dominance (FOSD) shifts of the income distribution. Hence, our model provides sufficient conditions for the Robinson effect to hold when income distribution is hit by a FOSD shock – it being the case (as shown in section 3) that such a shock may not in general lower market elasticity, even though the price elasticity of the individual demand is decreasing in income.

The paper is organized as follows. In the next section a simple general framework is developed to study the relationship between income distribution and the elasticity of market demand. In Section 3 the main result of the paper is presented, which identifies sufficient conditions on the income distribution for the 'Robinson effect' to take place, when the income distribution is hit by shocks in the first-order stochastic-dominance sense. These conditions are satisfied by a wide range of commonly used distributions. Section 4 offers some concluding remarks.

2 Income distribution and demand elasticity

In this section we present a partial equilibrium framework to assess the role of income distribution and the effects of distribution changes on market demand, when income is the only source of heterogeneity.

Consumers differ only in income, and their behavior is described by a continuous standard Marshallian demand curve $q(p, y)$, where the prices of commodities other than q are held fixed throughout. Each agent is accordingly identified by his income $y \in Y = (y_m, y_M)$, where $0 < y_m < y_M \leq \infty$. The good q is normal, that is (letting subscripts denote derivatives) $q_y(p, y) > 0$ and $q_p(p, y) < 0$, for all $(p, y) \in P \times Y$, where P is a subset of non-negative reals. A natural specification might be $P = (0, p_M)$, with p_M satisfying $q(p_M, y_M) = 0$: it would be the choking price for the highest income consumers (in the limit, if $y_M = \infty$). For any $p \in P$, one clearly has $\lim_{y \to y_M} q(p, y) > \lim_{y \to y_m} q(p, y) \geq 0$.

Income is continuously distributed according to the density $f(y, \theta) > 0$, where $\theta \in \Theta$ is a real parameter of the distribution. In the next section it will measure a FOSD shock. The income distribution $F : Y \times \Theta \to [0, 1]$ is obviously defined by

$$F(y, \theta) = \int_{y_m}^{y} f(x, \theta) dx \tag{1}$$

Clearly, $F_\theta(y_M, \theta) = 0$, since by definition $F(y_M, \cdot) = 1$ for all θ. Aggregate (mean) market demand is

$$Q(p, \theta) = \int_{y_m}^{y_M} q(p, y) f(y, \theta) dy \tag{2}$$

A natural question is, what happens to market demand when the income distribution shifts, following a change in θ. Trivially,

$$Q_\theta(p,\theta) = \int_{y_m}^{y_M} q(p,y) f_\theta(y,\theta) dy$$

which, by standard results (e.g., Hirshleifer and Riley, 1992, ch.3), will be positive if θ is a FOSD shift, since q is increasing in y; while it will be positive or negative, depending on convexity or concavity of Engel curves, if θ measures a mean preserving, second order stochastic dominance shift of the distribution.

The focus of our paper, however, is what happens to market demand *elasticity* when the income distribution changes. Let $\eta(p,y)$ be the (positive) demand elasticity along the individual demand curve $q(p,y)$. It is straightforward to derive the market demand elasticity H satisfying

$$H(p,\theta) = \int_{y_m}^{y_M} \eta(p,y) \varphi(y,p,\theta) dy \tag{3}$$

where $\varphi(y,p,\theta)$ is defined by

$$\varphi(y,p,\theta) = \frac{q(p,y) f(y,\theta)}{Q(p,\theta)} \tag{4}$$

so that, quite naturally, market elasticity is a weighted average of individual elasticities. Given $p \in P$, $\varphi > 0$ is the density describing how market demand is distributed across income classes. The corresponding cumulative distribution is

$$\Phi(y,p,\theta) = \int_{y_m}^{y} \varphi(x,p,\theta) dx \tag{5}$$

such that $\Phi(y_M,\cdot,\cdot) = 1$. In particular, we note that by writing out the whole expression,

$$\Phi(y,p,\theta) = \frac{1}{Q(p,\theta)} \int_{y_m}^{y} q(p,x) f(x,\theta) dx$$

$\Phi(y,p,\theta)$ has the form of a Lorenz curve, since Q is the average value of q.

We gather in the next proposition two noteworthy, albeit quite intuitive, general properties of $\Phi(y,p,\theta)$.

Proposition 1 *(a) For given $(p,\theta) \in P \times \Theta$, $\Phi(y,p,\theta)$ dominates stochastically $F(y,\theta)$ in the first order sense, that is $\Phi(y,p,\theta) \leq F(y,\theta)$ for all $y \in Y$, with strict inequality somewhere; (b) If $\eta_y(p,y) < 0$ for all $y \in Y$, an increase in p affects $\Phi(y,p,\theta)$ as a first order stochastic dominance shock, i.e., $\Phi_p(y,p,\theta) \leq 0$ for all $y \in Y$, with strict inequality somewhere.*

Proof. (a) Using definitions (1) and (5), for given (p, θ) we have

$$F(y,\theta) - \Phi(y,p,\theta) = \int_{y_m}^{y} f(x,\theta)\left(1 - \frac{q(p,x)}{Q(p,\theta)}\right) dx \equiv Z(y)$$

say. Note that $Z(y_m) = Z(y_M) = 0$, while $Z_y = [1 - q(p,y)/Qp,\theta)]f(y,\theta)$. Since $f(y,\theta)$ is positive, Q is an average and q is monotonically increasing in y, there is only one value $\overline{y}$ of y such that $q(p,\overline{y}) = Q$, which is the only maximum of Z. There follows that $Z > 0$ for all $y \in Y$, since it is increasing (decreasing) around y_m (y_M). Hence, $\Phi(y,p,\theta) - F(y,\theta) = -Z(y,p,\theta) < 0$.

(b) By writing out the derivative of (5) with respect to p, we get

$$\Phi_p(y,p,\theta) = -\frac{Q_p(p,\theta)}{Q(p,\theta)}\Phi(y,p,\theta) - \frac{1}{p}\int_{y_m}^{y} \eta(p,y)\varphi(y,p,\theta)dy$$

after some rearrangement. Now multiply through by $p > 0$ and use (3) to obtain the following condition for $\Phi_p(y,p,\theta) < 0$:

$$K(y) \equiv \int_{y_m}^{y} (H(p,\theta) - \eta(p,y))\varphi(y,p,\theta)dy < 0$$

where K is defined for given (p,θ). Clearly, $K(y_m) = 0$, and $K(y_M) = 0$ by (3). Since $H(p,\theta)$ is an average of $\eta(p,y)$ and $\eta_y(p,y) < 0$, the derivative $K_y = (H(p,\theta) - \eta(p,y))\varphi(y,p,\theta)$ is increasing in y and vanishes at $y = \widetilde{y}$ such that $\eta(p,\widetilde{y}) = H(p,\theta)$, which is a minimum. This implies that $K(y) < 0$ for all y, and hence $\Phi_p < 0$. ■

These properties hold in general – in particular, as is obvious, they do not depend on θ. Property (a) implies that $\mu(\theta) = \int_{y_m}^{y_M} xf(x,\theta)dx < m(p,\theta) = \int_{y_m}^{y_M} x\varphi(x,p,\theta)dx$ for all $p \in P$: the average income weighted by the demand share of each income class on overall demand, is higher than mean income (i.e., average income weighted by the income share of each income class on overall income): this follows naturally from the commodity being normal. By property (b), following and increase in p, the implied decrease in demand is such that the degree of income heterogeneity among buyers increases – in the sense that demand is more unevenly distributed across income classes; also, the share of high income buyers on overall demand increases, which, though naturally to be expected, may be empirically not trivial, and in some circumstances significant from a welfare point of view.[2]

[2]This applies, e.g., to commodities like pharmaceuticals or health services, where the issue of price controls and availability for low income consumers may be relevant. Gertler *et al.* (1987) provide some empirical evidence in this respect.

Equation (3) makes it clear that, when working on elasticity, the crucial question is how shifts in F translate themselves into shifts in Φ: that is, how changes in income distribution affect the income distribution of market demand (or its Lorenz curve). We now turn to the case where an exogenous shock generates a FOSD shift to the income distribution.

3 First order stochastic dominance

In this section we enquire about the effects of a FOSD shock to the income distribution: hence, we interpret θ as an index of FOSD and impose that $F_\theta(y, \theta) \leq 0$ for all $y \in Y$ (with strict inequality somewhere), which implies that aggregate (average) income is increasing in θ, $\mu_\theta(\theta) > 0$. As individual demand $q(p, y)$ is increasing in income y, this also immediately implies that $Q_\theta(p, \theta) > 0$: not surprisingly, a FOSD shock increases demand at all prices.[3]

But how about elasticity? In principle, there is no reason to expect that Robinson's assumption on preferences (an increase in individual income affects negatively the price elasticity of individual demand) delivers a negative relationship between aggregate income and the price elasticity of market demand. The following example shows that an increase in mean income may leave market elasticity unaltered, even though the elasticity of individual demand is decreasing in individual income.

Let the consumer's demand for commodity q be

$$q(p, y) = \max\left\{1 - \frac{p}{y}, 0\right\}$$

such that its elasticity (whenever the consumer buys the commodity) is $\eta(p, y) = p/(y - p)$, which is positive and clearly decreasing in income.[4] Let now the latter be distributed across consumers as a standard exponential,

$$f(y, \theta) = e^{-(y-\theta)}$$

with $y_m = \theta$ and $y_M = \infty$. An increase in $\theta > 0$ amounts to a FOSD shock, which increases linearly aggregate (mean) income.[5] We show in the Appendix that in this case the aggregate demand function takes the form

$$Q(p, \theta) = G(p)e^\theta$$

[3] For a simple proof, see e.g. Hirshleifer and Riley (1992, ch.3).

[4] This demand function can be rationalized as deriving from a separable utility function (see, e.g., Tirole, 1989, p.144).

[5] Indeed, it is easily seen that $\mu(\theta) = 1 + \theta$, and that $F_\theta(y, \theta) = -e^{-y+\theta} < 0$.