allowing preference differences between skilled and unskilled workers. ${ }^{3}$
Section 3 shows that the introduction of this assumption may affect the results of the interplay of agglomeration and dispersion forces in determining the equilibrium outcomes, and Section 4 more deeply discusses the preference and competition effects on prices determined by changes in the localization of workers and firms, underlining that the heterogeneity in preferences we introduce may be responsible for the emergence of stable asymmetric equilibria. Finally, Section 5 concludes.

## 2 The model with heterogeneous preferences

We consider a model with two regions, indexed with $r$ and $s$, endowed with two factors/workers, which are distinguished between skilled interregionally mobile workers, indexed with $H$, and unskilled interregionally immobile workers, indexed with $L$. The total number of skilled workers is $H$, while each region is endowed with $L / 2$ unskilled workers. Workers consume $M$ varieties of a modern manufactured good, with each variety denoted by suffix $i$ and consumed in the quantity $q_{i}$, and the quantity $q_{0}$ of a traditional good (the numeraire of the model). Moreover, workers' preferences are represented by the following quadratic utility function:

$$
\begin{equation*}
U\left(q_{0} ; q_{i}, i \in[0, M]\right)=\alpha_{j} \int_{0}^{M} q_{i} d i-\frac{\beta_{j}-\delta_{j}}{2} \int_{0}^{M} q_{i}^{2} d i-\frac{\delta_{j}}{2}\left(\int_{0}^{M} q_{i} d i\right)^{2}+q_{0} \tag{1}
\end{equation*}
$$

with $j=H, L, \alpha_{j}>0$ and $\beta_{j}>\delta_{j}>0$.
The total number (mass) of produced varieties $M$, is the sum of the $n_{r}$ varieties produced in region $r$ and the $n_{s}$ varieties produced in region $s$. Parameters $\alpha_{j}, \beta_{j}$ and $\delta_{j}$ describe workers' preferences. Particularly, parameter $\alpha_{j}$ expresses the intensity of the preference for the differentiated good with respect to the traditional good, and the two parameters $\beta_{j}$ and $\delta_{j}$, with $\beta_{j}>\delta_{j}$, express the intensity of the preference of consumers of type $j$ for differentiation in the consumption of the modern good. Hence, for any given value of $\beta_{j}$, parameter $\delta_{j}$ underlines the degree of

[^0]substitutability between varieties and the higher $\delta_{j}$ is, the higher the degree of substitutability of varieties is.

It is straightforward to notice that the setup we consider only differs from that originally proposed by Ottaviano et al. (2002) in the fact that we introduce the suffix $j$ that characterizes parameters in (1). This suffix draws attention to the fact that skilled and unskilled workers have different preferences. In the rest of the paper we show this simple extension of the original framework may give rise to some interesting results, given that prices will show a new kind of dependence on the spatial distribution of workers and firms, and given that this will allow us to identify a new force related to the demand side that can be at work in determining the regional distribution of the economic activity.

Each worker maximizes (1) given its budget constraint

$$
\begin{equation*}
\int_{0}^{M} p_{i} q_{i} d i+q_{0}=w_{j}+\bar{q}_{0} \tag{2}
\end{equation*}
$$

where $w_{j}$ represents the wage of the worker of type $j$ and $\bar{q}_{0}$ is the endowment of the numeraire of each individual. ${ }^{4}$

The demand function for each variety produced in region $z$ of any worker $j$ located in region $v$ is

$$
\begin{equation*}
q_{z v}^{j}\left(p_{z v}\right)=a_{j}-\left(b_{j}+d_{j} M\right) p_{z v}+d_{j} P_{v} \tag{3}
\end{equation*}
$$

where $v, z=r, s$. The first element in the suffix of quantities and prices expresses the location of producers, while the second, the location of the worker who demands the good. Moreover, the new parameters are obtained in the following way: $a_{j}=\alpha_{j} /\left[\left(\beta_{j}+(M-1) \delta_{j}\right], b_{j}=1 /\left[\beta_{j}+(M-1) \delta_{j}\right]\right.$ and $d_{j}=\delta_{j} /\left(\beta_{j}-\delta_{j}\right)\left[\beta_{j}+(M-1) \delta_{j}\right] .{ }^{5} \quad$ Finally, $P_{z}$ is the price indexes prevailing in region $z$, which, given the symmetry of all firms in a particular region, is

$$
\begin{equation*}
P_{z}=n_{z} p_{z z}+n_{v} p_{v z} \tag{4}
\end{equation*}
$$

[^1]In order to simplify the notation, we drop the suffix $L$ in the three parameters, $\alpha_{L}, \beta_{L}$ and $\delta_{L}$, which refer to unskilled workers and we assume that parameters referred to skilled workers $H$ are proportional to those of unskilled workers, with the factor of proportionality given by $\rho>0$. Therefore, we have that

$$
\begin{align*}
\alpha_{H} & =\alpha_{L} / \rho=\alpha / \rho  \tag{5}\\
\beta_{H} & =\beta_{L} / \rho=\beta / \rho \\
\delta_{H} & =\delta_{L} / \rho=\delta / \rho
\end{align*}
$$

Moreover, from (5) and the definitions of $a_{j}, b_{j}$ and $d_{j}$, it is easily verified that

$$
\begin{equation*}
a_{H}=a_{L}=a ; \quad b_{H}=\rho b_{L} \quad \text { and } \quad d_{H}=\rho d_{L} \tag{6}
\end{equation*}
$$

These simple assumptions allow us to introduce a particular kind of workers' preference heterogeneity, sufficiently simple to handle because it requires that parameters referring to skilled workers are proportional to those of unskilled workers. It would certainly be more general to consider the case in which these parameters were different, without necessarily being proportional. However, as it will later appear, this simplification alone is sufficient to complicate the analysis enough to suggest to avoid making matters worse with a more general framework with different and not necessarily proportional parameters. Hence, we choose to adopt the simplification in (5), since we already obtain some interesting results with it, and given that it can be considered as a particular case of a more general one, in which the results of the former would continue to hold under particular conditions. ${ }^{6}$

In Fig. 1 we plot the inverse demand function for a variety produced in region $z$ of the $j-t h$ worker located in region $v$, that is

$$
\begin{equation*}
p_{z v}=\frac{a-q_{z v}^{j}\left(p_{z v}\right)}{\rho(b+d M)}+\frac{d P_{v}}{(b+d M)} \tag{7}
\end{equation*}
$$

[^2]with $v, z=r, s .{ }^{7} \quad$ In particular, Fig. 1 contains the graphics of two inverse demand functions which are drawn for two different values of $\rho$, that is $\rho_{1}<\rho_{2}$. We note that the two curves intersect in $I$ when $q_{z v}=a$. Moreover, as the graphics show, any increase in the preference for the manufactured good and variety in its consumption, which reduces $\rho$, produces a clockwise rotation of the demand curve around $I$. In particular, we observe that when the preference parameter $\rho$ goes to zero because of a very strong preference for differentiation that tends to annihilate any substitutability between varieties, then
\[

$$
\begin{equation*}
\lim _{\rho \rightarrow 0} q_{z v}^{j}\left(p_{z v}\right)=a \tag{8}
\end{equation*}
$$

\]

## Insert figure 1 about here

As we have already stated, in many of our comments, we refer to the case in which $\rho<1$, which corresponds to the case in which skilled workers have a stronger preference for the modern good and variety in its consumption. These assumptions imply that skilled workers' elasticity of demand is smaller than that of unskilled workers. To justify the assumptions that skilled workers' preference for the modern good is stronger than that of unskilled workers, we may consider that skilled workers' incomes are usually higher than those of unskilled workers. Therefore, by assuming $\rho<1$ we may in some sense reflect Joan Robinson's (1969) thought that increases in agents' incomes make individuals demand less elastic. Moreover, we may justify the fact that skilled workers have a stronger preference for variety in the consumption of the modern good, by observing, for instance, that skilled workers are the ones who produce the differentiated modern goods and, therefore, they are more able to appreciate this differentiation.

Let us define with $\lambda_{r}$ the fraction of skilled workers in region $r$. We notice that each representative firm which produces in region $r$ sells on the local market the quantity

$$
\begin{equation*}
q_{r r}\left(p_{r r}\right)=q_{r r}^{L}\left(p_{r r}\right) \frac{L}{2}+q_{r r}^{H}\left(p_{r r}\right) \lambda_{r} H \tag{9}
\end{equation*}
$$

[^3]The quantity sold on the foreign market is instead

$$
\begin{equation*}
q_{r s}\left(p_{r s}\right)=q_{r s}^{L}\left(p_{r s}\right) \frac{L}{2}+q_{r s}^{H}\left(p_{r s}\right)\left(1-\lambda_{r}\right) H \tag{10}
\end{equation*}
$$

Similar expressions can be obtained for firms that produce in region $s$.
Operating profits of a representative firm which produces in $r$ are obtained by adding operating profits which derive from sales in $r, \pi_{r r}$, to those derived from sales in $s, \pi_{r s}$, which are, respectively,

$$
\begin{equation*}
\pi_{r r}=p_{r r} q_{r r} \text { and } \pi_{r s}=\left(p_{r s}-t\right) q_{r s} \tag{11}
\end{equation*}
$$

The production cost of each firm in region $z=r, s$ is generated by the fixed cost that firms have to sustain in order to employ $f$ skilled workers and are given by

$$
\begin{equation*}
T C_{r}=f w_{r} \tag{12}
\end{equation*}
$$

Therefore, pure profits $\pi_{r}$ of the representative firm which produces in region $r$ are

$$
\begin{equation*}
\pi_{r}=\pi_{r r}+\pi_{r s}-f w_{r} \tag{13}
\end{equation*}
$$

Finally, the assumption of full employment of workers implies that

$$
\begin{equation*}
H_{r}=\lambda_{r} H=n_{r} f \quad \text { and } \quad H_{s}=\left(1-\lambda_{r}\right) H=n_{s} f \tag{14}
\end{equation*}
$$

## 3 Preference differences and equilibrium outcomes

In this section we derive equilibrium prices and quantities and skilled workers' indirect utility functions used to evaluate the stability properties of the different potential outcomes. First of all, from the first order conditions for the maximization of profits, we obtain the following equilibrium price for varieties sold at home

$$
\begin{equation*}
p_{z z}^{*}\left(\lambda_{z}, \rho\right)=\frac{t d_{L}\left(\frac{L}{2}+\rho \lambda_{z} H\right)\left(1-\lambda_{z}\right) M+2 a\left(\frac{L}{2}+\lambda_{z} H\right)}{2\left(2 b_{L}+d_{L} M\right)\left(\frac{L}{2}+\rho \lambda_{z} H\right)} \tag{15}
\end{equation*}
$$

where $z=r, s$. The asterisk always denotes equilibrium values.


Fig. 1


[^0]:    ${ }^{3}$ We choose to work with this model because of its tractability. Moreover, we notice that Tabuchi and Thisse (2001) also adopt this structure.

[^1]:    ${ }^{4}$ As usual, the individual endowment of the numeraire is supposed to be sufficiently large to have a positive consumption of the traditional good in equilibrium for each individual.
    ${ }^{5}$ See, for instance, Ottaviano et al. (2002) and Fujita and Thisse (2002).

[^2]:    ${ }^{6}$ The nature of our results would be the same when parameters for skilled workers are all lower (higher) than those for unskilled workers. However, for any other case different from ours it would be possible to compute equilibrium results, even though for their interpretation we should use simulations.

[^3]:    ${ }^{7}$ It is clear that given our assumption, the demand function of unskilled workers corresponds to the case in which $\rho=1$.

