consumer lying between the two firms. As is well known, the price-location problem is a two-stage game in which at the first stage the firms choose their location and at the second stage choose their prices. The game is simultaneous.

The optimal firms' behaviour obviously differs according to the value of w. The results in terms of optimal locations are well known in the literature when $w = 1$ and when $w = 0$: in the unconstrained Hotelling game with a uniform distribution of consumers the firms maximize profits by locating at $-1/4$ e $5/4$ (Lambertini, 1994); moreover, Tabuchi and Thisse (1995) demonstrate that with a triangular distribution two asymmetric equilibria arise, $\left(-\sqrt{6}/9, 5\sqrt{6}/18\right)$ and $\left(1-5\sqrt{6}/18, 1+\sqrt{6}/9\right)$. The following analysis will focus on the price-location equilibria for intermediate values of the parameter w, i.e. when the density becomes trapezoidal.

3 Consumer concentration and equilibrium prices and locations

We look for a subgame perfect equilibrium through backward induction, solving first for the prices and then for the locations as a function of the exogenous parameter w and the optimal prices determined in the first stage. Notice that if firm 1 and 2 set a price respectively equal to p_1 and p_2 being located respectively in a and b, the above hypotheses on transportation costs, unit demand and full market coverage imply that the marginal consumer's location is

$$z = \left(\frac{1}{2}\left[\frac{p_2 - p_1}{b-a} + b - a\right] + a\right) \tag{1}$$

Clearly, given the shape of our density, the firms' reaction functions in both stages of the game will be different according to the fact that the firms know that their behaviour implies that the marginal consumer lies in the 'central interval' or in the two external intervals, i.e. $z \in \left[\frac{1-w}{2}, \frac{1+w}{2}\right]$ - central interval - or $z \in \left[0, \frac{1-w}{2}\right)$ and $z \in .\left(\frac{1+w}{2}, 1\right]$ - external interval. We solve the model under both conjectures and verify under which conditions one or more equilibria exist in which conjectures are fulfilled. Notice that, given the simmetry of the density, the possible existence of a subgame perfect equilibrium such that $z \in \left[0, \frac{1-w}{2}\right)$ implies the existence of a specular equilibrium, with the marginal consumer lying in a specular position within the interval $\left(\frac{1+w}{2}, 1\right]$. This allows to restrict the analysis to one external area only.

3.1 The marginal consumer lies in the central interval

Given the hypothesis of unit consumers' demand and given our normalization, the market demand for each good corresponds to its market share. Therefore, the demand for the two firms are respectively:

$$\begin{aligned} q_1 &= F(z,w) \\ q_2 &= 1 - F(z,w) \end{aligned}$$

where F is the cumulative function of f. As long as $z \in \left[\frac{1-w}{2}, \frac{1+w}{2}\right]$,

$$q_1(z,w) = \frac{2z - \frac{1}{2}(1+w)}{1+w}$$

so that, by substituting (1), we get:

$$q_1 = \left(\frac{\frac{p_2 - p_1}{b-a} + b + a + \frac{1}{2}(w-1)}{1+w} \right) \tag{2}$$

The demand accruing to the firm 2 will be:

$$q_2 = \left[1 - \left(\frac{\frac{p_2 - p_1}{b-a} + b + a + \frac{1}{2}(w-1)}{1+w} \right) \right] \tag{3}$$

Since there are no production costs, the profit functions of the two firms are:

$$\pi_1 = \left[p_1 \left(\frac{\frac{p_2 - p_1}{b-a} + b + a + \frac{1}{2}(w-1)}{1+w} \right) \right] \tag{4}$$

$$\pi_2 = \left[p_2 \left(1 - \frac{\frac{p_2 - p_1}{b-a} + b + a + \frac{1}{2}(w-1)}{1+w} \right) \right] \tag{5}$$

3.1.1 The price stage

By differentiating the firms' profit functions and solving the first order condition with respect to prices, we find the following reaction functions:

$$\begin{aligned} p_1 &= \frac{1}{2}\left[p_2 + b^2 - a^2 - \frac{1}{2}(a + b + w(b-a)) \right] \\ p_2 &= \frac{1}{2}\left[p_1 - b^2 + a^2 - \frac{1}{2}(3(a+b) + w(b-a)) \right] \end{aligned}$$

The Nash equilibrium in prices is therefore:

$$p_1 = \frac{1}{3}\left(b^2 - a^2\right) + \frac{1}{6}(b-a) + \frac{1}{2}w(b-a) \tag{6}$$

$$p_2 = \frac{1}{3}\left(a^2 - b^2\right) + \frac{5}{6}(b-a) + \frac{1}{2}w(b-a) \tag{7}$$

3.1.2 The location stage

Substituting the optimal prices in (4) and (5) , profits are expressed as a function of locations and w:

$$\pi_1^* = \frac{1}{6}\left[\frac{1}{3}\left(b^2-a^2\right)+\frac{1}{6}(b-a)+\frac{1}{2}w(b-a)\right]\frac{2(b+a)+1+3w}{1+w}$$
$$\pi_2^* = \frac{1}{6}\left[\frac{5}{6}(b-a)+\frac{1}{2}w(b-a)-\frac{1}{3}\left(b^2-a^2\right)\right]\frac{5+3w-2(a+b)}{1+w}$$

The first and second order condition for profit maximization are satisfied for

$$a=-\frac{1}{6}-\frac{1}{2}w+\frac{1}{3}b \tag{8}$$

$$b=\frac{1}{3}a+\frac{5}{6}+\frac{1}{2}w \tag{9}$$

The solution of the system (8) and (9) gives the optimal symmetric locations $a^*=\frac{1}{8}-\frac{3}{8}w$ e $b^*=\frac{7}{8}+\frac{3}{8}w$. If firms locate in a^* and b^*, their optimal prices are $p_1^*=p_2^*=\frac{3}{8}+\frac{3}{4}w+\frac{3}{8}w^2$ and the indifferent consumer is located in $\frac{1}{2}$: the conjecture that the indifferent consumer lies in the central area is fulfilled. We can therefore establish the following proposition:

Proposition 1 *For all values of $w\in(0,1]$ there exist a subgame perfect symmetric Nash equilibrium in prices and locations.*

Notice that the optimal locations coincide with those identified in Lambertini (1994), $a=-\frac{1}{4}$ e $b=\frac{5}{4}$, when $w=1$. The optimal prices are increasing in w: a higher degree of concentration around the midlle (lower w) induces firms to move inwards in order to match the tastes of a growing share of consumers: the more concentrated is the consumer distribution, the less the firms differentiate their products.. This reduced differentiation strenghten price competition. The overall equilibrium shows clearly a dominance of the demand effect: the advantage of acquiring the consumers in the central area dominates the advantage of softening competition through a large product differentiation.

3.2 The marginal consumer lies in one of the external intervals

Now we want to verify whether there exist subgame perfect equilibria, such that the marginal consumer falls in the left external interval $\left[0,\frac{1-w}{2}\right]$. In this interval the density function's slope is $\frac{4}{1-w^2}$. Hence, as $z\in\left[0,\frac{1-w}{2}\right]$, the demand for firm 1 is:

$$q_1(z,w)=\frac{z^2}{\frac{1}{2}(1+w)(1-w)}$$

Substituting (1) in the above expression we obtain the following demand functions in terms of the locations and prices:

$$
\begin{aligned}
q_1 &= \left(\frac{\left(\frac{1}{2} \left[\frac{p_2 - p_1}{b-a} + b - a \right] + a \right)^2}{\frac{1}{2}(1+w)(1-w)} \right) \\
q_2 &= \left[1 - \frac{\left(\frac{1}{2} \left[\frac{p_2 - p_1}{b-a} + b - a \right] + a \right)^2}{\frac{1}{2}(1+w)(1-w)} \right]
\end{aligned}
$$

The profit functions are:

$$
\begin{aligned}
\pi_1 &= p_1 \left(\frac{\left(\frac{1}{2} \left[\frac{p_2 - p_1}{b-a} + b - a \right] + a \right)^2}{(1+w)\left(\frac{1}{2} - \frac{1}{2}w\right)} \right) \\
\pi_2 &= p_2 \left[1 - \frac{\left(\frac{1}{2} \left[\frac{p_2 - p_1}{b-a} + b - a \right] + a \right)^2}{(1+w)\left(\frac{1}{2} - \frac{1}{2}w\right)} \right]
\end{aligned}
$$

3.2.1 The price stage

The first and second order conditions for profit maximization with respect to firm 1's price are satisfied by the following reaction function:[1]

$$p_1 = \frac{1}{3}\left(p_2 + b^2 - a^2\right) \tag{10}$$

As far as firm 2 is concerned, the first and second order conditions are satisfied by the reaction function:[2]

$$p_2 = -\frac{2}{3}b^2 + \frac{2}{3}a^2 + \frac{2}{3}p_1 + \frac{1}{3}\sqrt{\left[p_1 + (a^2 - b^2)\right]^2 + \left[6\left(b^2 + a^2\right) - 12ba\right]\left(1 - w^2\right)} \tag{11}$$

[1] The first order condition is satisfied also by $p_1 = \left(p_2 + b^2 - a^2\right)$. However, at this solution the second order condition for a maximum is not satisfied for $w < 1$.

[2] Again, we have two solutions satisfying the FOC. The other solution

$$p_2^2 = -\frac{2}{3}b^2 + \frac{2}{3}a^2 + \frac{2}{3}p_1 - \frac{1}{3}\sqrt{\left[p_1 + (a^2 - b^2)\right]^2 + \left[6\left(b^2 + a^2\right) - 12ba\right]\left(1 - w^2\right)}$$

does not satisfies the second order condition.

The solution of the system (10) and (11) gives the following Nash equilibrium in prices:

$$\begin{aligned} p_1 &= \rho\,(b-a) \\ p_2 &= 3\rho b - 3\rho a - b^2 + a^2 \end{aligned}$$

where ρ is a root of the polynomial $8x^2 - 2\,(b+a)\,x + \left(w^2-1\right)$.

The existence of two solutions demonstrates that the reaction functions intersect twice. Since the two roots of the polynomial are

$$\begin{aligned} x_1 &= -\frac{1}{8}\,(a+b) + \frac{1}{8}\sqrt{(a+b)^2 + 8\,(1-w^2)} \\ x_2 &= -\frac{1}{8}\,(a+b) - \frac{1}{8}\sqrt{(a+b)^2 + 8\,(1-w^2)} \end{aligned}$$

we may establish that these intersections occur at the following two price couples:

$$p_1 = -\frac{1}{8}\,(a-b)\left(a+b-\sqrt{(a+b)^2+8\,(1-w^2)}\right) \tag{12}$$

$$p_2 = -\frac{3}{8}\left(a+b-\sqrt{(a+b)^2+8\,(1-w^2)}\right)(a-b)+a^2-b^2 \tag{13}$$

using solution x_1, or

$$p_1 = -\frac{1}{8}\,(a-b)\left(a+b+\sqrt{(a+b)^2+8\,(1-w^2)}\right) \tag{14}$$

$$p_2 = -\frac{3}{8}\,(a-b)\left(a+b+\sqrt{(a+b)^2+8\,(1-w^2)}\right)+a^2-b^2 \tag{15}$$

using solution x_2. It must be noticed, however, that only (14) and (15) entail positive prices at equilibrium for both firms. Therefore this is the only economically meaningful economic solution to the price game.

3.2.2 The location stage

The profit functions calculated at the optimal prices are:

$$\pi_1 = \frac{1}{32}\,(a-b)\,\frac{\left(a+b+\sqrt{(a+b)^2+8\,(1-w^2)}\right)^3}{(1+w)\,(-1+w)}$$

$$\pi_2 = \left[-\tfrac{3}{8}\left(a+b+\sqrt{(a+b)^2+8\,(1-w^2)}\right)(a-b)+a^2-b^2\right]$$
$$\left[1-\frac{\left(\frac{1}{2}\left[\frac{\left[-\frac{3}{8}\left(a+b+\sqrt{(a+b)^2+8(1-w^2)}\right)(a-b)+a^2-b^2\right]+\left[\frac{1}{8}(a-b)\left(a+b+\sqrt{(a+b)^2+8(1-w^2)}\right)\right]}{b-a}+b-a\right]+a\right)^2}{\frac{1}{2}(1-w^2)}\right]$$

By differentiating firm 1's profits with respect to its location, we get the following first order condition:

$$\tfrac{1}{32}\left(a+b+\sqrt{(a+b)^2+8\left(1-w^2\right)}\right)^3 \frac{\sqrt{(a+b)^2+8\left(1-w^2\right)}+3\,(a-b)}{\left(w^2-1\right)\sqrt{(a+b)^2+8\left(1-w^2\right)}}=0$$

which gives the optimal location:[3]

$$a=\frac{5}{4}b-\frac{1}{4}\sqrt{9b^2+16\left(1-w^2\right)} \tag{16}$$

If we now differentiate firm 2's profits with respect to its location, and substitute the reaction function (16), tedious calculations (see the Appendix) show that we can identify the following acceptable solution:

$$b=\frac{5}{18}\sqrt{6\left(1-w^2\right)} \tag{17}$$

Using (17) into (16) we have

$$a=-\frac{1}{9}\sqrt{6\left(1-w^2\right)} \tag{18}$$

Therefore, equations (17) and (18) give the optimal locations as a function of w, under the conjecture that the marginal consumer lies in the interval $\left[0,\dfrac{1-w}{2}\right)$.

We now have to verify whether there is a range of w such that this conjecture is actually fulfilled. We first notice that when $w=0$ - i.e. when the density describing the consumers' preferences is a symmetric triangle - the equations (17) and (18) collapse to $a=-\frac{1}{9}\sqrt{6}$ and $b=\frac{5}{18}\sqrt{6}$, that correspond exactly to Tabuchi and Thisse's solutions. In general, when evaluated at the optimal locations (17) and (18), the price equations (14) and (15) become respectively:

$$p_1 = \frac{7}{18}\left(1-w^2\right) \tag{19}$$

$$p_2 = \frac{7}{9}\left(1-w^2\right) \tag{20}$$

By substituting in (17)-(20) into (1), we find the marginal consumer's location as a function of w:

$$z=\frac{1}{2}\frac{\left(1-w^2\right)}{\sqrt{6\left(1-w^2\right)}}+\frac{1}{12}\sqrt{6\left(1-w^2\right)}$$

[3] The above FOC has two solutions. The other is $a=\frac{5}{4}b+\frac{1}{4}\sqrt{9b^2+16\left(1-w^2\right)}$, which does not satisfy the condition $a<b$.

This allows us to establish that the firms' conjectures generating the asymmetric equilibrium (17)-(20) are fulfilled if

$$\frac{1}{2}\frac{\left(1-w^2\right)}{\sqrt{6\left(1-w^2\right)}}+\frac{1}{12}\sqrt{6\left(1-w^2\right)}<\frac{1-w}{2}$$

i.e., if $w<\frac{1}{5}$. By a similar reasoning, it can be proved that, under the same condition on w, a specular asymmetric equilibrium exists, with the marginal consumer lying in the right external interval, with firms located respectively at $a=1-5\sqrt{6\left(1-w^2\right)}/18$ and $b=\sqrt{6\left(1-w^2\right)}/9$. We can therefore establish the following proposition:

Proposition 2 *For $0<w<\frac{1}{5}$ there exist three subgame perfect Nash equilibria in prices and locations, a symmetric equilibrium and two asymmetric ones.*

Notice that in the asymmetric equilibria one firm locates outside the market area, while the other locates in the external interval opposite to that in which lies the marginal consumer. As w increases in the admissible range $\left[0,\frac{1}{5}\right)$, both firms move inwards. Given w, the firm locating within the market area may charge higher prices and enjoy higher profits.

It may be interesting to ask what happens when $w=\frac{1}{5}$. In this case, the asymmetric equilibria defined above make the marginal consumer fall in $\frac{2}{5}$, or specularly in $\frac{3}{5}$, i.e. in correspondence of the hedges of the density function. This is a situation similar to that Tabuchi and Thisse describe with respect to a possible symmetric equilibrium: since the density is not differentiable, the reaction functions are indeed discontinuous.

Let us assume that the solution (17)-(20) holds for $w=\frac{1}{5}$. Then the following results would apply:

$$\begin{aligned} a &= -\frac{4}{15},\ b=\frac{2}{3} \\ p_1 &= \frac{28}{75},\ p_2=\frac{56}{75},\ z=\frac{2}{5} \\ \pi_1 &= \frac{28}{225},\ \pi_2=\frac{112}{225} \end{aligned}$$

In order to ensure that it is indeed an equilibrium, we must exclude the profitability of unilateral deviations from the candidate equilibrium location, in correspondence of the admissible prices for such a location. Let us define the alternative location for firm 1:

$$a'=-\frac{4}{15}+\epsilon$$

If firm 1 locates in a', while firm 2 locates in $\frac{2}{3}$, the marginal consumer lies in the central interval, and the price rules (6)-(7) apply, so that

$$\begin{aligned} p_1 &= \frac{268}{675}-\frac{4}{15}\epsilon-\frac{1}{3}\left(-\frac{4}{15}+\epsilon\right)^2 \\ p_2 &= \frac{488}{675}-\frac{14}{15}\epsilon+\frac{1}{3}\left(-\frac{4}{15}+\epsilon\right)^2 \end{aligned}$$

When evaluated at these prices, and at $a' = -\frac{4}{15} + \epsilon$, and $b = \frac{2}{3}$, the profits of firm 1 turn out to be

$$\frac{5}{6}\left(\frac{268}{675} - \frac{4}{15}\epsilon - \frac{1}{3}\left(-\frac{4}{15} + \epsilon\right)^2\right)\left(\frac{\frac{44}{135} - \frac{2}{3}\epsilon + \frac{2}{3}\left(-\frac{4}{15} + \epsilon\right)^2}{\frac{14}{15} - \epsilon} + \epsilon\right) > \frac{28}{225}$$

for arbitrarily small positive values of ϵ. This is enough to prove that, for $w = \frac{1}{5}$, the solutions (17)-(20) are not subgame perfect equilibria and allows us to establish that for $w = 1/5$ there exists only a subgame perfect symmetric Nash equilibrium in prices and locations, defined by equations (6)-(9).

4 Remarks and conclusions

In this paper we have analysed the effects of the consumers' concentration towards the middle of the space of product characteristics, in a a model of horizontal differentiation with quadratic transportation costs. The consumers' density is assumed to be symmetric and trapezoidal; if the size of the market is normalized to 1, this allows to consider the lenght of the shortest base as a mean preserving spread of consumers' preferences. Clearly, the traditional uniform distribution and a symmetric triangular distribution can be nested into this setup as limit cases.

We have proved that as far as the shortest base is positive - i.e. the distribution is differentiable at 1/2 - a symmetric subgame perfect Nash equilibrium exists in the two stage price-location game. The result we achieve is rather intuitive: starting from the optimal solution obtained under the standard uniform distribution, as preferences become more concentrated around the middle, both firms move inwards and reduce the degree of product differentiation. This clearly reinforces price competition and results in lower equilibrium prices. This result is consistent with a more general intuition that homogeneity of consumers might have important implications in terms of reducing the firms'market power (Benassi, Chirco, and Scrimitore, 2002).

Moreover, our discussion shows that the asymmetric equilibria identified by Tabuchi and Thisse may coexist with the above symmetric equilibrium. For a relevant range of values of our mean preserving spread parameter - when preferences become sufficiently concentrated - two asymmetric subgame perfect equilibria appear, with one firm producing a relatively 'average' product, and the other firm choosing to locate outside the characteristics space. Once one firm decides to produce a product which meets the taste of the large share of consumers located around the middle, the other firm finds it optimal to avoid a destructive price competition by choosing a product with 'extreme' and 'out of market' characteristics. However, this peculiar location choice requires that a low price is charged, in order to capture at least the consumers located at the nearest tail of the distribution. This solution is such that as w increases within its admissible range - the distribution becomes more dispersed - both