$$T_i = 0.23\ (x - d_1 - d_2) \qquad \text{if } 12.214 < x \leq 114$$

$$T_i = 0.23\ (x - d_2) \qquad \text{if } 114 < x \leq 193.627$$

$$T_i = 0.23\ (193.627 - d_2) + 0.33\ (x - 193.627) \qquad \text{if } x > 193.627$$

From the result of Baldini et al. the total tax burden loss is 30.47 billion of Euro (approximately 100% more than the Executive estimates).

Already looking at the charts **3** and **4** by means of the *equivalent* household post tax income notion, it notes that all population shares gain by the fall in the tax liabilities and the average tax rates behavior along the income parade continues to depart from proportionality, but in a different way, probably becoming structurally less progressive (post tax income gains higher for the top deciles).

This is confirmed by the Reynolds-Smolensky index.

	Gini pre	- Gini post	$= \Pi^{RS}$	$= \Pi^{K}$	$* g /(1- g)$	- R
Old Tax	0.3777	0.3403	0.0374	0.2171	0.1801	0.001699
Tax Ref.	0.3777	0.3530	0.0247	0.1992	0.1283	0.000857

Sources: Bosi et al. 2002

These results show a fall in Π^{RS} index according to the decrease of g (total tax ratio) and Π^{K}. The lower reranking effect is not sufficient to overcome these reductions.

In conclusion even if the Executive objective is to pay more attention to the low and middle-income group, the researchers affirm that 55% of the total tax burden loss goes to incomes higher than 50000 Euro (not more than 5% of the total population). Together with the negative variation of Π^{RS} index, this allows to affirm that the redistribution continues to go from the richer group to the poorer, but with a reduction of intensity[31].

Clearly, any analysis based on redistributive indexes may be accepted according to a *given* F(x), that is a given distribution of pre tax (*equivalent*) incomes.

4 - Non-Progression Neutral Tax Cut: Conclusive Remarks

In the literature, outcomes about the vertical distance between L^1_{X-T} and L^2_{X-T} for all p (where L^2_{X-T} is the Concentration curve for the post reform post tax incomes) and then about

[31] - According to the fact that this tax reform does not lead to a negative income tax, 20% of poorest families gain only 2% of the total tax burden fall.

distributional effects, have been obtained taking into account the pre tax income distribution in two different ways.

According to Jakobsson (1976, proposition 1) it is possible to state that

$$RP^2 \leq RP^1 \quad \forall x \quad \text{iff} \quad L^2_{X-T} \geq L^1_{X-T} \qquad \textit{for every pre tax distribution } F(x)$$

In order to investigate the distributional effect of different schedules only the information about magnitudes of marginal and average rates is required.

On the other hand consistent with Hemming and Keen (1983, proposition 1), when two income taxes (T^2,T^1) raise the same revenue[32]

> "an income tax (T^2) is more progressive than another (T^1) *for a given pre tax distribution F(x)* *if and only if* the post tax income function ($x - T^2$) single crosses from above the post tax income function ($x - T^1$) on some interval Y = [x_{min}, x^{max}]".

In the former case it obtains a global result and it can define a comparison between any two schedules by using their specifications, verifying if the residual progression is increased or not for all x.

In the latter, relative distributional implications of alternative taxes can be defined also if it verifies a single-crossing condition between the two post tax income schedules conditional on the pre tax distribution[33]. For $g^1 = g^2$, the two curves must intersect - a pure-redistributive case - as a result the only remaining trouble is just find which is the curve that single crosses the other from above.

Hence when this distribution is such that tax schedules are equal-yield also for Hemming and Keen the only necessary information comes from the form of T^i (i = 1,2, in our case).

In contrast when non-equal yield taxes have to be compared, the post tax schedules may not necessarily cross. In such a case Hemming and Keen were able to define a transformation of post tax schedules that involves a new type of single-crossing condition.

Let $[x - t^2(x)]$ be the post tax reform income schedule, $[x - t^1(x)]$ the old schedule, L^2_{X-T} and L^1_{X-T} their respective Lorenz curves[34]. The condition of Lorenz domination (LD) of new schedule over the old one, for the equal-yield case is, for all *v* those belong to Y

$$\int_{x_{min}}^{v} [x - t^2(x)]\, f(x)\, dx \Big/ \int_Y [x - t^2(x)]\, f(x)\, dx \quad \geq \quad \int_{x_{min}}^{v} [x - t^1(x)]\, f(x)\, dx \Big/ \int_Y [x - t^1(x)]\, f(x)\, dx$$

---

32 - This proposition may be adapted if it wish to follow Kakwani (1977) approach, relating progressivity to the distribution of the tax burden.

33 - If the elasticity condition of Jakobsson theorem holds, it is well known that this implies at most a single crossing, while the reverse is not true.

34 - To simplify we assume L^i_{X-T} (i= 1,2) as a Lorenz curve.

By assuming $g^1 = g^2$, $\int_y [x - t^2(x)]\, f(x)\, dx = \int_y [x - t^1(x)]\, f(x)\, dx$

Hence only the numerators are relevant for Hemming and Keen's proof.

By following a normalization procedure for both sides it can deal, at least for the sufficiency condition, with a non equal-yield case ($g^1 \neq g^2$) as an equal-yield one, that is acting on the post tax income function in such a way to have

$$\int_{x_{min}}^{v} \{ [x - t^i(x)] / \int_y [u - t^i(u)]\, f(u)\, du \} f(x)\, dx \quad = \quad \int_{x_{min}}^{v} [x - t^i(x)]_N\, f(x)\, dx$$

where $[x - t^i(x)]_N = [x - t^i(x)] / \int_y [u - t^i(u)]\, f(u)\, du$ $\qquad i = 1, 2$

indicates the share of total net income associated with pre tax income x under $t^i(x)$ given F(x).

Making the same operation on the denominator:

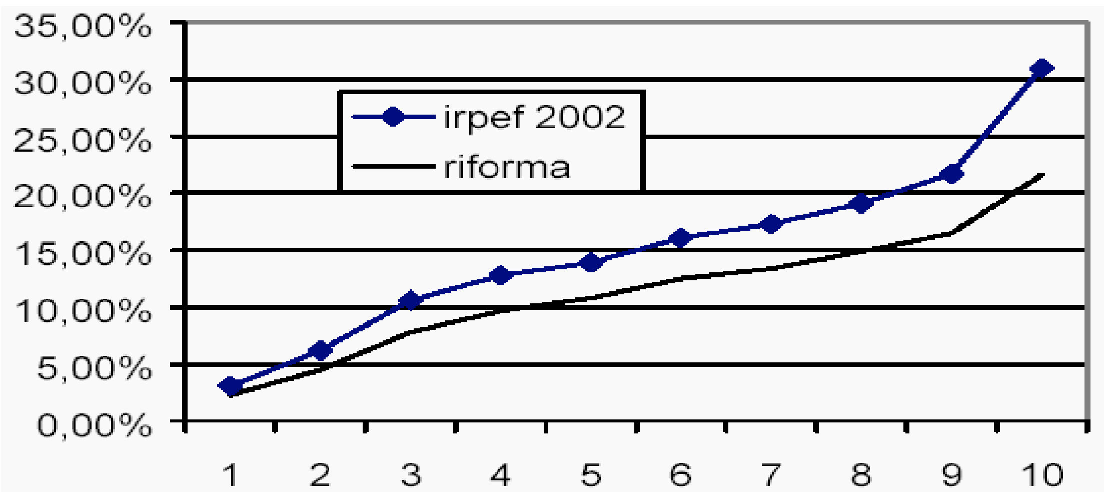
$$\int_y \{[x - t^i(x)] / \int_y [u - t^i(u)]\, f(u)\, du \}\, f(x)\, dx \quad = \quad [1 / (1 - g^i)] \int_y \{[x - t^i(x)]\, f(x)\, dx =$$
$$= [1 / (1 - g)]\, (1 - g) = 1 \qquad i = 1, 2$$

and it can rewrite the condition for LD as:

$$\int_{x_{min}}^{v} [x - t^2(x)]_N\, f(x)\, dx \quad \geq \quad \int_{x_{min}}^{v} [x - t^1(x)]_N\, f(x)\, dx \qquad \text{for all } v \in Y$$

In this way Hemming and Keen show how also in this case it is possible to make use of the same proof for the sufficiency, as when the total tax ratios are equal. Their propositions is necessary with respect to the Italian tax reform under analysis. Even if I have not many data it may infer some interesting information just looking at this chart:

Average tax rates for household equivalent post tax income



Sources: *Baldini-Bosi-Matteuzzi 2002*

By the decrease of average rates for all the deciles, post tax reform household equivalent incomes may be considered higher than under the old schedule. This chart seems to involve a tax cut for all the families, increasing along the income parade: the progressivity changes.

On the other hand, for instance, with a residual progression neutral tax cut the gain in percentage terms has to be equal for every households, in accordance with:

$t^r(x) = t^1(x) - a\,[x - t^1(x)]$ (or $t^r(x) / x = t^1(x) / x - a\,[1 - t^1(x) / x]$ where $a > 0$)

and $x - t^r(x) = [1 + a]\,[x - t^1(x)]$

This residual progression neutral tax cut would push down the old piecewise linear curve for the *average* rates by the term

$$a\,[1 - t^1(x)/x]$$

The elasticity RP for every deciles would remain constant, $[1 - t'(x)] / [1 - t(x)/x]$, then also the slope: the vertical distance between the two curves would be equal.

If it imposes an *equal-yield* ($g^r = g^2$) residual progression neutral tax cut, this leads to an RP neutral curve of average rates that single crosses once from *below* the tax reform schedule of average rates ($t^2(x) / x$).

Moreover the Lorenz curves with respect to the old schedule and the RP neutral tax cut are exactly superimposes.

More generally it can be shown that $a = (g^1 - g^r) / (1 - g^1)$

Then
$$\begin{aligned} x - t^r(x) &= [x - t^1(x)] + [(g^1 - g^r) / (1 - g^1)]\,[x - t^1(x)] = \\ &= [x - t^1(x)]\,[\,1 + (g^1 - g^r) / (1 - g^1)] = \\ &= [x - t^1(x)]\,[(1 - g^r) / (1 - g^1)] \end{aligned}$$

Hence, by using the fact that $g^r = g^2$ it can compare the two schedules

$$x - t^2(x) \quad \text{with respect to} \quad [x - t^1(x)]\,[(1 - g^r) / (1 - g^1)] = x - t^r(x)$$

In this case the proposition changes and it can be shown that:

"If $[x - t^2(x)] / (1 - g^2)$ crosses $[x - t^1(x)] / (1 - g^1)$ once from above, then $L^2_{X-T}(p) \geq L^1_{X-T}(p)$ for all $p \in [0, 1]$"[35].

[35] - Where, as before, $[x - t^i(x)] / (1 - g^i) = [x - t^i(x)]_N$ $(i = 1,2)$, is the share of total net income associated with pre-tax income x under $t^i(x)$ given the $F(x)$.

In our case the reality seems to contradict this proposition because, like in a mirror, if the RP neutral curve of average rates single crosses once from *below* the tax reform schedule of average rates, the RP neutral post tax income curve single crosses once from *above* the tax reform schedule of post tax income, while to ensure LD for the distribution of net incomes under the tax reform over the distribution of net incomes under the old schedule, it should be the opposite: the Executive's policy seems apparent.

From another point of view, it finds that the necessary condition of Latham (1988, theorem 5) may be verified.

Latham states that (α^i = distribution of net incomes under the two schedules):

> "given that $g^2 < g^1$, α^2 L.D α^1 only if there exists an interval (x_{min}, x^*) over which t^1 (x) < t^2 (x), where $x_{min} < x^* < x^{max}$."

Reasonably, observing the chart this interval seems to exist, but it is not enough to ensure Lorenz dominance: in this case it appears that after-tax incomes are higher everywhere, as allowed by the theorem.

Hence, apparent results can be obtained by the use of these theorems. What the practitioner needs is either the functional form of the tax, or post tax, schedules, or these functional forms together with the total net income, as in Hemming and Keen's *non*-equal yield case.

To summarize:

- According to Baldini et al. (2002) the total tax burden loss is 30,47 billion of Euro (around 100% more than the Executive estimates);
- The inequality of the distribution of the net incomes increases with respect to the old distribution of after-tax incomes, as a result of both total tax ratio and disproportionality of tax burden (progression) reductions;
- Assessing by the analysis of this section, the result of a lower Reynolds-Smolensky index finds confirmation by the *unverified* Hemming-Keen's single-crossing condition, useful especially to work when it is not possible to assess effects by using the Jakobsson's elasticity condition;
- If one of the objectives of the Executive is to decrease the total tax ratio, a RP neutral tax cut for all could be regarded as a possibility to increase social welfare. By Baldini et al. (2002) there is evidence that other strategies are possible to avoid the progressivity reduction and, at the same time, to cut in the same way the total tax burden.

- All the results of these chapters are correct in a partial equilibrium framework: the distribution of pre tax income is assumed to be independent of the tax code in operation and, of course, it has to be convincing that - using a utility function where the (equivalent) incomes are the only input - a social welfare function such as

$$W_F = \int_0^y U(x)\, f(x)\, dx$$

is related with the actual well-being of the society.

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