coordination problems. However, these interactions may be more complex with decreasing returns of regional productivity levels that may appear when workers density is sufficiently high, as shown, for instance, in figure 1, where we represent productivity levels, a_r and a_v, as a function of regional skilled workers density, H_r.[6]

Insert figure 1 about here

3 Centripetal and centrifugal forces in the core-periphery equilibrium

In this section we evaluate the sustainability of full agglomeration equilibria of the modern sector in one region and we discuss how different parameters concur in the determination of the intensities of centripetal and centrifugal forces at work.

As usual, the agglomeration of all firms in region v is a sustainable equilibrium only if the ratio between the sales that a firm could realize by relocating its production in region r, Q_{mir}, and those required to break even, Q_{mir}^*, is smaller than 1, that is if:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \phi^{1+\frac{\sigma\mu}{\sigma-1}} \left[1 + \left(\frac{1}{\phi^2} - 1\right)\frac{(1-\mu)}{2}\right] < 1 \tag{21}$$

Expression (21) is derived considering the case in which real wages of skilled mobile workers are equal in the two regions in order to give them the incentive to work in both regions. It is well known that an expression similar to (21) can be derived if we assume that firms produce quantities that correspond to null profits, that is long run equilibrium quantities, and we examine if skilled workers have any incentive to move from the core v to the periphery r. Particularly, skilled workers do not move towards the periphery r when their real wage in the periphery r is smaller than in the core v. Therefore, the core periphery outcome with agglomeration in v is sustainable when

$$\varpi_{hr}^{\sigma} = a_v^{\sigma\mu} \left(\frac{a_v}{a_r}\right)^{1-\sigma} \phi^{1+\frac{\sigma\mu}{\sigma-1}} \left[1 + \left(\frac{1}{\phi^2} - 1\right)\frac{(1-\mu)}{2}\right] < a_v^{\sigma\mu} = \varpi_{hv}^{\sigma} \tag{22}$$

[6] The function for a_r is $a_r = 1 + 0.2H_r(1-H_r)(H_r - 1/2)$, and for a_v is $a_v = 1 + 0.2H_v(1-H_v)(H_v - 1/2)$ with $H_v = 1 - H_r$.

or equivalently when

$$\left(\frac{\varpi_{hr}}{\varpi_{hv}}\right)^{\sigma}=\left(\frac{a_v}{a_r}\right)^{1-\sigma}\phi^{1+\frac{\sigma\mu}{\sigma-1}}\left[1+\left(\frac{1}{\phi^2}-1\right)\frac{(1-\mu)}{2}\right]<1 \tag{23}$$

As it is well-known, inequality in (23) coincides with that in (21).

We note that if the technological advantage of the core region v is sufficiently high, full agglomeration of the modern sector in v is sustainable for all freeness of trade levels, ϕ.

Without interregional technological differences, that is with $a_v = a_r = 1$, (21) coincides with the expression derived in the standard Economic Geography model. When we introduce potential technological differences in (21), we find the additional term

$$\left(\frac{a_v}{a_r}\right)^{1-\sigma} \tag{24}$$

Let us define terms in (21), or equivalently in (23), in the following way

$$\gamma=\left(\frac{a_v}{a_r}\right)^{1-\sigma}, \qquad \chi=\phi^{1+\frac{\sigma\mu}{\sigma-1}} \quad \text{and} \quad \delta=\left(\frac{1}{\phi^2}-1\right)\frac{(1-\mu)}{2} \tag{25}$$

where $\gamma, \delta > 0$ and $0 < \chi < 1$.

Hence, we rewrite expression (21) as follows

$$\gamma\chi\left(1+\delta\right)<1 \tag{26}$$

If we apply a logarithmic transformation to (26), we may state what follows:

Proposition 1 *Full agglomeration of the modern sector in region v is a sustainable equilibrium if and only if*

$$\ln\gamma+\ln\chi+\ln\left(1+\delta\right)<0 \tag{27}$$

Vice versa, full agglomeration of the modern sector in region v is not sustainable if the inequality sign in (27) is replaced by $\geq$.

Proposition 2 *Expression (27) can be used to assess the "intensities" of centripetal and centrifugal forces, and to evaluate how these intensities are affected by parameters.*

First, let us consider the standard economic geography model with $a_v = a_v = 1$ and, therefore, $\ln\gamma = 0$. We observe that when trade is free, $\phi = 1$, then $\chi = 1$ and $\delta = 0$. Increasing trade costs, which corresponds to decreasing ϕ, will always imply a negative value of $\ln\chi$ given that, for $\phi \in (0,1)$, it is always true that $0 < \chi < 1$. The fact that $\ln \chi$ is always negative for $\phi \in (0,1)$, suggests that it can be used as an index of the intensity of all *traditional centripetal (agglomeration) forces*, which act in the standard economic geography model when the modern sector is fully agglomerated in one region. Indeed, when trade costs are positive, expression $\ln \chi$ is always negative, and its negative value tends to decrease the left side of inequality (27) representing the total contribution of traditional agglomeration forces when the modern sector is fully agglomerated in one region. We suggest that we can use $-\ln \chi$ as a *direct index* of traditional centripetal forces because the higher is its value, the smaller is $\ln \chi$ and, consequently, the left side of (27) tends to be smaller.[7] Moreover, we note that χ is increasing in ϕ, and to higher χ values correspond lower absolute values of $\ln \chi$. Therefore, the intensity of traditional centripetal forces in correspondence with full agglomeration equilibrium tends to decrease when economic integration increases.

Term $\ln(1+\delta)$ is always positive for $\phi \in (0,1)$, because $\delta > 0$. Therefore, we suggest that $\ln(1+\delta)$ may be used as an index of the intensity of *traditional centrifugal (dispersion) forces*, which act in the standard economic geography model when the modern sector is fully agglomerated in a region, v in our example. Indeed, for given values of the other two addends in (27), an increase in $\ln(1+\delta)$ contributes to raising the left side of (27), and therefore it tends to destabilize the agglomerated equilibrium. Moreover, it is easy to verify that δ is decreasing in ϕ and, for this reason, we may write that the traditional centrifugal forces' intensity in correspondence with full agglomeration equilibrium tends to decrease when the degree of economic integration increases.

Both indexes $-\ln \chi$ and $\ln(1+\delta)$ are referred to traditional agglomeration and dispersion forces, given that they can be derived from the standard Economic Geography model without interregional technological differences when $a_v = a_v = 1$. For this reason, we define these indexes as *fixed-technology forces indexes*.

[7] Obviously, to have a negative left side of expression (27) the other terms, γ and δ, should be null, or sufficiently low in absolute values.

Finally, the third term in (27), that is $\ln\gamma$, represents the contribution of agglomeration or dispersion forces that are not in the standard Economic Geography model. These forces are determined by the value of technological differences (a_v/a_v) and, for this reason, they are defined as *variable-technology forces*. Unless we know relative regional productivity levels, it is impossible a priory to define if $\ln\gamma$ represents the additional contribution of an agglomeration force or of a dispersion force. In fact, this term can be either greater or less than zero, depending on whether region v is more or less productive in the modern sector than region r. When region v is more productive than region r (with $a_v > a_r$), $\ln\gamma$ represents an index of the intensity of a *variable-technology centripetal (agglomeration) force*. On the contrary, when region v is less productive than region r (with $a_v < a_r$), $\ln\gamma$ represents an index of the intensity of a *variable-technology centrifugal (dispersion) force*. Obviously, this term vanishes if the two regions have reached the same technological development level, as in the standard core-periphery model with $a_r = a_v = 1$. We notice that the force described by $\ln\gamma$ has a Ricardian nature.

The particular indexes just identified can be useful for evaluating the contribution of all the above mentioned forces, fixed-technology or variable-technology, to the stability outcome of the core-periphery equilibrium. Moreover, they may be used to evaluate how the intensities of fixed-technology or variable-technology forces vary with the parameters in the model. This is summarized in Table 1, in which we report the sign of the derivative of all terms $-\ln\chi$, $\ln(1+\delta)$, $\ln\gamma$ and $-\ln\gamma$, which respectively represent the magnitude of centripetal and centrifugal fixed-technology and variable-technology forces, with respect to parameters listed in the first column. We note that if $a_v > a_r$, technological differences give rise to a *centripetal* variable-technology force because region v is not only the core in which all manufacturing production is concentrated, but it is also the more developed region. In this case, to measure the intensity of the force we use $-\ln\gamma$ (in order to have a positive index) and we have to refer to the third column of Table 1. On the contrary, when $a_v < a_r$, the region in which manufacturing production is concentrated, v, is less developed than the periphery r. Hence, when we evaluate the variable-technology forces with agglomeration in region v, we must consider that they assume a *centrifugal* nature, given that v is relatively less productive than r in the manufacturing sector. Therefore, to measure the intensity of variable-technology centrifugal forces in this case we use $\ln \gamma$, and we refer to the forth column of Table 1. Zeros in Table 1 denote the case in which parameters have no effect on the intensity of a particular force.

	FIXED-TECHNOLOGY FORCES INTENSITY		VARIABLE-TECHNOLOGY FORCES INTENSITY	
	CENTRIPETAL	CENTRIFUGAL	CENTRIPETAL IF $a_v > a_r$	CENTRIFUGAL IF $a_v < a_r$
	$-\ln \chi$	$\ln(1+\delta)$	$-\ln \gamma$	$\ln \gamma$
ϕ	$-$	$-$	0	0
μ	$+$	$-$	0	0
σ	$-$	0	$+$	$+$
a_v	0	0	$+$	$-$
a_r	0	0	$-$	$+$

Table 1

As parameters change, γ, χ and δ change, reflecting the fact that the intensities of agglomeration and dispersion forces are modified. Let us, for instance, consider fixed-technology forces. We may synthesize our findings in the following way.

Proposition 3 *Fixed-technology centripetal forces intensity increases when the degree of freeness of trade (ϕ) and the elasticity of substitution between industrial varieties (σ) decrease, and when the expenditure share on industrial goods (μ) increases.*

Proposition 3 suggests that an increase in the degree of freeness of trade, that is, a decrease in trade costs, gives to mobile workers a smaller incentive to stay in the core, because trade costs saved when working in the core are smaller. On the other hand, if trade costs increase, the intensity of agglomeration forces also increases. This positive relationship between trade costs and the intensity of the fixed-technology agglomeration force is also found in the case of the symmetric equilibrium.[8] Instead, when the elasticity of substitution, σ, increases, competition among

[8] See Baldwin et al. [1].

different firms increases and increasing returns to scale are less intensively exploited therefore producing a smaller incentive for firms to stay in the core region. Finally, higher μ values imply for workers that choose to work in the core region the opportunity to avoid trade costs on a wider share of expenditure on manufactured consumption goods. Therefore, when μ increases, the incentive to stay in the core becomes stronger.

Proposition 4 *The intensity of fixed-technology centrifugal forces increases when the degree of freeness of trade (ϕ) and the expenditure share on manufacturing or industrial good (μ) decrease.*

These fixed-technology centrifugal forces are originated either by the demand of immobile workers in the periphery, or by the more intensive competition that firms must face in the core. If the level of freeness of trade is high, there is not a strong incentive for firms to relocate their production in the periphery in order to satisfy unskilled workers' demand in the same region, because low trade costs allow these firms to continue to produce in the core where they may exploit the wider local market dimensions and then export in the periphery with low trade costs. On the contrary, high trade costs strengthen dispersion forces. These results confirm the ones found for the symmetric equilibrium within the standard Krugman's [6] core periphery model.[9]

If we consider parameter μ, the incentive to satisfy the peripheral demand by means of local production is smaller, the smaller the share of expenditure that workers devote to manufacturing varieties is.

When we analyze variable-technology forces intensities, which determine $\ln \gamma$, we distinguish two cases: (1) the core region is also the more developed region with $a_v > a_r$; (2) the more productive region is the peripheral region r in which, for a sort of perverse specialization, there is no production of the manufactured goods with $a_v < a_r$. We may therefore state what follows:

Proposition 5 *The active variable-technology force is the centripetal one if $a_v > a_r$, or the centrifugal one if, instead, $a_v < a_r$.*

Proposition 6 *For given values of all other factors, an increase in the elasticity of substitution (σ) provides incentives for more intensive exploitation of technological differences, that is the technological advantage of a region, strengthening active variable-technology forces.*

[9] See Baldwin et al. [1].

Particularly, if $a_v > a_r$, an increase in the value of σ strengthens variable-technology forces which are active in this case, that is centripetal forces, allowing a better exploitation of technological advantage of the core region v. On the contrary, if $a_v < a_r$, an increase of σ strengthens variable-technology forces active in this particular case, that is centrifugal forces, with a better exploitation of the technological advantage of periphery r.

Proposition 7 *The active variable technology forces are strengthened by an increase in the already existent technological gap and, on the contrary, they are weakened by a reduction in the regional productivity gap.*

Differentiating (26) in the neighborhood of ϕ^S, we obtain some standard and some new results. First of all, we find that

$$\frac{d\phi^S}{d\mu} < 0 \tag{28}$$

As summarized in table 1, this result derives from the fact that when μ increases, both centripetal forces are intensified and centrifugal forces are weakened. This standard economic geography result is presented in a new light for full agglomeration equilibria, by allowing us to distinguish the effects that changes in μ have on fixed-technology centripetal and centrifugal forces.

Moreover, we find that the following results hold:[10]

$$\frac{d\phi^S}{da_v} < 0 \quad \text{and} \quad \frac{d\phi^S}{da_r} > 0 \tag{29}$$

From expression (29), we may point out that an increase in a_v, which corresponds to an increase in the technological gap in favor of the core v (when this is already the leader in the development process), or a reduction of the technological lag of the core v (when, for some reason, there exists an adverse specialization that leads the less productive region to be the core region in which manufacturing is agglomerated), is translated into a reinforcement of the sustainability of production concentration in v. Indeed, full agglomeration of the modern sector in v becomes sustainable even for smaller levels of the freeness of trade, with ϕ^S decreasing. On the contrary,

[10] Standard economic geography models do not include productivity differential in their assumptions. However, a few exceptions exist such as the model by Ricci [13] and by Forslid and Wooton [3].

agglomeration in v is weakened when ϕ^S increases, and this happens when region r reduces its technological gap with respect to the core v (when the core v is the more productive region), or when r increases its technological advantage (in the case in which the core v is the less productive region).

A further novelty is found in the sign of the following expression

$$\frac{d\phi^S}{d\sigma}$$

Indeed, $d\phi^S/d\sigma$ is unambiguously positive, as in standard economic geography models, only if the core region is the less productive region. In this case, an increase in σ corresponds to a reduction of the intensity of fixed-technology centripetal forces and to an increase in the intensity of variable-technology centrifugal forces. Therefore, when $a_v < a_r$ it is always true that

$$\frac{d\phi^S}{d\sigma} > 0 \tag{30}$$

On the contrary, if the core v is also the more productive region, as it is more likely to occur, an increase in σ is reflected in a reduction of the intensity of fixed-technology centripetal forces, and in an increase in the intensity of variable-technology centripetal forces. We can clearly state which of the two opposite effects prevail, when we know the values of the parameters in the model. However, we may write that the effect on variable technology forces is stronger with

$$\frac{d\phi^S}{d\sigma} < 0 \tag{31}$$

when, for given σ, ϕ^S and μ, the productivity level available for firms in v is such that

$$a_v > a_r \left(\phi^S\right)^{\frac{\mu}{(\sigma-1)^2}} \tag{32}$$

Given that $\phi \in [0, 1]$, (32) is always satisfied and the relationship between ϕ^S and σ is negative.

Finally, we note that the changes summarized in Table 1, could be helpful to identify whether, when using data on a particular agglomerated outcome, the agglomeration is driven by pecuniary externalities in the standard economic geography model or by the geographically localized externalities considered in the paper when $\kappa > 0$. If, for instance, a negative relationship is found between agglomeration and σ, then agglomeration is driven by pecuniary externalities. On the contrary, when a positive relationship between is found between agglomeration and σ, then agglomeration is driven by geographically localized externalities.

We notice that the modified version of the standard economic geography that we present confirms the finding by Venables [15] that with Ricardian differences there could be equilibria characterized by the localization of sectors in the region in which they have a comparative disadvantage. In fact, if $a_r > a_v$, agglomeration in region v that has a comparative advantage in agriculture may be sustainable for intermediate levels of integration, as is shown in figure 2.

Insert figure 2 about here

However, this may happen only if the disadvantage is not too wide.[11] Moreover, even in the case of a small lag, agglomeration in the "wrong" region v is never sustainable for high and low level of freeness of trade. In fact, when $\phi = 1$ and when $\phi \to 0$, it can be easily verified that $\varpi_{hr} > \varpi_{hv}$, and therefore agglomeration in the less developed region v is not sustainable. As a consequence, we find that when the two regions are sufficiently integrated, the comparative advantages dominate and production localization reflects comparative advantages with manufacturing production agglomerated in the more productive region, while the agricultural good is produced in both regions. A similar result is obtained by Forslid and Wooton ([3]) who find that "when trade barriers are sufficiently low, comparative advantage takes the upper hand, pulling workers and production from the core to the other region". However, their results are different, since comparative advantage in their case acts as a dispersion force, because it is considered within the manufacturing sector, and it boosts a symmetric stable outcome. In our case, instead, the

[11] Following Baldwin et al. ([1], p. 50) we can show that expression (23) can be rewritten as $f(\phi) = (a_v/a_r)^{1-\sigma} \, \phi^{\frac{\sigma\mu}{\sigma-1}-1} \dfrac{\phi^2(1+\mu)+(1-\mu)}{2} - 1 < 0$. $f(\phi)$ is such that: (1) $f(1) = (a_v/a_r)^{1-\sigma} - 1 \lesseqgtr 0$ and $f'(1) > 0$; (2) $f(0) \gtreqless 0$ when $\left(\frac{a_r}{a_v}\right)^{\sigma-1} \gtreqless \lim_{\phi\to 0}\left(\phi^{\frac{\rho-\mu}{\rho}} \frac{2}{(1-\mu)}\right)$, and $f'(0) < 0$; (3) $f(\phi)$ has a unique minimum; (4) $\partial f(\phi)/\partial a_v < 0$; and (5) $\partial f(\phi)/\partial a_r > 0$. If the technological disadvantage of the core region v is too high, $f(\phi)$ is always positive and agglomeration in v in never sustainable.

comparative advantage acts as an agglomeration force given that it has an intersectoral nature and favours a sustainable core-periphery outcome. We note what follows

Proposition 8 *When the manufacturing sector is agglomerated in the region with a technological disadvantage, an increase of trade costs enhances agglomeration if trade costs are small (ϕ is low) and dispersion if trade costs are intermediate (ϕ is intermediate). However, when the manufacturing sector is agglomerated in the region with a technological advantage, an increase of trade costs may only reduce agglomeration.*

Previous proposition recalls the results by Ricci ([13], p. 367), who, in a different framework, obtains that "if the large country has a comparative disadvantage, a rise in trade costs may enhance agglomeration".

Finally, we have not so far considered how the productivity differential gap is determined. One determinant could be the existence of geographically localized spillovers which may produce higher productivity levels in the region in which all skilled workers are concentrated. However, if a too high concentration of workers creates some problems of coordination in the organization of the production process, then this kind of congestion force at work would reverse the technological gap in favour of the region with the lowest concentration of workers.

4 Symmetric equilibrium stability

In this section we reclassify centripetal and centrifugal forces with respect to the symmetric equilibrium in order to take into account the fact that technological differences may exist. To evaluate the intensity of centripetal and centrifugal forces in the symmetric equilibrium we rewrite expression (15) in the following way:

$$R_r = w_{hr} = a_r^{\frac{\rho}{1-\rho}} \mu \left[\left(\frac{w_{hr}}{p_{mr}} \right)^{-\frac{\rho}{1-\rho}} (w_{hr} H_r + L) + \left(\frac{w_{hr}}{p_{mv}} \right)^{-\frac{\rho}{1-\rho}} \phi \left(w_{hv} H_v + L \right) \right] \tag{33}$$

R_r are equilibrium sales of a firm in region r, and an analogous expression, R_v, can be obtained for region v. We evaluate R_r in order to define different centripetal and centrifugal forces that are in action when the two regional economies are in the neighborhood of the symmetric equilibrium. Particularly, as in the previous section, we distinguish between fixed-technology (or traditional