position of the overall variance of a series into the contributions of harmonic waves. Thus it is possible to identify dominant cyclical structure in the series under analysis. We are especially interested in the "classical" business cycle structure: the Juglar cycle with a length of 7-10 years and the Kitchin cycle with a length of 3-5 years. The cyclical nature of the business cycle remains debated; modern researcher rather talk about "fluctuations" than "cycles" (Lucas, 1977). However, these phenomena seem to be robust and can be found not only in historical data (A'Hearn and Woitek, 2001), but also in modern economic time series (Reiter and Woitek, 1999). Identifying the relative importance of cyclical components in the GDP of Italian regions is a first step in determining whether there is an intra-national business cycle in Italy. The second step of the analysis will be to see whether there is an inter-relationship between regional cycles, and how this phenomenon changed over time.

2 Methodology

As stated in the introduction, we are interested in the classical business cycle, i.e. cycles with a length of 7-10 years (Juglar cylces), which are superiomposed by shorter, 3-5 year cyles (Kitchin cycles). To address the issues listed above, we decided to employ spectral analysis.[2] A stationary time series X_t can be decomposed into superimposed waves with frequencies $\omega \in [-\pi, \pi]$. The spectrum measures the (marginal) contribution of each wave to the overall variance. It is defined as the Fourier transform of the autocovariance

[2]See e.g. Harvey (1993), pp. 175-179, Granger and Newbold (1986), pp. 48-53, Brockwell and Davis (1991), pp. 434-443, Priestley (1981), vol. II, and Koopmans (1974), pp. 119-164.

function $\gamma_x, \tau = 0, \pm 1, \pm 2, \ldots$:

$$f_x(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \gamma_x(\tau) e^{-i\omega\tau};\ \omega \in [-\pi, \pi]. \tag{1}$$

Integrating the spectrum over the frequency band $[-\pi, \pi]$, we obtain the variance of the series:

$$\gamma_x(0) = \int_{-\pi}^{\pi} f_x(\omega) d\omega. \tag{2}$$

After dividing the spectrum by $\gamma_x(0)$, we can calculate the contribution of cyclical components in a frequency band $[\omega_1, \omega_2]$ to the overall variance by integrating over the interval (and multiplying by two). Thus it is possible to assess the relative importance of the cyclical components in the frequency bands of interest e.g. the classical Juglar and Kitchin cycle.

The multivariate spectrum of two stationary time series X_t and Y_t is defined as the Fourier transform of the covariance function $\mathbf{\Gamma}_{xy}(\tau), \tau = 0, \pm 1, \pm 2, \ldots$:

$$\mathbf{F}_{xy}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \mathbf{\Gamma}_{xy}(\tau) e^{-i\omega\tau};\ \omega \in [-\pi, \pi]. \tag{3}$$

The off-diagonal elements of the spectral density matrix $\mathbf{F}_{xy}(\omega)$, $f_{xy}(\omega)$, are called cross-spectra. The cross spectrum at frequency ω is a complex number and given by

$$f_{xy}(\omega) = c_{xy}(\omega) - i q_{xy}(\omega);\ \omega \in [-\pi, \pi], \tag{4}$$

where $c_{yx}(\omega)$ is the cospectrum and $q_{yx}(\omega)$ is the quadrature spectrum. The cospectrum measures the covariance between the "in-phase" components of

X_t and Y_t, whereas the quadrature spectrum measures the covariance between the "out-of-phase" components. Together with the univariate spectra, the cross spectrum can be used to calculate a measure similar to R^2 in linear regression analysis. This measure is the squared coherency $sc(\omega)$:

$$sc(\omega) = \frac{|f_{xy}(\omega)|^2}{f_x(\omega)f_y(\omega)}; \quad 0 \leq sc(\omega) \leq 1. \tag{5}$$

This measure assesses the degree of linear relationship betwee two series, frequency by frequency. If we are interested in the extent to which the variance of cyclical components of the series X_t in the frequency band $[\omega_1, \omega_2]$ can be attributed to corresponding cyclical components in series Y_t, we can use $sc(\omega)$ to decompose the fraction of overall variance in this interval into an explained and an unexplained part:

$$\int_{\omega_1}^{\omega_2} f_x(\omega)d\omega = \underbrace{\int_{\omega_1}^{\omega_2} sc(\omega)f_x(\omega)d\omega}_{\text{"explained" variance}} + \underbrace{\int_{\omega_1}^{\omega_2} f_u(\omega)d\omega}_{\text{"unexplained" variance}} \quad . \tag{6}$$

We will use this decomposition to compare the degree of linear relationship between cycles in different series for frequency intervals of interest, e.g. given by the Juglar and the Kitchin cycle.

As pointed out by Croux *et al.* (2001), a measure like the squared coherency presented above is not suited for analysing the comovement of time series, because it does not contain information about possible phase shift between cycles in the series X_t and Y_t. In this sense, the correlation coefficient in time domain is more informative, since it is calculated lag by lag, providing both information on the lead-lag structure and the degree of linear relationship between the two series. We can overcome this problem by also presenting the phase spectrum, but the phase spectrum is difficult to interpret, since it is only defined $\mod 2\pi$, and cannot easily be summarised over a frequency band like in the case of the explained variance.[3]

Croux *et al.* (2001) propose an alternative measure, the so-called dynamic correlation $\rho(\omega)$, which measures the correlation between the "in-phase" components of the two series at a frequency ω:

$$\rho(\omega) = \frac{c_{xy}(\omega)}{\sqrt{f_x(\omega) f_y(\omega)}}; \quad -1 \leq \rho(\omega) \leq 1. \tag{7}$$

Using

$$sc(\omega) = \frac{|f_{xy}(\omega)|^2}{f_x(\omega) f_y(\omega)} = \frac{c_{xy}(\omega)^2 + q_{xy}(\omega)^2}{f_x(\omega) f_y(\omega)}, \tag{5'}$$

we can use this idea to further decompose the expression in equation (6):

$$\begin{aligned}
&\int_{\omega_1}^{\omega_2} f_x(\omega) d\omega = \int_{\omega_1}^{\omega_2} sc(\omega) f_x(\omega) d\omega + \int_{\omega_1}^{\omega_2} f_u(\omega) d\omega = \\
&= \int_{\omega_1}^{\omega_2} \frac{c_{xy}(\omega)^2 + q_{xy}(\omega)^2}{f_x(\omega) f_y(\omega)} f_x(\omega) d\omega + \int_{\omega_1}^{\omega_2} f_u(\omega) d\omega = \\
&= \underbrace{\int_{\omega_1}^{\omega_2} \frac{c_{xy}(\omega)^2}{f_x(\omega) f_y(\omega)} f_x(\omega) d\omega}_{\text{"explained" variance (in-phase)}} + \underbrace{\int_{\omega_1}^{\omega_2} \frac{q_{xy}(\omega)^2}{f_x(\omega) f_y(\omega)} f_x(\omega) d\omega}_{\text{"explained" variance (out-of-phase)}} + \\
&+ \underbrace{\int_{\omega_1}^{\omega_2} f_u(\omega) d\omega}_{\text{"unexplained" variance}} .
\end{aligned} \tag{6'}$$

Thus, it is possible to decompose explained variance into the "in-phase" com-

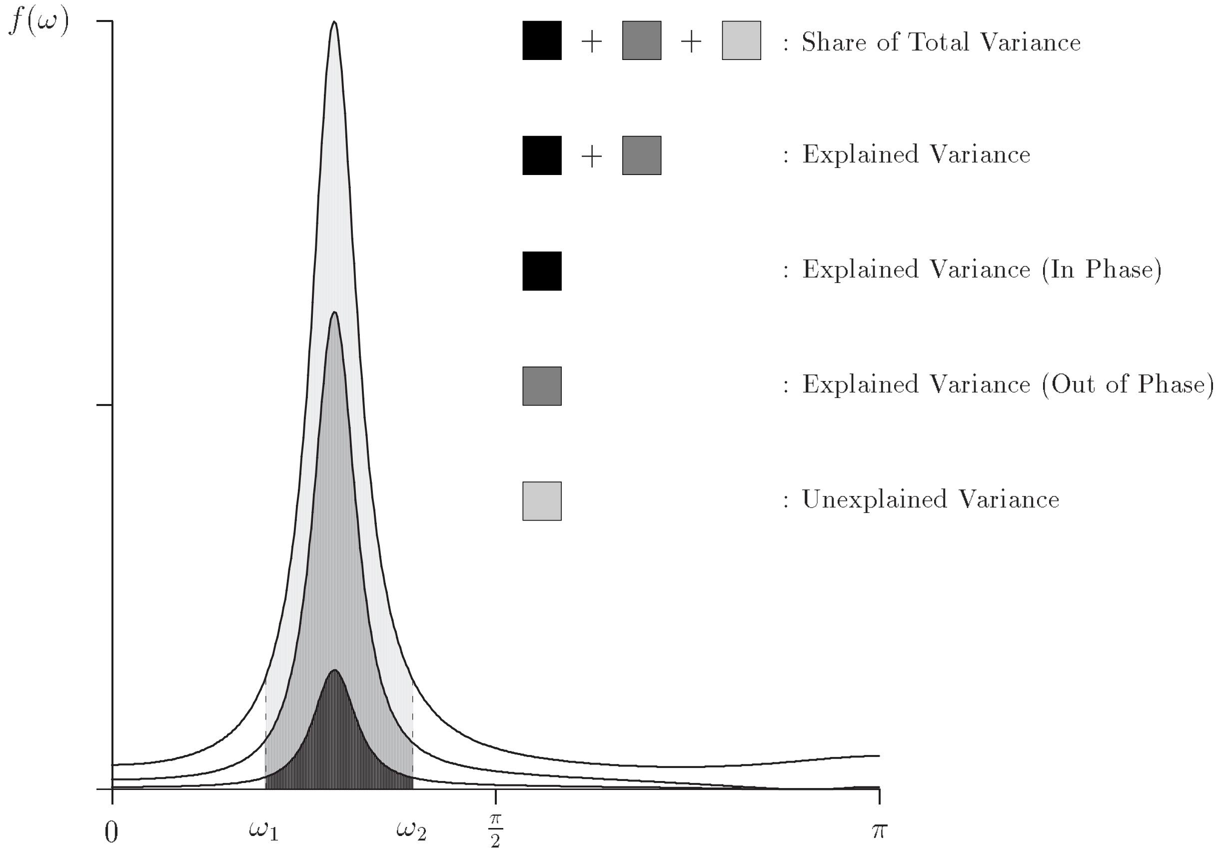
[3]The phase spectrum measures the phase shift between two cycles at frequency ω, and allows to judge the lead-lag relationship between the two series frequency by frequency:

$$\phi_{xy}(\omega) = -\arctan(q_{xy}(\omega)/c_{xy}(\omega)); \; \omega \in [-\pi, \pi].$$

The phase spectrum measures the phase lead of the series X over the series Y at a frequency ω. We will present the phase shift for the frequency where the univariate spectra reach there maximum.

ponent and the "out-of-phase" component, adding some information on the importance of the phase shift in a frequency interval to the R^2 interpretation in equation (6) above.

Figure 1: Variance Decomposition in the Frequency Domain



To estimate the spectra, we fit autoregressive models in the time domain, and calculate the spectra of the estimated models.[4] Assume a univariate AR

[4]This method is based on the seminal work by Burg (1967), who shows that the resulting spectrum is formally identical to a spectrum derived on the Maximum Entropy Principle. This is seen to be a more reasonable approach than the normally used periodogram estimator. The periodogram implies the assumption that all the covariances outside the smple period in the infinite sums in equation (1) and (3) are zero. Given that economic time series are notoriously short, this seems to be a problematic assumption(see

model of order p, with residual variance σ^2. The spectrum is given by

$$f(\omega) = \frac{1}{2\pi} \frac{\sigma^2}{\left|1 - \sum_{j=1}^{p} \alpha_j e^{-i\omega j}\right|^2}; \ \omega \in [-\pi, \pi]. \tag{8}$$

With a VAR model of order p, the spectral density matrix is given by

$$\mathbf{F}(\omega) = \frac{1}{2\pi} \mathbf{A}(\omega)^{-1} \mathbf{\Sigma} \mathbf{A}(\omega)^{-\star}; \ \omega \in [-\pi, \pi]. \tag{9}$$

$\mathbf{\Sigma}$ is the error variance-covariance matrix of the model, and $\boldsymbol{A}(\omega)$ is the Fourier transform of the matrix lag polynomial $\mathbf{A}(L) = \mathbf{I} - \mathbf{A}_1 L - \cdots - \mathbf{A}_p L^p$.[5] But before we can actually estimate the spectrum, we have to solve the problem that the series under consideration are not stationary. The problem we face here is that the widely used filtering methods cause artificial cyclical structure when applied to a series based on a data generating process different form the assumptions underlying the filter.[6] Following Canova (1998), we chose the pragmatic way of comparing the results for the difference filter, the Hodrick-Prescott filter (Hodrick and Prescott, 1980) and the Baxter-King filter (Baxter and King, 1999) with a modification proposed by Woitek (1998).

As stated in the introduction, we want to look at the change of the business cycle phenomenon over time. To do this, we reformulate the VAR model as state space model, treating the VAR parameters as time dependent. The

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the discussion in Priestley, 1981, p. 432, 604-607). For applications to economic time series, see e.g. Hillinger and Sebold-Bender (1992), Woitek (1996), and A'Hearn and Woitek (2001).

[5] L is the backshift operator; the superscript '$\star$' denotes the complex conjugate transpose.

[6] See the discussion in Cogley and Nason (1995), King and Rebelo (1993) and Harvey and Jaeger (1993).

starting point is a VAR of order p

$$\begin{aligned}
\mathbf{x}_t &= \mathbf{c} + \sum_{j=1}^{p} \mathbf{A}_j \mathbf{x}_{t-j} + \mathbf{u}_t = \\
&= \underbrace{\begin{pmatrix} \mathbf{c} & \mathbf{A}_1 & \dots & \mathbf{A}_p \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} 1 \\ \mathbf{x}_{t-1} \\ \vdots \\ \mathbf{x}_{t-p} \end{pmatrix}}_{\mathbf{Z}_{t-1}} + \mathbf{u}_t = \qquad (10) \\
&= \mathbf{A}\mathbf{Z}_{t-1} + \mathbf{u}_t; \; \mathbf{u}_t \sim iid\left(\mathbf{0}, \mathbf{H}\right).
\end{aligned}$$

Vectorizing the above equation, and allowing the parameters of the VAR to be time dependent, gives

$$\mathbf{X}_t = \left(\mathbf{Z}'_{t-1} \otimes \mathbf{I}\right) \underbrace{\text{vec}\mathbf{A}_{t-1}}_{\boldsymbol{\alpha}_{t-1}} + \mathbf{u}_t. \qquad (11)$$

which is the measurement equation in our state space version of equation (10).[7] The transition equation for the VAR parameters is given by

$$\boldsymbol{\alpha}_t = \mathbf{T}\boldsymbol{\alpha}_{t-1} + \boldsymbol{\eta}_t; \; \boldsymbol{\eta}_t \sim iid(\mathbf{0}, \mathbf{Q}). \qquad (12)$$

We assume the matrix $\mathbf{T}$ to be a diagonal matrix with elements $\rho = 0.9$ on the diagonal, forcing the time path for the parameters to be a damped AR(1) process. The elements in the covariance matrices $\mathbf{H}$ and $\mathbf{Q}$ are treated as hyperparameters, and the likelihood function derived based on the cumulated prediciton errors is maximised with respect to these parameters. The solution of this estimation procedure implies a time path for $\boldsymbol{\alpha}_t$, Thus allowing the

[7] For the following, see Harvey (1992).

spectral density matrix in equation (3) to be time dependent. We assume that the univariate spectra in equation (1) are constant, since we are not interested in the change of the length of the cycle in the first place. We want to use the time dependent cross spectra to derive a time dependent version of the explained variance and the phase shift, which enables us to judge the extent to which the regional business cycles move together over time.

3 Results

The first step in the analysis is to compare the univariate cyclical structure of the regional GDPs in the Centre-North and the Mezzogiorno.[8] Following Canova (1998), we judge the robustness of our results by comparing the outcome for three detrending methods: the difference filter, the Hodrick Prescott filter (Hodrick and Prescott, 1980), and the Baxter-King filter (Baxter and King, 1999) in a slightly modified version (Woitek, 1998). In addition, we also perform a significance test of the share of total variance.[9] The results of this exercise are displayed in Table 1.

[8]The series are annual, at 1990 prices. For the observation period 1951-1993, the data are from Paci and Saba (1998). Based on the data from Svimez (2000), we extended the series to include observations up to 2000.

[9]The distribution of the test statistic is constructed based on 1000 replications of a white-noise process.