# Appendix

In this Appendix we (a) present in more detail the example discussed in the text; and (b) present a simple general argument to show that, under a FOSD shock, knowing the sign of the relationship between price elasticity and income along the individual demand curve says nothing on the relationship between market elasticity and average income.

## (a) The example

The income distribution is a standard exponential, with density $f(y,\theta) = e^{-(y-\theta)}$, and cumulative distribution $F(y,\theta) = 1 - e^{-y+\theta}$, $y \in [\theta, \infty)$. As explained in the text (see also f.note 5), $\theta > 0$ is a FOSD parameter and mean income is $\mu(\theta) = 1+\theta$. We notice that, contrary to our assumption in Proposition 3, $F_\theta(y_m,\theta) = -1 < 0$ and $\pi(y,\theta)$ is independent of $\theta$. Indeed

$$\pi(y,\theta) = 1 + \frac{yf_y(y,\theta)}{f(y,\theta)} = 1 + \frac{-ye^{-(y-\theta)}}{e^{-(y-\theta)}} = 1 - y$$

As to the individual demand function, we have $q(p,y) = \max\left\{1 - \frac{p}{y}, 0\right\}$, so that aggregate demand is

$$Q(p,\theta) = \int_\theta^\infty \max\left\{1 - \frac{p}{y}, 0\right\} e^{-(y-\theta)} dy \tag{A.1}$$

Assume now that $p > \theta$. Then (A.1) becomes

$$Q(p,\theta) = \int_p^\infty \left(1 - \frac{p}{y}\right) e^{-(y-\theta)} dy$$

which gives $Q(p,\theta) = (1 - pA(p)\,e^p)e^{-p+\theta}$ where $A(p) = \int_p^\infty x^{-1}e^x dx$ is a decreasing positive function of $p$. Clearly, this can be written as $Q(p,\theta) = G(p)e^\theta$ (which is isoelastic in $\theta$), with $G(p) = e^{-p} - pA(p)$.

## (b) A simple argument

Assume $\theta$ is a FOSD shock to the income distribution, such that $F_\theta(y,\theta) \leq 0$ (strictly somewhere) for all $y \in Y$, which implies $Q_\theta(p,\theta) > 0$ for all $p \in P$. Upon differentiation, a necessary and sufficient condition for $H_\theta(p,\theta) < 0$ is that

$$-pQ_{p\theta}(p,\theta) < Q_\theta(p,\theta)H(p,\theta)$$

where subscripts denote (cross) partials and (obviously) $p$, $Q_\theta(p,\theta)$, and $H(p,\theta)$ are all positive. We now show that $\eta_y(p,y)<0$ implies $Q_{p\theta}(p,\theta)<0$, which means that the LHS is itself positive: some specific assumption on $F(y,\theta)$ is accordingly required beyond FOSD, to ensure that $\eta_y(p,y)<0$ implies $H_\theta(p,\theta)<0$.

Integration by parts yields

$$Q_{p\theta}(p,\theta) = -\int_{y_m}^{y_M} q_{py}(p,y)F_\theta(y,\theta)dy$$

Since $\eta_y(p,y)<0$ implies trivially $-pq_{py}(p,y)<q_y(p,y)\eta(p,y)$ for all $y$, there follows that

$$pQ_{p\theta}(p,\theta) = -p\int_{y_m}^{y_M} q_{py}(p,y)F_\theta(y,\theta)dy < \int_{y_m}^{y_M} q_y(p,y)\eta(p,y)F_\theta(y,\theta)dy < 0$$

the last inequality deriving from $q_y(p,y)$ and $\eta(p,y)$ being both positive for all $y$, while $F_\theta(y,\theta)\leq 0$ (strictly somewhere) by the definition of FOSD. Since $p$ is obviously positive, this implies $Q_{p\theta}(p,\theta)<0$.