mu)} \geq \left(\frac{a_v}{a_r}\right)^{\sigma-1} \tag{28}$$

When τ^{**}.exists, agglomeration of the manufacturing sector in region v is unsustainable for $\tau \leqslant \tau^{**}$. In other words, the high nominal wage of unskilled workers in region v is not supported by a sufficiently high level of technology available for firms in this region. We may compare this result with that of Puga (1999), who considers only one category of workers who can or cannot be interregionally mobile. While in Puga (1999) labor mobility implies a monotonic relationship between the sustainability of agglomeration and the levels of trade costs, in our work, the introduction of two types of workers, characterized by different mobility assumptions, allows us to show

[11] The graphic is obtained for the following parameter values: $\gamma = 0.1$, $\mu = 0.4$, $\mu_c = 0.6$, $\sigma = 5$, $a_v = a_r = 1$.

that *the existence of an immobile factor may give rise to a non-monotonic relationship.*[12] In fact, we may come across the $\cap$-shaped relationship found by Venables (1996) when $\mu_c > \mu_c^*$. When this is so, the existence of an immobile factor leads to the dispersion of the economic activity for high level of integration, because firms find it profitable to produce in the periphery, where the wages of unskilled workers are lower. However, from expression (28) we may conclude that this happens only if the technological advantage of the core region v is not too large, and if the wages of unskilled workers in the core are too high in relation to the technological gap $(a_v - a_r)$.

Finally, following Puga (1999), numerical simulations for (24) and (26) suggest the following results:

$$\frac{\partial \tau^*}{\partial \sigma} < 0 \qquad \frac{\partial \tau^*}{\partial \mu} > 0 \qquad \frac{\partial \tau^*}{\partial \gamma} > 0 \qquad \frac{\partial \tau^*}{\partial a_v} > 0$$

and

$$\frac{\partial \tau^*}{\partial \mu_c} \gtrless 0 \qquad if \qquad \mu_c \lessgtr \mu_c^*$$

4.2 Stable Symmetric Equilibrium

The free entry and exit condition implies that the number of manufacturing firms in region r (n_r) increases (decreases) when profits in the region are positive (negative).[13] Therefore, the evolution of the mass of firms in region r is given by:

$$\dot{n}_r = \delta \pi_{ir} \tag{29}$$

where δ is a positive constant.

Let us rewrite equation (29) as follows:

$$\dot{n} = \delta \pi_i \tag{30}$$

where $\pi_i = \begin{bmatrix} \pi_{in} \\ \pi_{is} \end{bmatrix}$ and $n = \begin{bmatrix} n_n \\ n_s \end{bmatrix}$.

[12] This is possible only if τ^{**} exists.

[13] See Puga (1999).

For given values of regional technological levels (a_r), the economy can be considered at a short run or at a long run equilibrium. When the variables are at their short run equilibrium values, the number of firms in each region is not necessarily at its long run equilibrium value, because profits of firms may be positive or negative. Therefore, we may divide the variables of the models between what we call "slow" and "fast" variables. That is, the number of firms in a region (n_n and n_s) is a *slow variable*, while the other variables of the model are referred to as *fast variables*.[14] This distinction underlines that, to carry out the stability analysis, we suppose that fast variables have already reached their short run equilibrium values (which depend on slow variables values) and move along them, while slow variables move towards their long run equilibrium values.[15]

This distinction allows us to rewrite expression (30) in the neighborhood of a long run equilibrium as follows:

$$\dot{n} = \delta u(n) \equiv z(n) \tag{31}$$

In fact, in appendix B we show that this is possible if profits in a neighborhood of a long run equilibrium can be expressed as a function of the number of firms n:

$$\pi_i = u(n)$$

Differentiating $\dot{n} = z(n)$ and taking Taylor's expansion of the first order evaluated at the equilibrium values for n (denoted by $*$) yields:

$$\partial\dot{n} = \dot{n} = z(n^*) + \frac{\partial z}{\partial n}(n^*)(n - n^*)$$

Given that $z(n^*) = 0$, this expression becomes:

$$\dot{n} = \frac{\partial z}{\partial n}(n^*)(n - n^*)$$

where the matrix $\frac{\partial z}{\partial n}(n^*)$ is the Jacobian matrix for equation (30) for given values of a_n and a_s.

[14] With the exception of $a_n = 1$ and a_s which, in this section, are considered given.

[15] See Boggio (1986, 1999). It should be noted that while we assume that fast variables are asymptotically stable. A more rigourous approach should prove it rather than assume it.

Let the Jacobian matrix evaluated at the equilibrium be:

$$J_1^* = \frac{\partial z}{\partial n}(n^*) = \delta \frac{\partial u}{\partial n}(n^*) = \delta M$$

where $M = \frac{\partial u}{\partial n}(n^*)$.

In appendix C we show how to compute matrix J_1 and we give the symmetric equilibrium solutions. It must be noted that this equilibrium is possible only if the levels of technology are the same in the two regions ($a_n = a_s = 1$).

Matrix J_1^* is symmetric when evaluated at the symmetric equilibrium and its eigenvalues are equal to the two eigenvalues λ_1 and λ_2 of matrix M multiplied by δ.[16] Moreover, we observe that the eigenvalues of matrices J_1^* and M at the symmetric equilibrium are real numbers because the two matrices are symmetric.

Let us define the level of the free-ness of trade t, that is, the level of integration between the two regions, as:[17]

$$t \equiv \tau^{1-\sigma}$$

where $0 \leqslant t \leqslant 1$. For a given level of the elasticity of substitution σ, the free-ness of trade increases (decreases) when trade costs decrease (increase).

Given the complexity of the eigenvalues, we are not able to find a closed-form solution for the range of trade costs for which the symmetric equilibrium is stable. However, numerical simulations are helpful to illustrate *how different level of trade costs and parameters may affect the stability of the symmetric equilibrium.*

When $a_n = a_s = 1$, simulations show that the symmetric equilibrium is stable for low levels of the free-ness of trade t, that is, for low levels of integration between the two regions. Figures 5a-b and Figures 6a-b plot, respectively, the two eigenvalues λ_1 and λ_2 from different angles when t

[16] Since δ is a scalar different from zero and $\delta \in C$, and $J_1 = \delta\ M$, if the eigenvalues of M are λ_1 and λ_2, the eigenvalues of J_1 are $\delta\lambda_1$ and $\delta\lambda_2$. See Lütkepohl (1996).

[17] See Baldwin and Forslid (2000).

and μ_c, change, for given values of the other parameters.[18] These figures show the complexity of eigenvalues evaluation. Nevertheless, we observe that both eigenvalues are negative, and, therefore, the symmetric equilibrium is stable, when the level of free-ness of trade t is low (that is trade costs τ are high) and the consumption share of expenditures on manufactured good μ_c is low. In fact, in this case centripetal forces are weaker than the centrifugal ones because pecuniary externalities may not be intensively exploited by firms and consumers, given the small share of consumers' expenditures on manufacturing goods and the high levels of trade costs. However, if the free-ness of trade t increases (trade costs decrease), for given low levels of μ_c, centripetal forces become strong enough to make the symmetric equilibrium unstable, and the eigenvalue λ_1 becomes positive. Moreover, a further increase in the free-ness of trade may yield a symmetric equilibrium that is again stable for the range of parameters for which the centrifugal forces are stronger than the centripetal ones. Figures 5a-b and Figures 6a-b allow us to point out that the symmetric equilibrium also becomes unstable if, for given trade costs values (τ), the share of the manufacturing good in expenditures (μ_c) increases. However, if this share becomes too high, and is associated with relatively high level of integration (t), manufacturing firms spread out and are uniformly distributed between the two regions, because the symmetric equilibrium becomes stable again.

Insert Figure 5a about here

Insert Figure 5b about here

Insert Figure 6a about here

Insert Figure 6b about here

[18] The two eigenvalues are computed for the following parameter values: $\sigma = 3$, $\gamma = 0.1$, $\mu = 0.3$, $\bar{L}_n = \bar{L}_s = 1$, $a_n = a_s = 1$, $\lambda_r = \lambda_v = 1$ and $\bar{H} = 1$.
Following the literature on the home bias, we adopt $\lambda_r = \lambda_v = 1$.

5 Stability and technological evolution

In this section, the stability analysis of the two types of equilibrium considered above is extended to the case in which the regional levels of technology (a_r) may change over time according to equation (23).

5.1 Agglomeration in region v

When the relative regional levels of technology may change, agglomeration in region v may be a sustainable equilibrium not only when (23) implies that the technological gap between the two regions gradually increases but also when it shrinks.

Let us first consider the case in which (23) entails a continuous increase in the technological gap ($a_v - a_r$) between the two regions. In this case, starting from a situation in which the level of the technology of firms in region v is not smaller than that in region r ($a_v \geq a_r$), agglomeration in region v is a sustainable equilibrium in the following two subcases: (i) trade costs are so high ($\tau > \bar{\tau}$) that they do not allow firms in the lagging region r to observe and assimilate potential knowledge spill-overs from the more productive manufacturing firms in region v and (ii) the lagging region's capabilities to assimilate potential technology spill-overs are too small because trade costs are too high for the given initial technological gap between the two regions ($a_v - a_r > \sqrt{\psi}$). In both cases, *firms in the lagging region are unable to recover from their lagged position during the technological development process, and agglomeration of the manufacturing sector in the leading region is the only sustainable equilibrium.* In fact, equation (23) implies that the technological advantage of region v continuously increases over time, and that a_v/a_r tends to infinity.

Let us rewrite the sustainability conditions for agglomeration in region v given by (24) and (26) as:

$$\frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} k$$

where k depends on the parameters. Since $\sigma > 1$, we observe that:

$$\lim_{a_v \to \infty} \frac{Q_{mir}}{Q^*_{mir}} = \lim_{a_r \to 0} \frac{Q_{mir}}{Q^*_{mir}} = 0 \tag{32}$$

Hence, when the relative technological advantage of region v increases continually over time, as in the two above-mentioned cases, agglomeration in region v is the only sustainable equilibrium because $Q_{mir}/Q^*_{mir} = 0 < 1$ for all admissible τ. In fact, in these two cases the agglomeration forces are strengthened because firms in region v become more and more productive compared to those in region r.

Figure 7 shows, for instance, what happens when the assimilation of interregional technology spill-overs is impeded because (23) implies that the relative technological level a_v/a_r continues to increase over time (with $\psi = 0$).[19] Particularly, Figure 7 shows that if a_v/a_r increases from the initial value 1 to 1.2, and then to 1.4, the curve Q_{mir}/Q^*_{mir} is shifted downward and to the right strengthening centripetal forces in region v.

Insert Figure 7 about here

Therefore, (23) and (32) imply that, when trade costs are higher than $\bar{\tau}$, the symmetric equilibrium is unstable. An exogenous increase in the level of technology a_v would result in a continuous increase of the technological advantage of region v, leading to the agglomeration of the modern sector in this region for any value of the trade costs.

Finally, we must consider the case in which the technological gap between the two regions can be closed, because learning capabilities are high enough to allow technology spill-overs to take place for the given initial technological gap ($\sqrt{\psi} > a_v - a_r$). In this case, agglomeration in region v may be a sustainable equilibrium only if $Q_{mir}/Q^*_{mir} < 1$ when the process of recovery of the technological lag is completed. Hence, in this case even though knowledge spill-overs take place, pecuniary externalities are strong enough to foster the concentration of firms in region v. On the

[19] The graphics are obtained for the following parameter values: $\gamma = 0.1$, $\mu = 0.4$, $\mu_c = 0.6$, $\sigma = 5$. Note that since $\mu_c > \mu_c^*$, $w_{lv} > 1$ and in region v is produced only the manufacturing good.

contrary, if $Q_{mir}/Q^*_{mir} \geq 1$ when the technological gap is closed, agglomeration of the modern sector is unsustainable, because centripetal forces are weaker than centrifugal forces.

5.2 Symmetric equilibrium

To study the stability of the symmetric equilibrium when technological change is possible, we must consider three differential equations. Two of them are given by expression (31), and the third is given by equation (23), which describes the change in the technology.

In this case, the Jacobian matrix is given by:

$$J_2 = \begin{bmatrix} \frac{\partial \dot{n}_n}{\partial n_n} & \frac{\partial \dot{n}_n}{\partial n_s} & \frac{\partial \dot{n}_n}{\partial a_s} \\ \frac{\partial \dot{n}_s}{\partial n_n} & \frac{\partial \dot{n}_s}{\partial n_s} & \frac{\partial \dot{n}_s}{\partial a_s} \\ \frac{\partial \dot{a}_s}{\partial n_n} & \frac{\partial \dot{a}_s}{\partial n_s} & \frac{\partial \dot{a}_s}{\partial a_s} \end{bmatrix}$$

Since $\frac{\partial \dot{a}_s}{\partial n_n} = \frac{\partial \dot{a}_s}{\partial n_s} = 0$, the Jacobian matrix (evaluated at the symmetric equilibrium denoted by $*$) is given by:

$$J_2^* = \begin{bmatrix} J_1^* & \left(\frac{\partial \dot{n}}{\partial a_s}\right)^* \\ 0 & -\psi \end{bmatrix}$$

Matrix J_2^* is a decomposable matrix and, therefore, two of its eigenvalues are given by the eigenvalues of matrix J_1^*, that is, $\delta\lambda_1$ and $\delta\lambda_2$, and the third eigenvalue is given by:

$$-\psi = -c(\bar{\tau} - \tau)$$

The symmetric equilibrium is stable when the three eigenvalues of matrix J_2^* are negative. As pointed out in section 4.2, the eigenvalues of matrix J_1^* are both negative - for a given level of integration between the two regions - when centrifugal forces for the given technology levels are stronger than centripetal forces.[20] The third eigenvalue $(-\psi)$, instead, is negative when trade costs are sufficiently low to allow for successful technology spill-over processes, that is, when $\tau < \bar{\tau}$.

Table 1 reveals that the symmetric equilibrium is always unstable for low levels of economic integration, that is, for the highest range of trade costs values. More precisely, when $\tau > \bar{\tau}$, namely

[20] The symmetric equilibrium technology levels are $a_s = a_n = 1$

when the level of trade costs is higher than the value below which knowledge spill-overs may be assimilated by firms in the lagging region, the symmetric equilibrium is unstable.

Insert Table 1 about here

The fact that the symmetric equilibrium is unstable for the highest range of trade costs values is not in line with the general results of economic geography. While existing models predict that dispersion is always a stable equilibrium for high trade costs, the present work shows that this is not always the case. If trade costs are high enough, the lagging region cannot benefit from any potential knowledge spill-overs process, and the initial technological gap increases over time leading to the agglomeration of the manufacturing sector in the leading region, which is a sustainable equilibrium.

By contrast, even when the two regions are sufficiently integrated ($\tau < \bar{\tau}$), and therefore the process of technological catching up may be implemented through learning by interacting processes because $\psi > 0$, the other centripetal forces may be strong enough to make the dispersion of the economic activity unstable. This is the case when at least one of the eigenvalues $\delta\lambda_1$ and $\delta\lambda_2$ of J_2^* is positive.

Finally, if the economy is at the symmetric equilibrium, and this equilibrium is stable, technology may still evolve since this equilibrium is compatible with equal exogenous growth rates of a_r for the two regions (*steady state equilibrium*).

To summarize, we observe that *trade costs play a different role in the process of knowledge spill-overs and in the process of interaction between the standard centripetal and centripetal forces* (for given technology development levels). More precisely, from the point of view of technology spill-overs processes, a reduction in trade costs, when they are high, may enhance the recovery of the less developed region, but from the other point of view (that is, the interplay of centripetal and centrifugal forces evaluated at fixed technology levels), the reduction in trade costs may strengthen centripetal forces. As a consequence, in this case, *there is a trade-off in the role played* *by trade costs in knowledge spill-overs processes and in determining the result of the conflict between standard fixed-technology centripetal and centripetal forces.*

6 Conclusion

The model we develop in the first part of this paper can be considered as a generalization of the Core-Periphery models by Krugman (1991b) and Krugman and Venables (1995). More precisely, to derive both of them we have to assume that the regional technological levels are the same for the two regions ($a_s = a_n = 1$). Hence, Krugman's model corresponds to the case in which manufacturing firms employ only skilled mobile workers ($I_{mir} = H_r$). Krugman and Venables' model, instead, assume that firms employ only intermediate manufacturing varieties and immobile workers, who correspond to unskilled workers in the present model ($I_{mir} = L_{mir}^{1-\mu} D_{mir}^{\mu} / \left[(1-\mu)^{1-\mu} \mu^{\mu} \right]$).

For given technological levels, we find that full agglomeration of the manufacturing sector in a region is unsustainable for high trade costs because centrifugal forces are stronger than centripetal ones. By contrast, full agglomeration may be an equilibrium for low trade costs. Moreover, the introduction of two types of workers, characterized by different mobility assumptions, allows us to show that the existence of an immobile factor may give rise to a non-monotonic relationship between the sustainability of agglomeration and the levels of trade costs. In fact, we may encounter the $\cap$-shaped relationship found by Venables (1996) when parameter values are such that the wages of unskilled workers are higher in the region in which the agglomeration of the modern sector takes place.[21] When this is so, the existence of an immobile factor may lead to the dispersion of the economic activity for high level of integration because firms find it profitable to produce in the periphery where the wages of unskilled workers are lower than in the core region. However, we show that this happens only if the technological advantage of the core region is not too large, and if the wages of unskilled workers in the core are too high in relation to the technological gap.

For given equal technological levels, we find that the traditional result of a stable symmetric

[21] In particular, this may happen when the share of consumers' expenditure on manufactures μ_c is higher than the threshold value μ_c^*.

by trade costs in knowledge spill-overs processes and in determining the result of the conflict between standard fixed-technology centripetal and centripetal forces.

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For given technological levels, we find that full agglomeration of the manufacturing sector in a region is unsustainable for high trade costs because centrifugal forces are stronger than centripetal ones. By contrast, full agglomeration may be an equilibrium for low trade costs. Moreover, the introduction of two types of workers, characterized by different mobility assumptions, allows us to show that the existence of an immobile factor may give rise to a non-monotonic relationship between the sustainability of agglomeration and the levels of trade costs. In fact, we may encounter the $\cap$-shaped relationship found by Venables (1996) when parameter values are such that the wages of unskilled workers are higher in the region in which the agglomeration of the modern sector takes place.[21] When this is so, the existence of an immobile factor may lead to the dispersion of the economic activity for high level of integration because firms find it profitable to produce in the periphery where the wages of unskilled workers are lower than in the core region. However, we show that this happens only if the technological advantage of the core region is not too large, and if the wages of unskilled workers in the core are too high in relation to the technological gap.

For given equal technological levels, we find that the traditional result of a stable symmetric

[21] In particular, this may happen when the share of consumers' expenditure on manufactures μ_c is higher than the threshold value μ_c^*.

equilibrium for high trade costs holds. Moreover, we show that when the technological advantage of a region is very high with respect to the other region, full agglomeration of manufacturing in the leading region is sustainable even for the highest values of trade costs.

When we allow for technological change and potential knowledge spill-overs, we enrich the analysis by considering new forces that may modify the above-mentioned results obtained with fixed-technology centripetal/centrifugal forces. More precisely, when obstacles to interacting, proxied by trade costs, are high, the symmetric equilibrium becomes unstable and centripetal forces induce the agglomeration of the manufacturing sector in the more developed region. Besides, low trade costs may yield either the agglomeration in the more productive region, or the dispersion of the modern sector.

In particular, the symmetric equilibrium can be attained only if the lagging region can complete a catching up process with the leading region. Hence, we show that the symmetric equilibrium is unstable when trade costs are too high, because firms in the lagging region cannot benefit from the potential knowledge spill-overs from the leading region. In this case, firms in the less developed region do not have enough opportunities to interact with firms in the leading region and, therefore, they are unable to assimilate the more productive technologies used by the latter. As a result, the technological gap between the two regions increases, and the manufacturing sector ends up being completely concentrated in the leading region. By contrast, when trade costs are sufficiently low, firms in the lagging region can benefit from knowledge spill-overs and the symmetric equilibrium may be stable if all the centripetal forces are weaker than the centrifugal ones.

Moreover, we find that, for intermediate trade costs values, the symmetric equilibrium can be stable, provided that the initial technological gap between the two regions is not too wide. In fact, when it is very wide, firms in the lagging region are unable to assimilate the potential knowledge spill-overs. When this is so, the agglomeration of the manufacturing sector in the leading region is the only sustainable equilibrium.

To sum up, there is a trade-off in the role played by trade costs in knowledge spill-over processes, and in determining the result of the conflict among the other fixed-technology centripetal and centrifugal forces. The results of this trade-off depend on which of the effects produced by different trade costs levels prevail. Particularly, if trade costs are very high, manufacturing ends up being completely agglomerated in the region that has an initial technological advantage, because firms in the lagging regions are unable to benefit from the interregional potential knowledge spill-overs. Thus, we like to stress that our results reverse the usual conclusion of New Economic Geography models that high trade costs favors economic dispersion by showing that high trade costs favor the agglomeration of firms in the more productive region.

Appendix A. Sustainability of agglomeration of the manufacturing sector in region v.

Agglomeration of the manufacturing sector in region v is an equilibrium if sales of a (potential) deviant firm relocating in region r are less than the level required to break even, that is if:

$$Q_{mir} < Q^*_{mir}$$

Let us consider as given the regional levels of the technology a_r and a_v.

A manufacturing firm has positive (negative) profits if its production is higher (lower) than the amount required to break even, Q^*_{mir}, that is given by

$$Q^*_{mir} = a_r$$

where $r = s, n$.

Let us consider the case in which the manufacturing sector is fully agglomerated in region v and a firm that is a potential deviant in region r. This firm decides to start its production in region r if the demand that it faces by producing in region r is higher than (or equal to) the amount Q^*_{mir} required to break even by producing in this region. The relationship between the two regional break even quantities is:

$$Q^*_{mir} = \frac{a_r}{a_v} Q^*_{miv} \tag{33}$$

where $r, v = n, s$ and $v \neq r$.

When the manufacturing sector is fully agglomerated in region v, the composite good price indexes in region v and in region r are respectively:

$$p_{mv} = n_v^{\frac{1}{1-\sigma}} p_v \qquad \text{and} \qquad p_{mr} = n_v^{\frac{1}{1-\sigma}} \tau p_v \tag{34}$$

The demand for variety i produced in region v is:

$$Q_{miv} = \frac{E_{mv} + E_{mr}}{n_v p_v} \tag{35}$$

Moreover, given the free entry and exit hypothesis for manufacturing firms, each firm in region v produces the quantity required to break even:

$$Q^*_{miv} = Q_{miv}$$

Hence, the zero profit level of output of the potential deviant firm in region r is given by:

$$Q^*_{mir} = \frac{a_r}{a_v} \frac{(E_{mv} + E_{mr})}{n_v p_v} \tag{36}$$

The demand for variety i produced by the potential deviant in r is:

$$Q_{mir} = p_r^{-\sigma} \left(\frac{1}{p_{mr}^{1-\sigma}} E_{mr} + \tau^{1-\sigma} \frac{1}{p_{mv}^{1-\sigma}} E_{mv} \right) \tag{37}$$

Substituting the price indexes from (34) into (37), it is possible to express the deviant firm's demand function as:

$$Q_{mir} = \left(\frac{p_r}{p_v} \right)^{-\sigma} \left(\frac{\tau^{\sigma-1} E_{mr} + \tau^{1-\sigma} E_{mv}}{n_v p_v} \right) \tag{38}$$

From (13) we can derive relative prices (p_r/p_v):

$$\frac{p_r}{p_v} = \frac{a_v}{a_r} \left(\frac{p_{mr}}{p_{mv}} \right)^{\mu} \left(\frac{w_{hr}}{w_{hv}} \right)^{\gamma} \left(\frac{w_{lr}}{w_{lv}} \right)^{1-\gamma-\mu}$$

From the regional price indexes of the composite good (34) we can derive the relative price index:

$$\frac{p_{mv}}{p_{mr}} = \tau^{-1}$$

In order to attract skilled workers in region r we know that the deviant firm has to offer them at least the same real wage that they gain in region v. Therefore, the following condition must hold:

$$\frac{w_{hr}}{w_{hv}} = \left(\frac{p_{mv}}{p_{mr}}\right)^{-\mu_c} = \tau^{\mu_c} \tag{39}$$

Moreover, the wage of unskilled workers in region r (w_{lr}) is equal to 1, because it is equal to the agricultural price (the numéraire of the model). Therefore, we may rewrite the ratio of the price of the varieties produced in the two regions as:

$$\frac{p_r}{p_v} = \frac{a_v}{a_r}\tau^{\mu+\gamma\mu_c}\left(\frac{1}{w_{lv}}\right)^{1-\gamma-\mu} \tag{40}$$

Substituting (40) into (38) and eliminating $n_v p_v$ yields the ratio between the demand for the potential deviant firm (Q_{mir}) and the break even amount for region r (Q^*_{mir}):

$$\frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma}\tau^{1-\sigma(1+\mu+\gamma\mu_c)}\left(\frac{1}{w_{lv}}\right)^{-\sigma(1-\gamma-\mu)}\left(\frac{\left(\tau^{2(\sigma-1)}-1\right)E_{mr}}{E_{mv}+E_{mr}}+1\right) \tag{41}$$

Then we compute expenditures on manufactures in both regions, in order to substitute $E_{mr}/\left(E_{mv}+E_{mr}\right)$ in the previous expression. When firms and skilled workers are fully agglomerated in the region v, these are respectively:

$$E_{mr} = \mu_c\bar{L} \tag{42}$$

and

$$E_{mv} = \mu_c\left(w_{lv}\bar{L}+w_{hv}\bar{H}\right)+\mu n_v p_v Q_{miv} \tag{43}$$

Moreover, given the free entry and exit condition, the wages of skilled workers in region v correspond to the share γ of total revenues of firms in v:

$$w_{hv}\bar{H} = \gamma n_v p_v Q_{miv} \tag{44}$$

Using (35), (43) and (44) expenditures on the composite manufactured good in region v are:

$$E_{mv} = \frac{\mu_c w_{lv}\bar{L}+\left(\mu_c\gamma+\mu\right)E_{mr}}{\left(1-\mu_c\gamma-\mu\right)} \tag{45}$$

Finally, substituting E_{mr} from (42) into (45), we obtain that E_{mv} is:

$$E_{mv} = \frac{\mu_c w_{lv} \bar{L} + (\mu_c \gamma + \mu)\, \mu_c\, \bar{L}}{(1 - \mu_c \gamma - \mu)} \tag{46}$$

Therefore:

$$\frac{E_{mr}}{E_{mv} + E_{mr}} = \frac{(1 - \mu_c \gamma - \mu)}{w_{lv} + 1} \tag{47}$$

Substituting $E_{mr}/(E_{mv} + E_{mr})$ from (47) into (41), we obtain the following expression:

$$\frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_{sr}}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{1}{w_{lv}}\right)^{-\sigma(1-\gamma-\mu)} \left(\frac{\left(\tau^{2(\sigma-1)}-1\right)(1-\mu_c\gamma-\mu)}{w_{lv}+1} + 1\right) \tag{48}$$

Finally, we evaluate the wage of unskilled workers in the core region (w_{lv}) in order to substitute it into (48). We observe that this wage may be either equal to 1 or higher than 1. Indeed, we point out that the wage of unskilled workers in region v can never be smaller than 1, because in this case the traditional good would not be produced in the peripheral region r, given that the production cost and, therefore, the price would be smaller in region v.

Hence, when the wages of unskilled workers are the same for the two regions ($w_{lv} = w_{lr}$), then the traditional good may be produced in both of them because $w_{lv} = p_{av} = p_{ar} = w_{lr}$. By contrast, the traditional good is not produced in the core region r when $w_{lv} > 1$.

To obtain the wage of unskilled workers we compute the sum of expenditures on the composite good in the two regions from (42) and (46):

$$E_{mv} + E_{mr} = \frac{\mu_c}{1 - \mu - \mu_c \gamma} \left(w_{lv} \bar{L} + \bar{L}\right) \tag{49}$$

where $\mu_c \neq (1 - \mu)/\gamma$.

We know that, because of the free entry and exit condition of firms, the total amount of wages paid to unskilled workers in region v is equal to the share $(1 - \mu - \gamma)$ of total revenues:

$$w_{lv} \bar{L} = (1 - \mu - \gamma) n_v p_v Q_{miv} \tag{50}$$

From (35), (49) and (50) we derive the wages of unskilled workers as a function of the parameters of the model:

$$w_{lv} = \frac{(1 - \mu - \gamma)\mu_c}{(1 - \mu)(1 - \mu_c)}$$

where $\mu, \mu_c \neq 1, \mu_c \neq (1-\mu)/\gamma$.

Hence, $w_{lv} > 1$ when:

$$\mu_c > \frac{1-\mu}{2(1-\mu)-\gamma} = \mu_c^*$$

where $\gamma \neq 2(1-\mu)$. Wages cannot be lower than 1 because in this case the traditional good would be produced in region v and not in region r.

On the contrary, when $0 < \mu_c \leqslant \mu_c^*$, the wages of unskilled workers in the core v must be equal to 1 if the traditional good is produced in the periphery r.

Therefore we may have the two following cases.

When $0 < \mu_c \leqslant \mu_c^*$, agglomeration of manufacturing firms in region v is an equilibrium if the ratio Q_{mir}/Q_{mir}^* from (41) is smaller than 1:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{\left(\tau^{2(\sigma-1)}-1\right)(1-\mu_c\gamma-\mu)}{2} + 1\right) < 1 \tag{51}$$

Otherwise, when $1 > \mu_c > \mu_c^*$, agglomeration of the manufacturing sector in region v is an equilibrium when:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{(1-\mu)(1-\mu_c)}{(1-\mu-\gamma)\mu_c}\right)^{-\sigma(1-\gamma-\mu)} \left[\left(\tau^{2(\sigma-1)} - 1\right)(1-\mu)(1-\mu_c) + 1\right] < 1 \tag{52}$$

Appendix B.

To prove that profits in a neighborhood of a long run equilibrium can be written as a function of the number of firms n

$$\pi_i = u(n) \tag{53}$$

it is necessary to determine the short run equilibrium, which is defined as a set of solutions to equations (54)-(58) below, once n_n and n_s are given. We express them in matrix form. To this end, variables without suffix r define vectors, variables with superscript $\tilde{}$ are 2x2 diagonal matrix with the i-th element of the corresponding vector in position (i,i) and zeros off the diagonal. Matrix T is: $T = \begin{bmatrix} 1 & \tau^{1-\sigma} \\ \tau^{1-\sigma} & 1 \end{bmatrix}$.

Substituting manufacturing prices p_r from (13), expenditures on the manufacturing good E_{mr} from (4), (5) and (10), manufacturing quantities Q_{mir} from (15), and production cost TC_{mir} from (9), into (12), (3), (20), (18), (19) and (21), for $r = n, s$, and given the normalizations of α and β, we obtain a system of 10 equations. The solutions of the system (54)-(58) is given by the set of the 10 "fast" variables (π_{in}, π_{is}, p_{mn}, p_{ms}, w_{hn}, w_{hs}, w_{ls}, $w_{\ln}$, H_n, H_s) for given values of the slow variables n_n and n_s.

In matrix form, the equilibrium is obtained by solving the following system:

- two manufacturing good market short run equilibrium conditions:

$$a + \sigma \tilde{a} \tilde{p}_m^{-\mu} \tilde{w}_l^{-1+\mu+\gamma} \tilde{w}_h^{-\gamma} \pi_i = \tag{54}$$

$$= \mu_c \tilde{a}^{\sigma} \tilde{p}_m^{-\sigma\mu} \tilde{w}_l^{-\sigma(1-\mu-\gamma)} \tilde{w}_h^{-\sigma\gamma} T \tilde{p}_m^{\sigma-1} \left(\tilde{L} w_l + \tilde{H} w_h + \tilde{n} \pi_i \right) +$$
$$+ \mu \tilde{a}^{\sigma} \tilde{p}_m^{-\sigma\mu} \tilde{w}_l^{-\sigma(1-\mu-\gamma)} \tilde{w}_h^{-\sigma\gamma} T \tilde{p}_m^{\sigma-1} \tilde{n} \left[(\sigma - 1) \pi_i + \tilde{w}_l^{(1-\mu-\gamma)} \tilde{p}_m^{\mu} w_h^{\gamma} \right]$$

- two composite good price indices:

$$0 = p_m^{1-\sigma} - T \tilde{a}^{(-1+\sigma)} \tilde{p}_m^{\mu(1-\sigma)} \tilde{w}_h^{\gamma(1-\sigma)} \tilde{w}_l^{(1-\gamma-\mu)(1-\sigma)} n \tag{55}$$

- two functions that express total wages of skilled workers in the two regions:

$$0 = \tilde{w}_h H - \gamma \tilde{n} \left[(\sigma - 1) \pi_i + \tilde{p}_m^{\mu} \tilde{w}_l^{1-\gamma-\mu} w_h^{\gamma} \right] \tag{56}$$

- skilled labor market equilibrium condition together with the condition of equal real wages for the two regions:

$$\begin{bmatrix} \frac{w_{hn}}{p_{mn}^{\mu_c}} \\ H_n + H_s \end{bmatrix} = \begin{bmatrix} \frac{w_{hs}}{p_{ms}^{\mu_c}} \\ \bar{H} \end{bmatrix} \tag{57}$$

- two unskilled labor market conditions:

$$\tilde{w}_l \bar{L} = (1-\gamma-\mu)\,\tilde{n}\left[(\sigma-1)\,\pi_i + \tilde{p}_m^{\mu}\tilde{w}_h^{\gamma}w_l^{1-\gamma-\mu}\right] + \tag{58}$$

$$+ (1-\mu_c)\,\Lambda(\tilde{w}_h H + \tilde{w}_l \bar{L} + \tilde{n}\pi_i)$$

where $\Lambda = \begin{bmatrix} \lambda_r & 1-\lambda_v \\ 1-\lambda_r & \lambda_v \end{bmatrix}$

Let:

1. $x = (p_{mn}, p_{ms}, w_{hn}, w_{hs}, w_{ls}, w_{\ln}, H_n, H_s)'$, a column vector of eight fast variables;

2. $y = (\pi_i', x')'$ the column vector of the ten fast variables;

3. G_k be a function from R^{12} to R with continuous derivative in the neighborhood of a long run equilibrium (LRE), such that $G_k(y,\ n) = 0$, with $k = 1, 2, ...10$ are the ten equations (54)-(58);

4. $G(y,n) \equiv (G_k(y,n))$.

If the $det = \left[\dfrac{\partial G_k}{\partial y_\iota}\right]_* \neq 0$, where y_ι is a generic element of y and $*$ means that the derivatives are evaluated at a LRE, equation $G(y,n) = 0$ allows us to define in a neighborhood of such LRE function u from R^2 to R^2 with continuous derivative such that

$$\pi_i = u(n)$$

The Jacobian matrix of u in a LRE is denoted by $\dfrac{\partial u}{\partial n}(n^*)$.

Finally, it should be noted that at the long run equilibrium values, that is, at long run equilibrium values of all fast and slow (n_n and n_s) variables, profits should be equal to zero ($\pi_{in} = \pi_{is} = 0$).

Appendix C.

In this appendix we show how we compute the Jacobian matrix evaluated at the long run equilibrium

$$J_1^* = \frac{\partial z}{\partial n}(n^*) = \delta\, M$$

where $M = \frac{\partial u}{\partial n}(n^*)$.

Define:

- the column vector $x = (p_{mn}, p_{ms}, w_{hn}, w_{hs}, w_{ls}, w_{\ln}, H_n, H_s)'$;

- and the two functions f and g, where:

f is defined from R^{12} to R^2 and is derived from the two equations (54) in appendix B, and is such that

$$f(\pi_i, n, x) = 0$$

g is defined from R^{12} to R^8 and is derived from the eight equations (55)-(58), and is such that

$$g(\pi_i, n, x) = 0$$

Total differentials of f and g are respectively given by (59) and (60):

$$A\ d\pi_i + B\ dn + C\ dx = 0 \tag{59}$$

$$D\ d\pi_i + E\ dn + F\ dx = 0 \tag{60}$$

where matrices A, B, C, D, E and F are evaluated at symmetric equilibrium values, that can be computed from the system of equation (54)-(58) and are given below.

Computing dx from (60),

$$dx = -F^{-1}(D\ d\pi_i + E\ dn)$$

and substituting it into (59), yields:

$$M = \frac{\partial \pi_i}{\partial n} = -(-CF^{-1}D + A)^{-1}(-C\,F^{-1}E + B)$$

Long run symmetric equilibrium values, which can be obtained only if technological development levels are equal in the two regions, $a_n = a_s = 1$, are:

$w_{lr} = 1; \qquad \pi_{ir} = 0; \qquad H_r = \frac{\bar{H}}{2}; \qquad w_{hr} = \frac{2\gamma\mu_c\bar{L}}{\bar{H}(1-\mu-\gamma\,\mu_c)};$

$n_r = (1+\tau^{1-\sigma})^{-\frac{\mu}{1-\sigma+\mu\sigma}} \left(\frac{\mu_c\bar{L}}{1-\mu_c\gamma-\mu}\right)^{\frac{(1-\sigma)(1-\mu-\gamma)}{1-\sigma+\mu\sigma}} \left(\frac{2\gamma}{\bar{H}}\right)^{\frac{\gamma(\sigma-1)}{1-\sigma+\mu\sigma}};$

$p_{mr} = (1+\tau^{1-\sigma})^{\frac{1}{1-\sigma+\mu\sigma}} \left(\frac{\mu_c\bar{L}}{1-\mu_c\gamma-\mu}\right)^{\frac{1-\gamma\sigma}{1-\sigma+\mu\sigma}} \left(\frac{2\gamma}{\bar{H}}\right)^{-\frac{\gamma\sigma}{1-\sigma+\mu\sigma}}$

where $r = n,\ s$.

Solutions are positive for $\mu_c < \frac{1-\mu}{\gamma}$.

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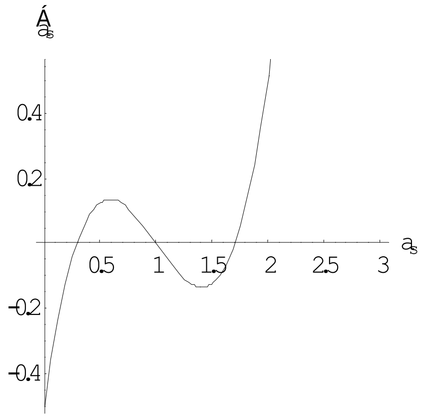


Figure 1

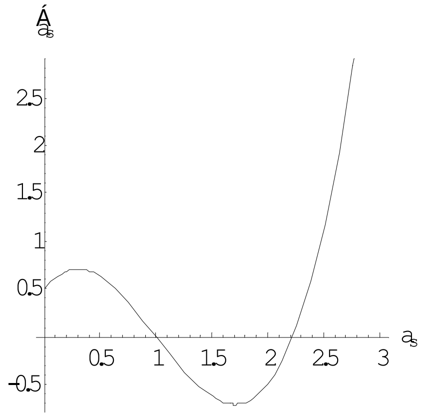


Figure 2

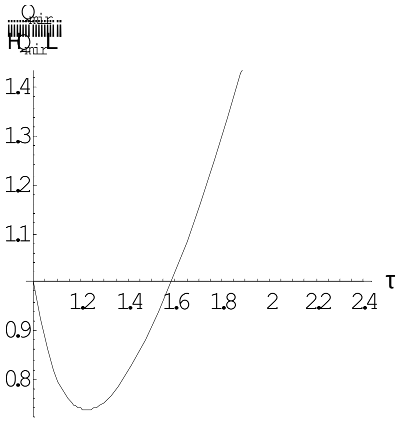
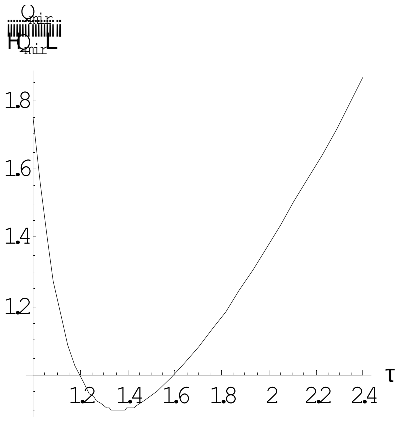


Figure 3



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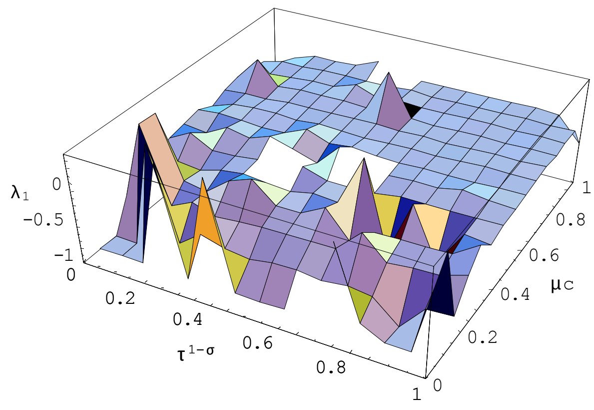

Figure 5a. Eigenvalue λ_1.

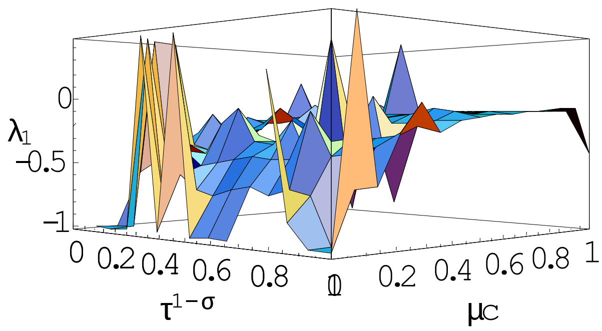

Figure 5b. Eigenvalue λ_1.

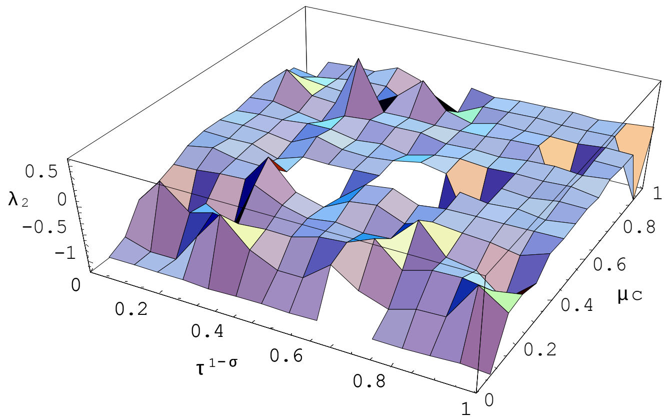

Figure 6a. Eigenvalue λ_2.

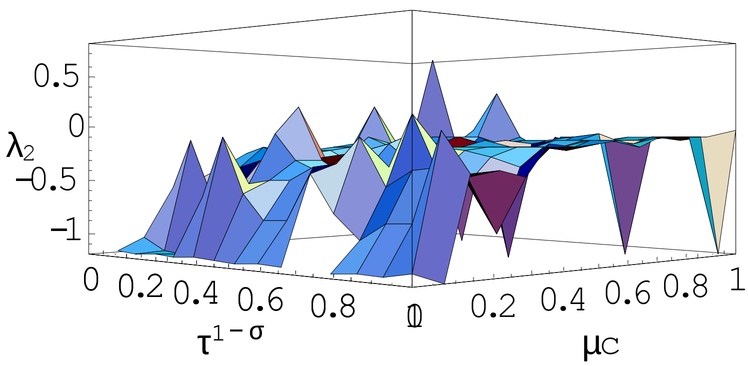

Figure 6b. Eigenvalue λ_2.

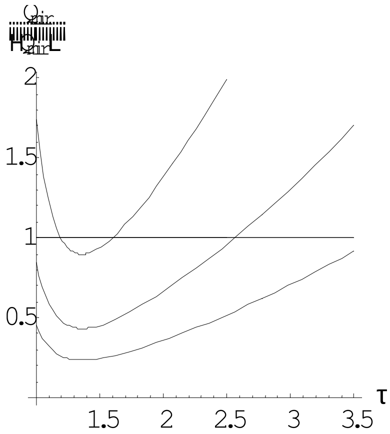

Figure 7

	if $\tau < \bar{\tau}$ ($-\psi < 0$)	if $\tau > \bar{\tau}$
$\delta\lambda_1 < 0$; $\delta\lambda_2 < 0$	Stable	Unstable
$\delta\lambda_1 < 0$; $\delta\lambda_2 > 0$ or $\delta\lambda_1 > 0$; $\delta\lambda_2 < 0$ or $\delta\lambda_1 > 0$; $\delta\lambda_2 > 0$	Unstable	Unstable

Table 1. Symmetric equilibrium for different trade costs (τ) values.