1 Introduction

In this work we extend the standard Economic Geography model by Krugman [6] in two ways: (1) we introduce potential interregional technological differences in productivity levels of skilled workers employed in the modern sector, and (2) we describe these differences as a function of skilled workers regional density and, therefore, of their migration processes.

First, we show that Krugman's results [6] are enriched by the consideration of potential international differences in productivity development levels, because in this way, we can give a more complete description of centripetal and centrifugal forces which, by means of their interactions, determine equilibrium stability properties. Moreover, we suggest a sufficiently simple way to evaluate the "intensity" of agglomeration and dispersion forces when full agglomeration equilibria are considered; an evaluation that has so far been considered rather complex if referred to these particular equilibria. This difficulty is lesser when we analyze the symmetric equilibrium, which corresponds to a uniform distribution of the economic activity. In fact, Baldwin et al. ([1], p. 45), state that "the CP [Core Periphery] model is astoundingly difficult to manipulate since the nominal wages are determined by equations that cannot be solved analytically" and that "at the symmetric equilibrium this difficulty is much attenuated. Due to the symmetry, all effects are equal and opposite". In this work, we still have the difficulties mentioned by Baldwin et. al. [1], but we suggest a sufficiently simple way to evaluate how different parameters concur to determine the intensity of centripetal and centrifugal forces. These forces will be distinguished between "fixed-technology", or pecuniary externality forces, and "variable-technology" forces.[1] While the former are the ones traditionally considered in Economic Geography models, the latter are not always taken into account in these models, and are so-called because they derive from regional productivity differentials.

This is not the first paper in which interregional technological differences are introduced in

[1] The distinction of fixed-technology and variable-technology forces has been introduced by Nocco [11].

economic geography models in manufacturing goods production. Indeed, they have already been introduced by Ricci [13], Forslid and Wooton [3], Venables [15] and Nocco [11]. In particular, Ricci [13], who compares Ricardian comparative advantages with absolute advantage, introduces interregional technological differences by considering interregional relative differences in variable amounts required to produce only two varieties of modern goods. However, Ricci [13] does not study how economic integration affects the results of the model with labour mobility. Forslid and Wooton [3] depart from the standard core-periphery model by Krugman [6] introducing interregional technological differences in fixed costs sustained by firms in the modern sector. Moreover, they consider also differences in the production costs of different varieties within each region. Venables [15] uses the framework he and Krugman developed in 1996 [10] with backward and forward linkages between upstream and downstream firms to study the interaction between comparative advantages and pecuniary externalities with different manufacturing sectors. Nocco [11] introduces interregional technological differences in total factor productivities in Puga's work [12] and considers interregional knowledge spillovers that act through trade, only when regions are sufficiently integrated and the technological gap is not too high relatively to learning capabilities of lagging countries. In the present work, we introduce interregional technological differences in Krugman's model [6].

In this paper we allow for technological differences in productivity levels of skilled workers employed in the modern sector, while there are no interregional differences in the production of the traditional or agricultural good. As a consequence, the more productive region in the manufacturing sector has a comparative advantage in manufacturing, while the other in agriculture. Forslid and Wooton [3] write that "comparative advantage will be a force that strengthens the tendencies for all manufacturing to agglomerate in one region". However, we show that not always this region is the right one, that is the region in which firms agglomerate may be the one with a comparative disadvantage in manufacturing.

Moreover, we introduce an additional agglomeration force, because we analyze how stability properties of equilibria may be affected when regional productivity levels depend upon skilled workers concentration (or density), and therefore upon their interregional movements. More specifically, in the paper we assume that a higher skilled workers density may give rise to higher regional manufacturing productivity levels. In this assumption we follow Krugman [8] and Ciccone and Hall [2]. In particular, Ciccone and Hall ([2], p. 68) find that: "increasing returns to density play a crucial role for explaining the large differences in average labor productivity across U.S. states. We estimate that doubling employment density in a county increases average labor productivity by 6 percent. This degree of locally increasing returns can explain more than half of the variation in labor productivity across U.S. states."[2]

This kind of relationship describes a positive externality of technological and geographically localized nature.[3] Introducing this externality in Krugman's model [6], we show that firms and workers' spatial distribution are influenced not only by the degree of economic integration between the two considered regions, but also by the size of the above mentioned geographical technological externality, which influences regional productivity levels.

The work is organized as follows. Section 2 introduces the model with potential interregional productivity differences in the modern, or manufacturing, sector. Section 3 presents the results for the sustainability of the symmetric equilibrium and introduces the indexes that may be used in this case to evaluate "intensities" of centripetal and centrifugal forces, distinguished between fixed-technology and variable-technology forces. Section 4 discusses symmetric equilibrium stability properties, and shows how these properties are modified when the positive relationship between skilled workers density and productivity levels exists. Conclusions are drawn in section 5.

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[2] Ciccone and Hall [2] explain the large differences in labor productivity across U.S. states by estimating the relationship between county employment density and productivity at the state level. In particular, they derive this relation from two models: "one based on local geographical externalities and the other on the diversity of local intermediate services" [2].

[3] See Scitovsky [14] for a classification of pecuniary and technological externalities.

2 The model

Let us consider two regions, or two countries, north, n, and south, s. Both regions are inhabited by L unskilled workers. Moreover, H skilled workers are interregionally mobile. Following Baldwin et al. [1], we adopt the following normalizations for the number of workers: $H = 1$ and $L = (1-\mu)/(2\mu)$. As usual, μ represents expenditure share on manufacturing or industrial goods, with $0 < \mu < 1$. We notice that every time we use suffix r, $r = n, s$, and that if both r and v are used in the same expression, $r, v = n, s$ and $r \neq v$.

Each worker j, skilled or unskilled, consumes a traditional (or agricultural) homogenous good, and many varieties of a modern (or manufactured, industrial) good, which are partly locally produced and partly imported. Preferences, identical for all workers, are described by the following utility function

$$U(Q_{mjr}, Q_{ajr}) = Q_{mjr}^{\mu} {Q_{ajr}}^{1-\mu} \tag{1}$$

where Q_{ajr} is the traditional good consumption by individual j in r, and Q_{mjr} is the modern composite good consumption, which includes all locally produced and imported varieties. The composite manufacturing good, Q_m, is obtained by the aggregation of all industrial varieties i produced by n_r firms in region r, and n_v firms in region v, with

$$Q_m = \left(\int_{i=1}^{n_r+n_v} Q_{mi}^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

$\sigma > 1$ is the elasticity of substitution between any pair of industrial varieties. Moreover, we remind that $\rho = \frac{\sigma-1}{\sigma}$ represents an inverse measure of preference intensity for variety in the consumption of manufactured goods. Each worker in region r maximizes (1) given the budget constraint:

$$p_{mr} Q_{mjr} + p_{ar} Q_{ajr} = y_{jr} \tag{3}$$

where p_{mr} and p_{ar} are, respectively, the price of the composite industrial good and of the agricultural good in region r, while y_{jr} is j^{th} worker's income in r. As usual, all firms in a particular region are symmetric. With iceberg costs for the industrial goods, τ have to be shipped in order to sell one unit of them in the other region. Therefore, the industrial price index in region r is

$$p_{mr} = \left(n_r p_r^{1-\sigma} + n_v \tau^{1-\sigma} p_v^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{4}$$

From now on, following Baldwin et al. [1], we define $\phi = \tau^{1-\sigma}$, with $\phi \in [0,1]$. ϕ is a measure of the "freeness" of trade, with ϕ equal to zero when trade costs are infinite, to one when they are null, and with ϕ that increases when trade costs decrease.

As usual, utility optimization yields demand for variety i produced in region r

$$Q_{mir} = p_r^{-\sigma}\left(\frac{1}{p_{mr}^{1-\sigma}} E_{mr} + \frac{1}{p_{mv}^{1-\sigma}} \phi E_{mv}\right) \tag{5}$$

where E_{mr} and E_{mv} are, respectively, expenditures on industrials goods in region r and in region v. Let us define w_h as skilled workers' wage, w_l as unskilled workers' wage and π_i as profits of firm i. Then, solving the utility maximization problem for each worker and aggregating expenditure on industrial goods in region r, we obtain regional expenditure on manufacturing goods

$$E_{mr} = \mu\left(w_{hr} H_r + w_{lr} L + n_r \pi_{ir}\right) \tag{6}$$

Expenditure levels in the agricultural good in region r and v, E_{ar} and E_{av}, are derived in a similar way

$$E_{ar} = (1-\mu)\left(w_{hr} H_r + w_{lr} L + n_r \pi_{ir}\right) \tag{7}$$

Skilled workers are interregionally mobile and employed in the production of industrial varieties. Unskilled workers are not mobile and they are employed in the production of the agricultural good. To produce one unit of the traditional good, one unit of unskilled worker is employed. Therefore, with perfect competition, each region r produces $Q_{ar} = L$ units of the traditional good. This good is homogeneous and it is exchanged without trade costs. Therefore, its price must be equal in the two regions, and, given that it is chosen as the numéraire, we have that

$$w_{lr} = w_{lv} = p_{ar} = p_{av} = 1 \tag{8}$$

Each industrial variety is obtained with increasing returns to scale, which are internal to firms and derive from a fixed cost of production. Specifically, to produce Q_{mir} units of the i^{th} variety, firms have to employ β/a_r units of skilled workers for each unit produced, and α units of skilled workers independent of the production level. The variable cost may differ between the two regions, given that a_r may be different from a_v. Parameter a_r may be used as a measure of skilled workers productivity in a particular region. Obviously, if region r is more productive than v, then $a_r > a_v$. Hence, the H_{mir} workers required by the i^{th} firm to produce Q_{mir} units of the industrial goods are

$$H_{mir} = \alpha + \frac{\beta}{a_r} Q_{mir} \tag{9}$$

The cost function for each firm i in region r is

$$TC_{mir} = w_{hr}(\alpha + \frac{\beta}{a_r} Q_{mir}) \tag{10}$$

Notice that the average production cost is decreasing in regional productivity level a_r. Moreover, for α and β we adopt the following normalizations: $\alpha = 1/\sigma$ and $\beta = (\sigma - 1)/\sigma$.[4] Each firm maximizes its profits by taking the price indices p_m as given, and sets the mill price p_r with a mark up over the marginal production cost

$$p_r = \frac{\sigma\beta}{(\sigma - 1)\, a_r} w_{hr} = \frac{w_{hr}}{a_r} \tag{11}$$

with the price paid by consumers located in region v equal to

$$p_v = \tau p_r$$

Profits realized by each firm i in region r are

$$\pi_{ir} = \frac{w_{hr}}{\sigma} \left(\frac{Q_{mir}}{a_r} - 1 \right) \tag{12}$$

From the previous expression we know that each firm i in region r produces

$$Q_{mir} = \frac{\sigma a_r \pi_{ir}}{w_{hr}} + a_r \tag{13}$$

[4] We follow Puga [12].

The industrial good market is imperfectly competitive, and it is characterized by a free entry and exit assumption for firms. Therefore, profits must be null in equilibrium, and the equilibrium production level for each firm in r is

$$Q_{mir}^{*} = a_r \tag{14}$$

The equilibrium production level is higher, the higher the regional skilled workers' productivity level is. Increases in workers' productivity levels are translated not only in increases of produced quantities, but also in regional production competitiveness levels, given that for a given wage rate, manufactured goods' prices decrease. Finally, we notice that if region r has a higher skilled workers' productivity level, with $a_r > a_v$, then it has a comparative advantage in the production of the manufacturing sector.

Equating (13) and (5), and substituting manufacturing expenditure values in both regions when profits are null from (6) and (7), we obtain the following equilibrium condition for each industrial variety

$$a_r = p_r^{-\sigma}\mu\left[\frac{1}{p_{mr}^{1-\sigma}}\left(w_{hr}H_r + L\right) + \frac{1}{p_{mv}^{1-\sigma}}\phi\left(w_{hv}H_v + L\right)\right] \tag{15}$$

Since skilled workers are interregionally mobile and never unemployed, it must be verified that

$$H_r + H_v = H = 1 \tag{16}$$

Skilled workers' real wages in r, ϖ_r, are

$$\varpi_r = \frac{w_{hr}}{p_{mr}^{\mu}} \tag{17}$$

Finally, we observe that total incomes produced in both regions r and v are, respectively,

$$Y_r = H_r w_{hr} + \frac{(1-\mu)}{2\mu} \quad \text{and} \quad Y_v = (1-H_r)w_{hv} + \frac{(1-\mu)}{2\mu} \tag{18}$$

As we can observe, the model so far described is the one proposed by Krugman [6], which has been modified in order to take into account potential interregional technological differences in skilled workers productivity levels when $a_r \neq a_v$.

The model is completed by the description of how regional productivity levels are determined. Equations that describe the values of a_r and a_v need to be continuous and differentiable around the symmetric equilibrium. Moreover, in the symmetric equilibrium both regions must be perfectly identical, because they are described by the same parameter values, and they have the same endogenous variable values. Particularly, in the symmetric equilibrium mobile workers and firms are uniformly distributed between the two regions, with $H_r = H_v = 1/2$, and regional productivity levels are equal, with $a_r = a_v = a$.

Following Krugman [8] we assume that labour productivity levels depend on the number of workers employed in a particular region. Particularly, we assume that regional productivity level, a_r, is a function of skilled worker density, H_r, with

$$a_r = f(H_r) \tag{19}$$

If skilled workers are uniformly distributed between the two regions, the productivity levels are equal with $a_r = a_v = a = 1$.[5]

Let us start from the symmetric equilibrium where $a_r = a_v = f(1/2) = 1$. Equation (19) tells us that if a certain number of skilled workers moves from the north to the south, the southern productivity level increases and, on the contrary, the northern productivity level decreases. Moreover, around the symmetric equilibrium it must be the case that for each region r

$$\left.\frac{\partial a_r}{\partial H_r}\right|_{H_r=H_v=\frac{1}{2}} = \kappa \tag{20}$$

Geographically localized externalities may have different sources. They may have positive nature, with $\kappa > 0$, if they derive from *knowledge spillovers processes* or *learning by interacting processes* that foster higher productivity levels where workers density is higher. Vice versa, they may also have a negative nature, with $\kappa < 0$, if they derive from phenomena of *congestions* or of

[5] Note that we normalize to 1 regional productivity levels when skilled workers are uniformly distributed.

coordination problems. However, these interactions may be more complex with decreasing returns of regional productivity levels that may appear when workers density is sufficiently high, as shown, for instance, in figure 1, where we represent productivity levels, a_r and a_v, as a function of regional skilled workers density, H_r.[6]

Insert figure 1 about here

3 Centripetal and centrifugal forces in the core-periphery equilibrium

In this section we evaluate the sustainability of full agglomeration equilibria of the modern sector in one region and we discuss how different parameters concur in the determination of the intensities of centripetal and centrifugal forces at work.

As usual, the agglomeration of all firms in region v is a sustainable equilibrium only if the ratio between the sales that a firm could realize by relocating its production in region r, Q_{mir}, and those required to break even, Q^*_{mir}, is smaller than 1, that is if:

$$\frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \phi^{1+\frac{\sigma\mu}{\sigma-1}} \left[1 + \left(\frac{1}{\phi^2} - 1\right)\frac{(1-\mu)}{2}\right] < 1 \tag{21}$$

Expression (21) is derived considering the case in which real wages of skilled mobile workers are equal in the two regions in order to give them the incentive to work in both regions. It is well known that an expression similar to (21) can be derived if we assume that firms produce quantities that correspond to null profits, that is long run equilibrium quantities, and we examine if skilled workers have any incentive to move from the core v to the periphery r. Particularly, skilled workers do not move towards the periphery r when their real wage in the periphery r is smaller than in the core v. Therefore, the core periphery outcome with agglomeration in v is sustainable when

$$\varpi^\sigma_{hr} = a_v^{\sigma\mu} \left(\frac{a_v}{a_r}\right)^{1-\sigma} \phi^{1+\frac{\sigma\mu}{\sigma-1}} \left[1 + \left(\frac{1}{\phi^2} - 1\right)\frac{(1-\mu)}{2}\right] < a_v^{\sigma\mu} = \varpi^\sigma_{hv} \tag{22}$$

[6] The function for a_r is $a_r = 1 + 0.2H_r(1 - H_r)(H_r - 1/2)$, and for a_v is $a_v = 1 + 0.2H_v(1 - H_v)(H_v - 1/2)$ with $H_v = 1 - H_r$.

or equivalently when

$$\left(\frac{\varpi_{hr}}{\varpi_{hv}}\right)^{\sigma}=\left(\frac{a_v}{a_r}\right)^{1-\sigma}\phi^{1+\frac{\sigma\mu}{\sigma-1}}\left[1+\left(\frac{1}{\phi^2}-1\right)\frac{(1-\mu)}{2}\right]<1 \tag{23}$$

As it is well-known, inequality in (23) coincides with that in (21).

We note that if the technological advantage of the core region v is sufficiently high, full agglomeration of the modern sector in v is sustainable for all freeness of trade levels, ϕ.

Without interregional technological differences, that is with $a_v = a_r = 1$, (21) coincides with the expression derived in the standard Economic Geography model. When we introduce potential technological differences in (21), we find the additional term

$$\left(\frac{a_v}{a_r}\right)^{1-\sigma} \tag{24}$$

Let us define terms in (21), or equivalently in (23), in the following way

$$\gamma=\left(\frac{a_v}{a_r}\right)^{1-\sigma}, \qquad \chi=\phi^{1+\frac{\sigma\mu}{\sigma-1}} \quad \text{and} \quad \delta=\left(\frac{1}{\phi^2}-1\right)\frac{(1-\mu)}{2} \tag{25}$$

where $\gamma, \delta > 0$ and $0 < \chi < 1$.

Hence, we rewrite expression (21) as follows

$$\gamma\chi\,(1+\delta)<1 \tag{26}$$

If we apply a logarithmic transformation to (26), we may state what follows:

Proposition 1 *Full agglomeration of the modern sector in region v is a sustainable equilibrium if and only if*

$$\ln\gamma+\ln\chi+\ln(1+\delta)<0 \tag{27}$$

Vice versa, full agglomeration of the modern sector in region v is not sustainable if the inequality sign in (27) is replaced by $\geq$.

Proposition 2 *Expression (27) can be used to assess the "intensities" of centripetal and centrifugal forces, and to evaluate how these intensities are affected by parameters.*

First, let us consider the standard economic geography model with $a_v = a_v = 1$ and, therefore, $\ln\gamma = 0$. We observe that when trade is free, $\phi = 1$, then $\chi = 1$ and $\delta = 0$. Increasing trade costs, which corresponds to decreasing ϕ, will always imply a negative value of $\ln\chi$ given that, for $\phi \in (0,1)$, it is always true that $0 < \chi < 1$. The fact that $\ln \chi$ is always negative for $\phi \in (0,1)$, suggests that it can be used as an index of the intensity of all *traditional centripetal (agglomeration) forces*, which act in the standard economic geography model when the modern sector is fully agglomerated in one region. Indeed, when trade costs are positive, expression $\ln \chi$ is always negative, and its negative value tends to decrease the left side of inequality (27) representing the total contribution of traditional agglomeration forces when the modern sector is fully agglomerated in one region. We suggest that we can use $-\ln \chi$ as a *direct index* of traditional centripetal forces because the higher is its value, the smaller is $\ln \chi$ and, consequently, the left side of (27) tends to be smaller.[7] Moreover, we note that χ is increasing in ϕ, and to higher χ values correspond lower absolute values of $\ln \chi$. Therefore, the intensity of traditional centripetal forces in correspondence with full agglomeration equilibrium tends to decrease when economic integration increases.

Term $\ln(1+\delta)$ is always positive for $\phi \in (0,1)$, because $\delta > 0$. Therefore, we suggest that $\ln(1+\delta)$ may be used as an index of the intensity of *traditional centrifugal (dispersion) forces*, which act in the standard economic geography model when the modern sector is fully agglomerated in a region, v in our example. Indeed, for given values of the other two addends in (27), an increase in $\ln(1+\delta)$ contributes to raising the left side of (27), and therefore it tends to destabilize the agglomerated equilibrium. Moreover, it is easy to verify that δ is decreasing in ϕ and, for this reason, we may write that the traditional centrifugal forces' intensity in correspondence with full agglomeration equilibrium tends to decrease when the degree of economic integration increases.

Both indexes $-\ln \chi$ and $\ln(1+\delta)$ are referred to traditional agglomeration and dispersion forces, given that they can be derived from the standard Economic Geography model without interregional technological differences when $a_v = a_v = 1$. For this reason, we define these indexes as *fixed-technology forces indexes.*

[7] Obviously, to have a negative left side of expression (27) the other terms, γ and δ, should be null, or sufficiently low in absolute values.

Finally, the third term in (27), that is $\ln \gamma$, represents the contribution of agglomeration or dispersion forces that are not in the standard Economic Geography model. These forces are determined by the value of technological differences (a_v/a_v) and, for this reason, they are defined as *variable-technology forces*. Unless we know relative regional productivity levels, it is impossible a priory to define if $\ln \gamma$ represents the additional contribution of an agglomeration force or of a dispersion force. In fact, this term can be either greater or less than zero, depending on whether region v is more or less productive in the modern sector than region r. When region v is more productive than region r (with $a_v > a_r$), $\ln \gamma$ represents an index of the intensity of a *variable-technology centripetal (agglomeration) force*. On the contrary, when region v is less productive than region r (with $a_v < a_r$), $\ln \gamma$ represents an index of the intensity of a *variable-technology centrifugal (dispersion) force*. Obviously, this term vanishes if the two regions have reached the same technological development level, as in the standard core-periphery model with $a_r = a_v = 1$. We notice that the force described by $\ln \gamma$ has a Ricardian nature.

The particular indexes just identified can be useful for evaluating the contribution of all the above mentioned forces, fixed-technology or variable-technology, to the stability outcome of the core-periphery equilibrium. Moreover, they may be used to evaluate how the intensities of fixed-technology or variable-technology forces vary with the parameters in the model. This is summarized in Table 1, in which we report the sign of the derivative of all terms $-\ln \chi$, $\ln(1+\delta)$, $\ln \gamma$ and $-\ln \gamma$, which respectively represent the magnitude of centripetal and centrifugal fixed-technology and variable-technology forces, with respect to parameters listed in the first column. We note that if $a_v > a_r$, technological differences give rise to a *centripetal* variable-technology force because region v is not only the core in which all manufacturing production is concentrated, but it is also the more developed region. In this case, to measure the intensity of the force we use $-\ln \gamma$ (in order to have a positive index) and we have to refer to the third column of Table 1. On the contrary, when $a_v < a_r$, the region in which manufacturing production is concentrated, v, is less developed than the periphery r. Hence, when we evaluate the variable-technology forces with agglomeration in region v, we must consider that they assume a *centrifugal* nature, given that v is relatively less productive than r in the manufacturing sector. Therefore, to measure the intensity of variable-technology centrifugal forces in this case we use $\ln \gamma$, and we refer to the forth column of Table 1. Zeros in Table 1 denote the case in which parameters have no effect on the intensity of a particular force.

	FIXED-TECHNOLOGY FORCES INTENSITY		VARIABLE-TECHNOLOGY FORCES INTENSITY	
	CENTRIPETAL	CENTRIFUGAL	CENTRIPETAL IF $a_v > a_r$	CENTRIFUGAL IF $a_v < a_r$
	$-\ln \chi$	$\ln(1+\delta)$	$-\ln \gamma$	$\ln \gamma$
ϕ	$-$	$-$	0	0
μ	$+$	$-$	0	0
σ	$-$	0	$+$	$+$
a_v	0	0	$+$	$-$
a_r	0	0	$-$	$+$

Table 1

As parameters change, γ, χ and δ change, reflecting the fact that the intensities of agglomeration and dispersion forces are modified. Let us, for instance, consider fixed-technology forces. We may synthesize our findings in the following way.

Proposition 3 *Fixed-technology centripetal forces intensity increases when the degree of freeness of trade (ϕ) and the elasticity of substitution between industrial varieties (σ) decrease, and when the expenditure share on industrial goods (μ) increases.*

Proposition 3 suggests that an increase in the degree of freeness of trade, that is, a decrease in trade costs, gives to mobile workers a smaller incentive to stay in the core, because trade costs saved when working in the core are smaller. On the other hand, if trade costs increase, the intensity of agglomeration forces also increases. This positive relationship between trade costs and the intensity of the fixed-technology agglomeration force is also found in the case of the symmetric equilibrium.[8] Instead, when the elasticity of substitution, σ, increases, competition among

[8] See Baldwin et al. [1].

different firms increases and increasing returns to scale are less intensively exploited therefore producing a smaller incentive for firms to stay in the core region. Finally, higher μ values imply for workers that choose to work in the core region the opportunity to avoid trade costs on a wider share of expenditure on manufactured consumption goods. Therefore, when μ increases, the incentive to stay in the core becomes stronger.

Proposition 4 *The intensity of fixed-technology centrifugal forces increases when the degree of freeness of trade (ϕ) and the expenditure share on manufacturing or industrial good (μ) decrease.*

These fixed-technology centrifugal forces are originated either by the demand of immobile workers in the periphery, or by the more intensive competition that firms must face in the core. If the level of freeness of trade is high, there is not a strong incentive for firms to relocate their production in the periphery in order to satisfy unskilled workers' demand in the same region, because low trade costs allow these firms to continue to produce in the core where they may exploit the wider local market dimensions and then export in the periphery with low trade costs. On the contrary, high trade costs strengthen dispersion forces. These results confirm the ones found for the symmetric equilibrium within the standard Krugman's [6] core periphery model.[9]

If we consider parameter μ, the incentive to satisfy the peripheral demand by means of local production is smaller, the smaller the share of expenditure that workers devote to manufacturing varieties is.

When we analyze variable-technology forces intensities, which determine $\ln \gamma$, we distinguish two cases: (1) the core region is also the more developed region with $a_v > a_r$; (2) the more productive region is the peripheral region r in which, for a sort of perverse specialization, there is no production of the manufactured goods with $a_v < a_r$. We may therefore state what follows:

Proposition 5 *The active variable-technology force is the centripetal one if $a_v > a_r$, or the centrifugal one if, instead, $a_v < a_r$.*

Proposition 6 *For given values of all other factors, an increase in the elasticity of substitution (σ) provides incentives for more intensive exploitation of technological differences, that is the technological advantage of a region, strengthening active variable-technology forces.*

[9] See Baldwin et al. [1].

Particularly, if $a_v > a_r$, an increase in the value of σ strengthens variable-technology forces which are active in this case, that is centripetal forces, allowing a better exploitation of technological advantage of the core region v. On the contrary, if $a_v < a_r$, an increase of σ strengthens variable-technology forces active in this particular case, that is centrifugal forces, with a better exploitation of the technological advantage of periphery r.

Proposition 7 *The active variable technology forces are strengthened by an increase in the already existent technological gap and, on the contrary, they are weakened by a reduction in the regional productivity gap.*

Differentiating (26) in the neighborhood of ϕ^S, we obtain some standard and some new results. First of all, we find that

$$\frac{d\phi^S}{d\mu} < 0 \tag{28}$$

As summarized in table 1, this result derives from the fact that when μ increases, both centripetal forces are intensified and centrifugal forces are weakened. This standard economic geography result is presented in a new light for full agglomeration equilibria, by allowing us to distinguish the effects that changes in μ have on fixed-technology centripetal and centrifugal forces.

Moreover, we find that the following results hold:[10]

$$\frac{d\phi^S}{da_v} < 0 \quad \text{and} \quad \frac{d\phi^S}{da_r} > 0 \tag{29}$$

From expression (29), we may point out that an increase in a_v, which corresponds to an increase in the technological gap in favor of the core v (when this is already the leader in the development process), or a reduction of the technological lag of the core v (when, for some reason, there exists an adverse specialization that leads the less productive region to be the core region in which manufacturing is agglomerated), is translated into a reinforcement of the sustainability of production concentration in v. Indeed, full agglomeration of the modern sector in v becomes sustainable even for smaller levels of the freeness of trade, with ϕ^S decreasing. On the contrary,

[10] Standard economic geography models do not include productivity differential in their assumptions. However, a few exceptions exist such as the model by Ricci [13] and by Forslid and Wooton [3].

agglomeration in v is weakened when ϕ^S increases, and this happens when region r reduces its technological gap with respect to the core v (when the core v is the more productive region), or when r increases its technological advantage (in the case in which the core v is the less productive region).

A further novelty is found in the sign of the following expression

$$\frac{d\phi^S}{d\sigma}$$

Indeed, $d\phi^S/d\sigma$ is unambiguously positive, as in standard economic geography models, only if the core region is the less productive region. In this case, an increase in σ corresponds to a reduction of the intensity of fixed-technology centripetal forces and to an increase in the intensity of variable-technology centrifugal forces. Therefore, when $a_v < a_r$ it is always true that

$$\frac{d\phi^S}{d\sigma} > 0 \tag{30}$$

On the contrary, if the core v is also the more productive region, as it is more likely to occur, an increase in σ is reflected in a reduction of the intensity of fixed-technology centripetal forces, and in an increase in the intensity of variable-technology centripetal forces. We can clearly state which of the two opposite effects prevail, when we know the values of the parameters in the model. However, we may write that the effect on variable technology forces is stronger with

$$\frac{d\phi^S}{d\sigma} < 0 \tag{31}$$

when, for given σ, ϕ^S and μ, the productivity level available for firms in v is such that

$$a_v > a_r \left(\phi^S\right)^{\frac{\mu}{(\sigma-1)^2}} \tag{32}$$

Given that $\phi \in [0,1]$, (32) is always satisfied and the relationship between ϕ^S and σ is negative.

Finally, we note that the changes summarized in Table 1, could be helpful to identify whether, when using data on a particular agglomerated outcome, the agglomeration is driven by pecuniary externalities in the standard economic geography model or by the geographically localized externalities considered in the paper when $\kappa > 0$. If, for instance, a negative relationship is found between agglomeration and σ, then agglomeration is driven by pecuniary externalities. On the contrary, when a positive relationship between is found between agglomeration and σ, then agglomeration is driven by geographically localized externalities.

We notice that the modified version of the standard economic geography that we present confirms the finding by Venables [15] that with Ricardian differences there could be equilibria characterized by the localization of sectors in the region in which they have a comparative disadvantage. In fact, if $a_r > a_v$, agglomeration in region v that has a comparative advantage in agriculture may be sustainable for intermediate levels of integration, as is shown in figure 2.

Insert figure 2 about here

However, this may happen only if the disadvantage is not too wide.[11] Moreover, even in the case of a small lag, agglomeration in the "wrong" region v is never sustainable for high and low level of freeness of trade. In fact, when $\phi = 1$ and when $\phi \to 0$, it can be easily verified that $\varpi_{hr} > \varpi_{hv}$, and therefore agglomeration in the less developed region v is not sustainable. As a consequence, we find that when the two regions are sufficiently integrated, the comparative advantages dominate and production localization reflects comparative advantages with manufacturing production agglomerated in the more productive region, while the agricultural good is produced in both regions. A similar result is obtained by Forslid and Wooton ([3]) who find that "when trade barriers are sufficiently low, comparative advantage takes the upper hand, pulling workers and production from the core to the other region". However, their results are different, since comparative advantage in their case acts as a dispersion force, because it is considered within the manufacturing sector, and it boosts a symmetric stable outcome. In our case, instead, the

[11] Following Baldwin et al. ([1], p. 50) we can show that expression (23) can be rewritten as $f(\phi) = (a_v/a_r)^{1-\sigma}\,\phi^{\frac{\sigma\mu}{\sigma-1}-1}\dfrac{\phi^2(1+\mu)+(1-\mu)}{2} - 1 < 0$. $f(\phi)$ is such that: (1) $f(1) = (a_v/a_r)^{1-\sigma} - 1 \lesseqqgtr 0$ and $f'(1) > 0$; (2) $f(0) \gtreqqless 0$ when $\left(\frac{a_r}{a_v}\right)^{\sigma-1} \gtreqqless \lim_{\phi\to 0}\left(\phi^{\frac{\rho-\mu}{\rho}}\frac{2}{(1-\mu)}\right)$, and $f'(0) < 0$; (3) $f(\phi)$ has a unique minimum; (4) $\partial f(\phi)/\partial a_v < 0$; and (5) $\partial f(\phi)/\partial a_r > 0$. If the technological disadvantage of the core region v is too high, $f(\phi)$ is always positive and agglomeration in v in never sustainable.

comparative advantage acts as an agglomeration force given that it has an intersectoral nature and favours a sustainable core-periphery outcome. We note what follows

Proposition 8 *When the manufacturing sector is agglomerated in the region with a technological disadvantage, an increase of trade costs enhances agglomeration if trade costs are small (ϕ is low) and dispersion if trade costs are intermediate (ϕ is intermediate). However, when the manufacturing sector is agglomerated in the region with a technological advantage, an increase of trade costs may only reduce agglomeration.*

Previous proposition recalls the results by Ricci ([13], p. 367), who, in a different framework, obtains that "if the large country has a comparative disadvantage, a rise in trade costs may enhance agglomeration".

Finally, we have not so far considered how the productivity differential gap is determined. One determinant could be the existence of geographically localized spillovers which may produce higher productivity levels in the region in which all skilled workers are concentrated. However, if a too high concentration of workers creates some problems of coordination in the organization of the production process, then this kind of congestion force at work would reverse the technological gap in favour of the region with the lowest concentration of workers.

4 Symmetric equilibrium stability

In this section we reclassify centripetal and centrifugal forces with respect to the symmetric equilibrium in order to take into account the fact that technological differences may exist. To evaluate the intensity of centripetal and centrifugal forces in the symmetric equilibrium we rewrite expression (15) in the following way:

$$R_r = w_{hr} = a_r^{\frac{\rho}{1-\rho}} \mu \left[\left(\frac{w_{hr}}{p_{mr}} \right)^{-\frac{\rho}{1-\rho}} (w_{hr} H_r + L) + \left(\frac{w_{hr}}{p_{mv}} \right)^{-\frac{\rho}{1-\rho}} \phi \left(w_{hv} H_v + L \right) \right] \tag{33}$$

R_r are equilibrium sales of a firm in region r, and an analogous expression, R_v, can be obtained for region v. We evaluate R_r in order to define different centripetal and centrifugal forces that are in action when the two regional economies are in the neighborhood of the symmetric equilibrium. Particularly, as in the previous section, we distinguish between fixed-technology (or traditional forces) and variable-technology (or non-traditional forces).[12] Starting from a symmetric equilibrium, if a technological gap arises, it needs to be closed in order to allow a return to the same. An initial departure from the perfectly symmetric situation described in the symmetric equilibrium, gives rise to traditional and less traditional agglomeration and dispersion forces. *Traditional* forces are identified by Baldwin et al. [1] as follows: two agglomeration forces, respectively called *market-access* and *cost of living effects*, and one dispersion force, the so-called *market-crowding effect*. These forces continue to act in our version of the model, and they may be commented following Baldwin et al. [1]. However, in our version, technological differences may add a further agglomeration force because they drive away the two economies from the symmetric equilibrium. It is, in fact, particularly easy to verify that when, for instance, a_r increases with respect to a_v, for given values of other factors in R_r and R_v, R_r increases with respect to R_v giving firms the incentive to move from region v to region r, increasing labor demand in r and, in so doing, encouraging a more intensive migration toward this region. We notice that the intensity of these forces increases for higher values of μ.

In order to study symmetric equilibrium stability in greater detail, we must remember that it requires all variables, endogenous and exogenous, and all parameters to be equal in the two regions. Specifically, from a technological point of view, this requires that $a_r = a_v = a$. Moreover, following Fujita et al. [4], we recall that in the neighborhood of the symmetric equilibrium, changes in the value of a regional variable are associated with changes of the same amount, but of the inverse sign, in the correspondent variable in the other region. For instance, a change in the number of skilled workers in a region, dH_r, is associated with the change $dH_v = -dH_r$ in the other region. This is still valid in our simple extension of the standard model, where we also need to consider that regional productivity level changes, described by (19), depend on the interregional distribution of skilled workers. It is easily verified that in the neighborhood of the symmetric equilibrium $da_r = -da_v$.

[12] Expression (33) is useful for comparing our results with the ones presented in Baldwin et al. [1].

From the choice of the numèraire and from the assumption on the traditional good, we know that

$$p_{ar} = p_{av} = w_{lr} = w_{lv} = 1 \tag{34}$$

After substituting prices from (11), we derive the first order Taylor expansion in the neighborhood of the symmetric equilibrium for: each manufacturing variety produced in both regions (15), the manufacturing price indexes (4), skilled workers' real wages (17), and total regional incomes (18). The resulting expressions are used to derive, after a number of appropriate substitutions, expression (35):

$$\begin{aligned} d\varpi_h \quad &= \quad \frac{2Zp_m^{-\mu}}{(1-\sigma)^2\Delta'}\left\{\mu(1-2\sigma) - Z\left[1-\sigma\left(1+\mu^2\right)\right]\right\} dH + \\ &\quad + \frac{p_m^{-\mu}}{(1-\sigma)\Delta'}\left\{Z^2\left(1-\sigma-\mu^2\right) + Z\mu + (\sigma-1)\right\}\frac{da}{a} \end{aligned} \tag{35}$$

where $Z = \frac{(1-\phi)}{(1+\phi)}$ and $\Delta' = \frac{(1-\sigma)Z^2 - Z\mu + \sigma}{(1-\sigma)}$. Z is an index of the "closedness" of trade with its value that range from 0, with free trade to 1, with autarchy. Expression (35) shows how regional real wages changes in the neighborhood of the symmetric equilibrium, $d\varpi_h$, depend on changes in the regional number of skilled workers dH, and on changes in technological development levels, da.

We have already noticed that in correspondence to the symmetric equilibrium we have: $a_r = a_v = a$ and we normalize $a = 1$. Therefore (35) can be rewritten as follows

$$\begin{aligned} d\varpi_h \quad &= \quad \frac{p_m^{-\mu}}{(1-\sigma)^2\Delta'}\{\left[2Z\mu(1-2\sigma) - 2Z^2\left(1-\sigma\left(1+\mu^2\right)\right)\right] dH + \\ &\quad +(1-\sigma)[Z^2\left(1-\sigma-\mu^2\right) + Z\mu + (\sigma-1)]da\} \end{aligned} \tag{36}$$

Finally, from the first order Taylor expansion in the neighborhood of the symmetric equilibrium for equation (20), we get

$$da = \kappa dH \tag{37}$$

Substituting (37) in (36) gives

$$d\varpi_h = \frac{p_m^{-\mu}}{(1-\sigma)^2\Delta'}\{\left[2Z\mu(1-2\sigma) - 2Z^2\left(1-\sigma\left(1+\mu^2\right)\right)\right] + \tag{38}$$
$$+(1-\sigma)[Z^2\left(1-\sigma-\mu^2\right) + Z\mu + (\sigma-1)]\kappa\}dH$$

Finally, we recall that $\rho = (\sigma - 1)/\sigma$, and we rewrite (38) as follows

$$\frac{d\varpi_h}{dH} = \frac{p_m^{-\mu}(1-\rho)}{\rho\left[1-(1-\rho)Z\mu-\rho Z^2\right]}\{2Z\left[\mu(1+\rho) - Z\left(\rho+\mu^2\right)\right] + \tag{39}$$
$$-\frac{\rho}{1-\rho}[Z^2\left(\rho+\mu^2(1-\rho)\right) - (1-\rho)Z\mu - \rho]\kappa\}$$

The symmetric equilibrium is stable if $d\varpi_h/dH$ is less than 0, and on the contrary it is unstable if it is greater than (or equal to) 0. We observe that the sign of $p_m^{-\mu}(1-\rho)/\rho$ in (39) is always positive. Hence, the sign of $d\varpi_h/dH$ depends on the sign of two terms: the denominator, which we call d, and the term in curly brackets, which we call g. These terms are respectively

$$d = 1 - (1-\rho)Z\mu - \rho Z^2 \tag{40}$$

and

$$g = g_0 - g_1 \tag{41}$$

with

$$g_0 = 2Z\left[\mu(1+\rho) - Z\left(\rho+\mu^2\right)\right] \quad \text{and} \quad g_1 = \frac{\rho}{1-\rho}[\left(\rho+\mu^2(1-\rho)\right)Z^2 - (1-\rho)\mu Z - \rho]\kappa \tag{42}$$

We can recall that parameters μ, ρ and Z assume values contained in the range $[0, 1]$. We consider how different values of Z determine the sign of d and g, which jointly affect the sign of $d\varpi_h/dH$, and, therefore, the stability properties of the symmetric equilibrium.

We note that for given values of μ and ρ, d is a parabola that opens downward, whose graphic is given in figure 3, where only the relevant range of Z values, that is $0 \leq Z \leq 1$, must be considered.

Insert figure 3 about here

If $Z = 0$, then $d = 1$. d has its maximum value for a negative value of Z. Hence, for $Z \in [0, 1]$, d is decreasing and its intercept with the axis of abscissas is $Z_d = \frac{-\mu(1-\rho)+\sqrt{\mu^2(1-\rho)^2+4\rho}}{2\rho}$. It is easy to show that $Z_d > 1$, given that for $Z = 1$, $d = (1 - \mu)(1 - \rho) > 0$.

Finally, to complete the study of the sign of $d\varpi_h/dH$, we need to discern two cases: case I and case II. In case I, a higher share of skilled workers in a region does not imply a higher (or lower) productivity level. In other words, there are no geographically localized technological externalities, and, thus, $\kappa = 0$. Instead, in case II, κ can be positive due to the existence of a technological positive (negative) externality that implies a higher (lower) productivity level of the region in which the skilled worker' share is higher. In particular, as it was stressed in the introduction, we will concentrate on the study in which, if there exists any technological externality, this is of the positive type, with $\kappa > 0$.

4.1 Case I: $\kappa = 0$.

When geographically localized technological externalities do not exist, that is when $\kappa = 0$, g coincides with g_0 given in expression (42). In the plane (g_0, Z), g_0 is a parabola that opens downward, and its graphic is represented in figure 4a.

Insert figure 4a about here

The intercepts of g_0 with the Z-axis are $Z = 0$ and $Z_0 = \frac{\mu(1+\rho)}{(\rho+\mu^2)} > 0$, while its maximum occurs at $Z = \frac{\mu(1+\rho)}{2(\rho+\mu^2)}$.

It can be proved that if $\rho > \mu$, that is if the "no black hole condition" identified by Fujita et al. [4] holds, then $Z_0 < 1$. Thus, we may enunciate the following proposition.

Proposition 9 *If $\kappa = 0$, the symmetric equilibrium is stable for $Z \in (Z_0, 1]$; it is unstable for $Z \in [0, Z_0]$, with $Z_0 = \frac{\mu(1+\rho)}{(\rho+\mu^2)}$.*

This means that the symmetric equilibrium is stable only if trade costs are high and the degree of integration is small, while it is unstable for low trade costs. These results are in line with those by Fujita et al. [4]. Moreover, we may remark that if $\kappa = 0$ and $a = 1$, expression (39) coincides with the expression by Fujita et al. [4] at page 73

$$\frac{d\varpi_h}{dH} = 2Zp_m^{-\mu}\left(\frac{1-\rho}{\rho}\right)\left[\frac{\mu(1+\rho) - Z\left(\rho+\mu^2\right)}{1 - Z\mu(1-\rho) - \rho Z^2}\right] \tag{43}$$

and, in this case, the symmetric equilibrium unstable for $Z \in [0, Z_0]$ and stable for $Z \in (Z_0, 1]$, with the break point level of trade costs $\tau^{\rho/(1-\rho)} = \frac{(\rho+\mu)(1+\mu)}{(\rho-\mu)(1-\mu)}$. In this work we show that these ranges change when $\kappa > 0$.

4.2 Case II: $\kappa > 0$

When positive geographically localized technological externalities exist, that is when $\kappa > 0$, in expression (39) g is given by g_0 minus g_1 in expression (42). We have already discussed the properties of g_0. g_1 is a parabola that opens upward. The intercepts of g_1 with the Z-axis are $Z = Z_1$ and $Z = Z_2$. We notice that $Z_1 > 1$ and $Z_2 < 0$, given that the following results hold: $g_1 = (\mu - 1)\,\mu\rho\kappa < 0$ when $Z = 1$; $g_1 = -\frac{\rho^2}{1-\rho}\kappa < 0$ when $Z = 0$, and that the minimum of g_1 is for $Z \in [0, 1]$. In fact, the slope of the parabola is negative (that is $-(1-\rho)\mu < 0$) when $Z = 0$, and it is positive (and equal to $\rho\,(1+\mu) + \rho - \mu + 2\mu^2(1-\rho) > 0$) when $Z = 1$.[13]

The graphic of g_1 is represented in figure 4b, where only $Z \in [0, 1]$ are the relevant values of the closedness of trade.

Insert figure 4b about here

We must remember that in this section we consider only parameter values for which the "no black hole condition" holds, with $\rho > \mu$.

Comparing g_1 with g_0 it is possible to define the sign of expression g in (39). g is always positive (negative) when Z is such that $g_0 > g_1$ ($g_0 < g_1$). We know that g_0 and g_1 cross only once when Z is positive, when $Z = Z^*$. Therefore, we may state that g is positive (negative) when $0 \leq Z < Z^*$ ($Z > Z^*$). However, we notice that Z^* can be higher or lower than 1. Clearly,

[13] Note that the "no black hole condition" identified by Fujita et al. [4] holds with $\rho > \mu$.

we are interested in defining the sign of g only for $Z \in [0,1]$, that is for the relevant values of the closedness of trade.

Proposition 10 *When $Z^* < 1$, the symmetric equilibrium is stable for $Z \in (Z^*, 1]$ and unstable for $Z \in [0, Z^*]$. When $Z^* \geq 1$, the symmetric equilibrium is always unstable.*

Consequently, to avoid the case in which the symmetric equilibrium is unstable for every value of Z, a new condition must be stated that we call "pro dispersion condition" and that will be explicitly defined in the following pages.

First, we want to underline that given the shape of the two parabolas g_0 and g_1, their intersection in Z^* may identify two subcases, respectively denoted A and B, according to the values of parameters in the model.[14]

Case 11 (A) *If parameter values are such that $g_0 < g_1$ when $Z = 1$, then $Z^* < 1$.*

Case 12 (B) *If parameter values are such that $g_0 > g_1$ when $Z = 1$, then $Z^* > 1$.*

Specifically, we must establish when $Z^* \lesseqgtr 1$. The sign of the inequality depends on the value of κ, the index of the geographically localized technological externalities. A necessary and sufficient condition that must be satisfied in order to have a range of Z values for which the symmetric equilibrium is stable is that $g_1 > g_0$ when $Z = 1$. It can be readily verified that $g_1(Z=1) > g_0(Z=1)$ if

$$\kappa < \kappa^* = \frac{2(\rho - \mu)}{\mu\rho} \tag{44}$$

Therefore a range of Z values for which the symmetric equilibrium is stable does exist, only if

$$\kappa < \kappa^* \text{ with } \kappa^* > 0 \tag{45}$$

Therefore, the above mentioned "pro dispersion condition" must hold with $\kappa < \kappa^*$ in order to have at least some value of trade costs for which the symmetric equilibrium is stable. Figure 5 shows the case in which the symmetric equilibrium may be stable (because $\kappa < \kappa^*$), while figure

[14] To compare our results with those by Krugman [6] and [7], we remind that we consider this traditional no black hole condition because we start from the point in which $a_r = a_v = 1$.

6 shows the case in which the symmetric equilibrium is always unstable (because $\kappa > \kappa^*$).

Insert figure 5 about here

Insert figure 6 about here

Comparing the sign of expression g with that of d, we may write what follows.

Proposition 13 *When $0 < \kappa < \kappa^*$, $Z^* < 1$ exists and the symmetric equilibrium is stable when $Z \in (Z^*, 1]$ (because $\frac{d\varpi_h}{dH} < 0$), and unstable when $Z \in [0, Z^*]$ (because $\frac{d\varpi_h}{dH} \geq 0$).*

Proposition 14 *When $\kappa > \kappa^*$, the symmetric equilibrium is unstable $\forall Z \in [0, 1]$ given that $Z^* > 1$ (because $\frac{d\varpi_h}{dH} > 0$).*

The range of Z for which the symmetric equilibrium is stable when $\kappa > 0$ is smaller than that for $\kappa = 0$. We may compare the ranges that we obtain for $\kappa = 0$ with those that correspond to $\kappa > 0$ and write the following proposition.

Proposition 15 *In general, when we consider the symmetric equilibrium and the productivity level is positively related to skilled workers density in the neighborhood of the symmetric equilibrium ($\kappa > 0$), the range of Z for which the symmetric equilibrium is stable is smaller than in the case in which the positive externality does not exist ($\kappa = 0$).*

We may comment on this result considering expression (33). The migration of a certain number of skilled workers move regional economies from the symmetric equilibrium in its neighborhood. Let us consider, for instance, the case in which a certain number of skilled workers moves from region v to region r. In this case the centrifugal force generated by the market crowding effect in region r is weaker than centripetal forces. Indeed, prices in the larger market r diminish because its productivity increases and skilled workers real wages increase in r, strengthening the intensity of centripetal forces and reducing the range of Z for which the symmetric equilibrium is stable. Thanks to the geographically localized externality generated when $\kappa > 0$, the intensity of centripetal forces increases with respect to the centrifugal one. Hence, the width of Z for which the symmetric equilibrium is stable is reduced with respect to the case in which this externality does not exist ($\kappa = 0$). The technological externality produced by a positive κ value does strengthen the variable-technology centripetal force reducing the width of the range of Z values for which the symmetric equilibrium is stable.

We notice that when κ increases, the width of the range for which the symmetric equilibrium is stable decreases. Moreover, it can be readily verified that

$$\frac{\partial \kappa^*}{\partial \mu} < 0 \qquad \text{and} \qquad \frac{\partial \kappa^*}{\partial \rho} > 0$$

Therefore, the symmetric equilibrium is more likely to be stable, the smaller the share of expenditures in manufacturing and the degree of product differentiation are.

Finally, let us show relative real wages as a function of workers share in region r, to simulate possible outcomes for different levels of trade costs. For this exercise we need to specify how productivity levels depend on workers density, and we use the following equation

$$a_r = 1 + [bH_r(1 - H_r) + c]\,(Hr - \frac{1}{2}) \tag{46}$$

with $b \geq 0$ and c that are shape parameters.[15] Figures 7a and 7b plot relative real wage (premium) in the two regions as a function of the workers' share in region r, when $b > 0$ and $c = 0$. Figure 7a represents the case in which $\tau = 2.11$ and shows that the symmetric equilibrium is stable either when $\kappa = 0$ or $\kappa > 0$. However, if trade costs are smaller with $\tau = 2$, the symmetric equilibrium is still stable in the case of no geographically localized externalities, but it is unstable with positive externalities (Figure 7b).[16]

Insert figures 7a,b about here

[15] Expression (46) is an ad hoc equation that has the following properties: if skilled workers are uniformly distributed between the two regions, regional manufacturing productivity levels are equal, that is $a_r = a_v = 1$. However, if a certain number of skilled workers migrate towards one of the two regions, let say for instance the north, the northern productivity level becomes higher than 1, while that of the south smaller than 1. Specifically, to have these results, $\kappa = b/4 + c$ must be positive. These assumptions reflect the fact that labor productivity becomes higher, the higher the number of skilled workers in that particular region is. This fact reflects positive geographically localized externalities. Increasing the number of skilled workers in a region, increases the density of these workers in the same region and, therefore, may increase knowledge spillovers among the same workers, and, in turn, this increases regional productivity. For this particular goal, c would be sufficient, and b could vanish. Moreover, if c were negative, the externality would be negative describing a congestion effect. However, this could not always be the case, given that productivity levels may decrease also when $\kappa > 0$, only if the number of workers becomes too high (at a level of $H_r > 1/2$). This phenomenon may take place because when the number of skilled workers in a regions becomes too high, it becomes more difficult to coordinate their production activity, or because congestion processes would reduce productivity levels. This is captured by coefficient $b > 0$. The range of admissible values for c is $(-2, 2)$ to avoid negative values of a_r.

Finally, when c is negative, and b is such that $\kappa = b/4 + c > 0$, congestions effects (described by c) may become so high that they involve a productivity level smaller than 1 when mobile workers are completely concentrated in a region. (See, for instance, figure 8)

[16] Figures 7a and 7b are drawn for: $b = 0.2$; $\sigma = 3.33$; $\mu = 0.3$.

Finally, if c is negative, and b is such that the geographically localized externality is positive in the neighborhood of the symmetric equilibrium, that is $\kappa = b/4 + c > 0$, congestions effects may become so strong to imply a productivity level smaller than 1 when mobile workers are completely concentrated in one region. In this case, figure 8 shows that while the symmetric equilibrium is stable for high trade costs ($\tau = 6$), it is unstable for lower trade costs ($\tau = 3$ and $\tau = 2$).[17]

Moreover, due to the existence of strong congestion effects, full agglomeration is never stable, while two asymmetric equilibria may be stable.

Insert figure 8 about here

5 Conclusion

This work re-examines Krugman model properties when interregional productivity differences may arise in the modern sector. This reassessment is achieved by means of the description of the intensities of centripetal and centrifugal forces which determine the sustainability of the full agglomeration equilibria of the modern sector.[18] We show how different parameters of the model concur to determine centripetal and centrifugal forces intensities, either in the case of "fixed-technology" or traditional forces, or in the case of "variable-technology" forces.

Moreover, our modified version of the standard economic geography model confirms the finding by Venables [15] that is with Ricardian differences there could exist equilibria characterized by the localization of sectors in the region in which they have a comparative disadvantage, even tough this could happen only for intermediate trade costs. However, we find that when the two regions are sufficiently integrated, the comparative advantage dominates and production localization reflects the comparative advantage with manufacturing production agglomerated in the more productive region, while the agricultural good is produced in both regions. A similar result is obtained by Forslid and Wooton ([3], p. *) who find that "when trade barriers are sufficiently low, comparative

[17] Figure 8 is drawn for: $\sigma = 3.33$; $\mu = 0.3$; $b = 9$; $c = -1$.

[18] Baldwin, Forslid, Martin, Ottaviano and Robert-Nicoud [1] stress that the evaluation of agglomeration and dispersion forces in fully agglomerated equilibria is rather difficult.

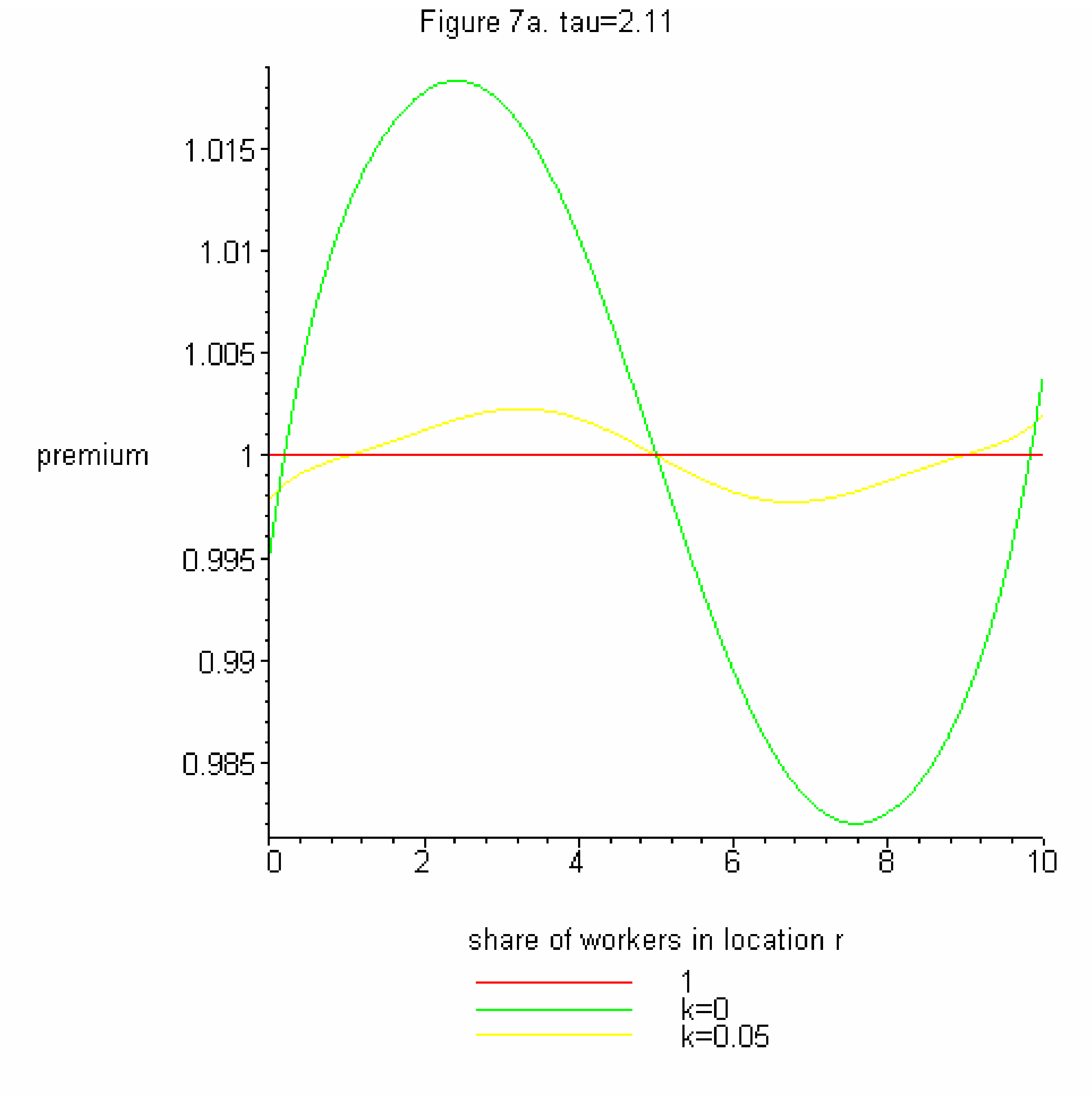
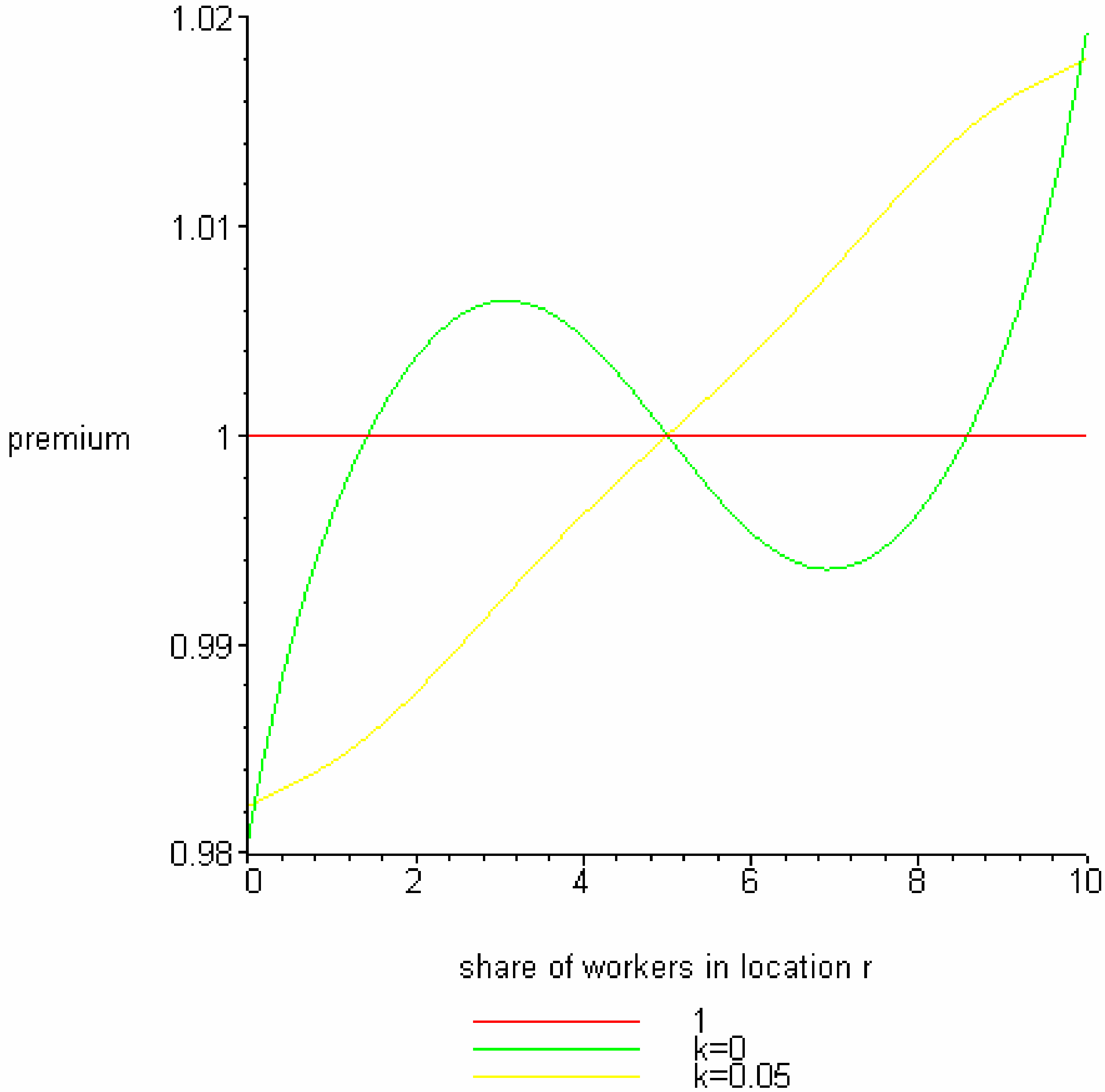


Figure 7a. tau=2.11
premium
1.015
1.01
1.005
1
0.995
0.99
0.985
0
2
4
6
8
10
share of workers in location r
1
k=0
k=0.05

Figure 7b. tau=2



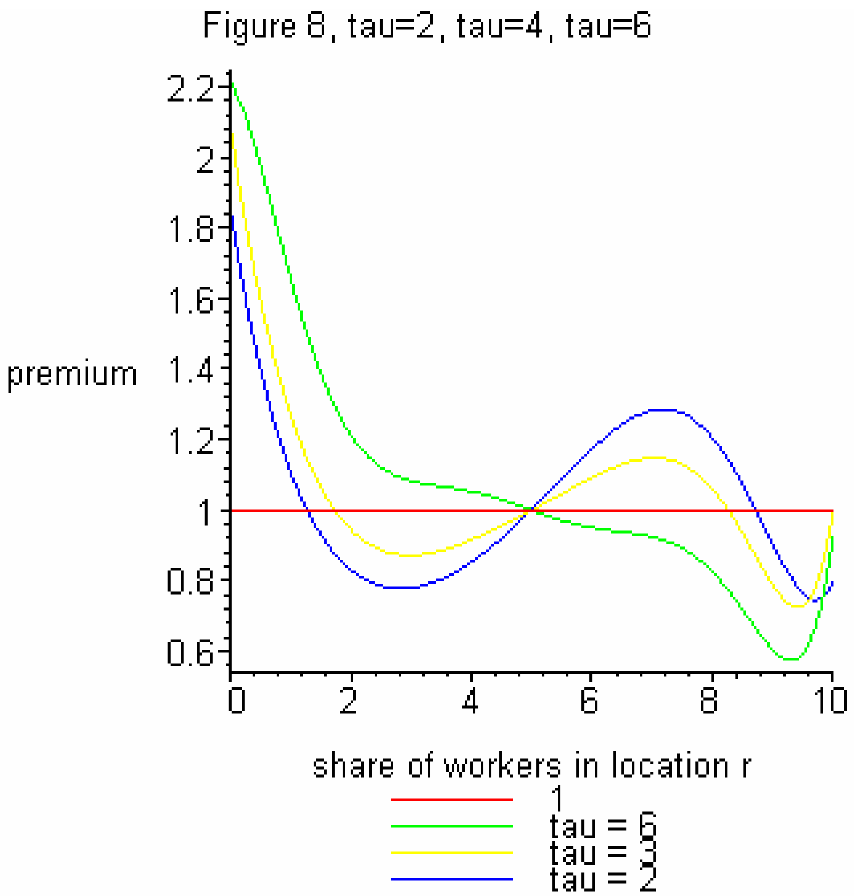


Figure 8, tau=2, tau=4, tau=6
premium
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
2
4
6
8
10
share of workers in location r
1
tau = 6
tau = 3
tau = 2

advantage takes the upper hand, pulling workers and production from the core to the other region". However, their results are different since comparative advantage in their case acts as a dispersion force and implies a symmetric stable outcome, while in our case it acts as an agglomeration force and implies a sustainable core-periphery outcome with production of the modern sector agglomerated in the more productive region.

Specifically, with potential technological differences, standard results may continue to hold. Particularly, when geographically localized knowledge spillovers are absent ($\kappa = 0$), the symmetric equilibrium can be attained only when interregional productivity levels are equal and the break point is the same as in the traditional model by Krugman [6]. In this case, the symmetric equilibrium is stable for low levels of integration, or high trade costs, and unstable for high integration levels. However, when $\kappa = 0$ the sustain point does coincide with the one found by Krugman only if manufacturing productivity levels are equal.

When regional modern sector productivity levels depend on skilled workers density ($\kappa > 0$), the range of closedness of trade for which the symmetric equilibrium is stable, changes. When the intensity of this externality increases (that is κ increases), the range of trade costs for which the symmetric equilibrium is stable, is reduced. Moreover, the positive technological externality generated by the higher productivity level in the region in which workers density is higher may even require an upper limit to its intensity in order to avoid the disappearance of the range of trade costs values for which the symmetric equilibrium is stable. This leads us to the definition of the pro dispersion condition that ensures the existence of such a range.

Finally, we note that the modified version of the standard economic geography presented in this paper could be useful for further studies on the evolution of interregional technological differences considered in a framework in which pecuniary externalities act.

Acknowledgement 16 *I am grateful to Gianmarco Ottaviano for helpful suggestions. The usual disclaimer applies.*

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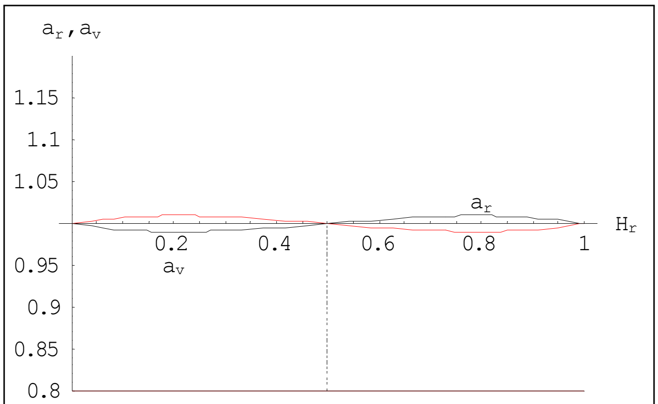

Figure 1

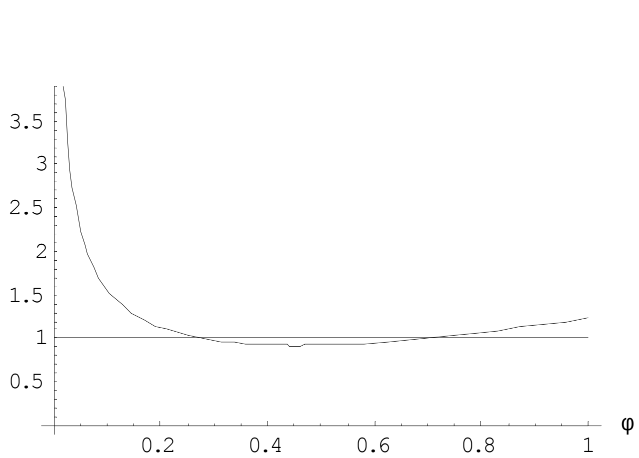

Figure 2. $\left(\frac{\varpi_{hr}}{\varpi_{hv}}\right)^{\sigma}$ when μ=0.3, σ=3, a_v=0.9 and a_r=1

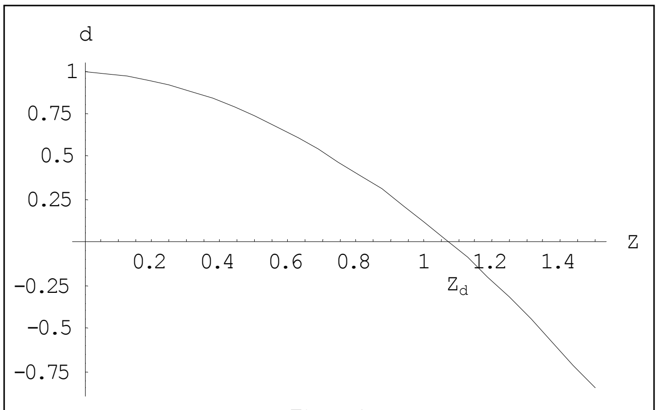

Figure 3

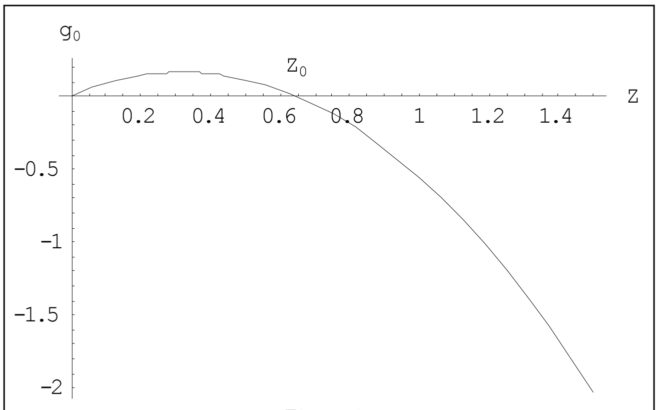

Figure 4a

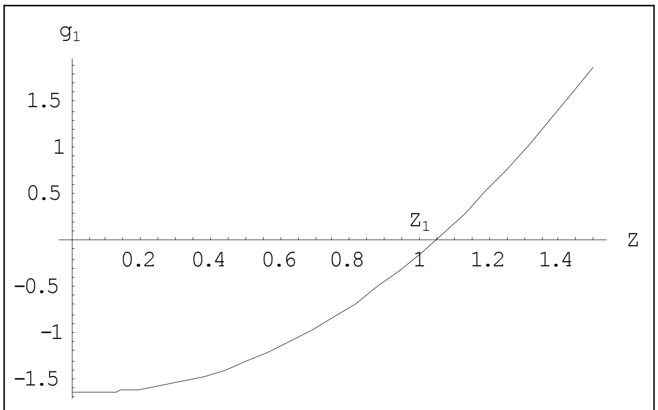

Figure 4b

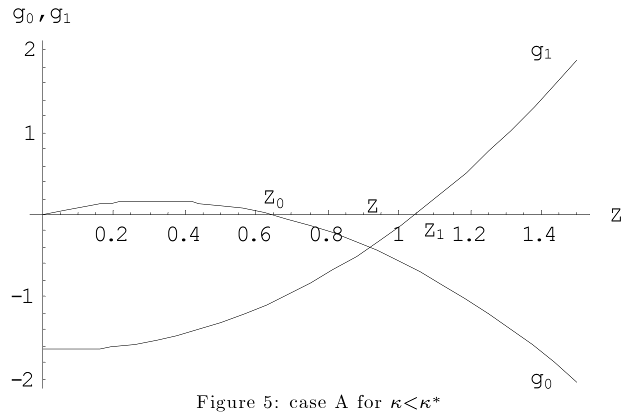

Figure 5: case A for $\kappa < \kappa^*$

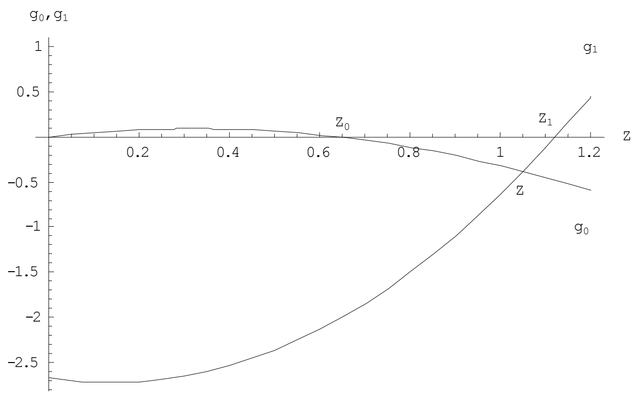

Figure 6: case B for $\kappa > \kappa^*$