Appendix

By solving with respect to b the first order condition for firm 2's profit maximization at the location stage, we obtain the following critical values

- $b = -a + \frac{3}{2}\sqrt{2\left(1-w^2\right)}$
- $b = -a - \frac{3}{2}\sqrt{2\left(1-w^2\right)}$
- $b = \rho$, where ρ is a root of the polynomial:

$$\begin{aligned} &4x^4 + 6x^3 a + \left(-30w^2 + 30\right)x^2 + \\ &+\left(-2a^3 + 21aw^2 - 21a\right)x - 2a^2 + 2a^2w^2 - 18 + 36w^2 - 18w^4 \end{aligned} \tag{A1}$$

Let us consider these solutions.

- Consider first the solution $b = -a + \frac{3}{2}\sqrt{2\left(1-w^2\right)}$. Given the reaction function of firm 1, we have to solve the following system in order to discuss the candidate optimal locations of firm 1 and firm 2:

  $$\begin{aligned} a &= \frac{5}{4}b - \frac{1}{4}\sqrt{\left(9b^2 + 16\left(1-w^2\right)\right)} \\ b &= -a + \frac{3}{2}\sqrt{2\left(1-w^2\right)} \end{aligned}$$

  Again we have two solutions: the couple of locations $a_1 = \frac{7}{6}\sqrt{2\left(1-w^2\right)}$ and $b_1 = \frac{1}{3}\sqrt{2\left(1-w^2\right)}$, and the couple $a_2 = \frac{1}{3}\sqrt{2\left(1-w^2\right)}$, and $b_2 = \frac{7}{6}\sqrt{2\left(1-w^2\right)}$. The first couple implies a value for b lower than a. This solution is therefore unacceptable. However, if eqts (14) and (15) were evaluated at the second couple, the marginal consumer would lie at $\frac{3}{4}\sqrt{2\left(1-w^2\right)}$. Since the disequation $\frac{3}{4}\sqrt{2\left(1-w^2\right)} > \dfrac{1-w}{2}$ is always satisfied, at this solution the firms' conjectures would not be fulfilled.

- It is easy to check that, if $b = -\left(a + \frac{3}{2}\sqrt{2\left(1-w^2\right)}\right)$, the values of b that solve the system of the two reaction functions are both smaller than a. This contradicts the assumption $a < b$.

We now consider the polynomial (A1). Its solutions obtained by substituting firm 1's optimal reply are:

$$\begin{aligned} x &= \sqrt{2\left(w^2-1\right)} \\ x &= -\frac{5}{18}\sqrt{6\left(1-w^2\right)} \\ x &= \frac{5}{18}\sqrt{6\left(1-w^2\right)} \end{aligned}$$

- We can immediately rule out the complex solution $x = \sqrt{2(w^2 - 1)}$.

- If $b = -\frac{5}{18}\sqrt{6(1 - w^2)}$, the optimal location of firm 1 is $a = -\frac{29}{36}\sqrt{6(1 - w^2)}$. In this case $a < b$, but both optimal solutions are negative and this contrasts again with the conjectures about the location of the marginal consumer.

- The last solution is indeed the only acceptable one. If $b = \frac{5}{18}\sqrt{6(1 - w^2)}$, then $a = -\frac{1}{9}\sqrt{6(1 - w^2)}$. Using these optimal locations into (14) and (15) we may verify that the marginal consumers is in the left external interval for $w < \frac{1}{5}$. Notice that this solution collapses to that obtained by Tabuchi and Thisse by setting $w = 0$.