incentive to acquire new consumers at the margin by keeping lower prices.[8] Demand increases and becomes more elastic simply because there are indeed new consumers entering the market, but also more consumers whose decision to enter or exit the market is now very sensible to small variations in prices. Notice that these observations are consistent with the fact that a positive comovement of demand and demand elasticity is observed only in $\widehat{B}$, i.e., it is peculiar of an intermediate portion of the demand curve, as defined by $\widehat{B}$. Moreover, they apply to whatever unimodal distribution, once concentration towards central income values is considered, and this explains the generality of our result. As a notable example, in the next section we apply the results of Propositions 1 and 2 to the lognormal distribution.

4 An example: income dispersion with lognormal distribution

Assume that income is distributed lognormally. This is a particularly remarkable case, since – as is well known – the lognormal distribution is perhaps the model most frequently used to describe actual income frequencies.[9]

We standardize mean income equal to unity, so that the density and distribution functions take the form[10]

$$f(y,\theta) = \frac{1}{y\sqrt{2\pi\ln\theta}}\exp\left(-\frac{\left(\ln y+\frac{1}{2}\ln\theta\right)^2}{2\ln\theta}\right)$$

$$F(y,\theta) = \int_0^y f(x,\theta)dx = \tfrac{1}{2}\left[1+\Phi\left(\tfrac{1}{4}\sqrt{2}\frac{2\ln y+\ln\theta}{\ln^{\left(\frac{1}{2}\right)}\theta}\right)\right]$$

[8] This may offer a general explanation for the empirical evidence discussed by Frankel and Gould (2001), who find a causal link running from income distribution in urban areas to retail prices: according to their estimates, greater inequality is indeed associated with an increase in retail prices paid by lower middle-class consumers.

[9] It is well known that the lognormal distribution fits satisfactorily the actual income distribution for central income values, while it is unsatisfactory in the tails, i.e. for extreme income values (for an evaluation of the empirical performance of various distributions, see e.g. Majumder and Chakravarty, 1990). Since the phenomenon we are interested in is peculiar of intermediate intervals, the lognormality assumption seems worth investigating. We recall that, if reservation prices are proportional to incomes, they also are lognormally distributed.

[10] Given a generic lognormal distribution $f(y,\theta) = (y\sqrt{2\pi\ln\theta})^{-1}\exp\left(-\frac{(\ln y-\zeta)^2}{2\ln\theta}\right)$, the mean is $\mu = e^{\zeta}\sqrt{\theta}$. Clearly, by imposing $\mu = 1$ one constrains the parameters θ and ζ according to the restriction $\varsigma = -\frac{1}{2}\ln\theta$. Note, in particular, that income variance is $\sigma^2 = \theta - 1 > 0$.

where $\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ is the so-called *error function* (Johnson *et al.* 1994, p.81). It can be checked that $\theta > 1$ is indeed a mean preserving spread, in that the conditions set out in (2) are satisfied. Also, single crossing of the cumulative distributions following a change in θ, as well as double crossing of the densities, are satisfied. In particular, for this lognormal distribution we have $y_A(\theta) = e^{-\frac{1}{2}\sqrt{\left(4\ln\theta + \ln^2\theta\right)}}$, $y_B(\theta) = \sqrt{\theta}$ and $y_C(\theta) = e^{\frac{1}{2}\sqrt{\left(4\ln\theta + \ln^2\theta\right)}}$. It is immediate to define the demand curve as

$$Q(p,\theta) = \tfrac{1}{2}\left[1 - \Phi\left(\tfrac{1}{4}\sqrt{2}\frac{2\ln p + \ln\theta}{\ln^{\left(\frac{1}{2}\right)}\theta}\right)\right]$$

the elasticity of which is

$$\eta(p,\theta) = \frac{\dfrac{1}{\sqrt{2\pi\ln\theta}}\exp\left(-\dfrac{\left(\ln p + \frac{1}{2}\ln\theta\right)^2}{2\ln\theta}\right)}{1 - \frac{1}{2}\left[1 + \Phi\left(\frac{1}{4}\sqrt{2}\frac{2\ln p + \ln\theta}{\ln^{\left(\frac{1}{2}\right)}\theta}\right)\right]}$$

The corresponding function $\eta_\theta(p,\theta)$ is derived in the Appendix, where it is shown to tend to minus infinity as $p \to \infty$. This function is in general analytically difficult to treat – and indeed one advantage of Proposition 2 is that it offers a simple, general characterization of its qualitative behaviour in terms of the income share elasticity. Actually, the Π function takes the simple form given by

$$\Pi(y,\theta) = -\frac{1}{2}\frac{2\ln y + \ln\theta}{\ln\theta}$$

It is easy to check that $\Pi_\theta = \ln y/(\theta\ln^2\theta)$, which is monotonically increasing in y and crosses zero at $y = \mu = 1$. Therefore we can establish that a mean preserving shock generates a unique area of positive comovement of demand and demand elasticity, the left boundary of which lies between $y_A(\theta) = e^{-\frac{1}{2}\sqrt{\left(4\ln\theta + \ln^2\theta\right)}}$ and 1, and the right boundary of which is $y_B(\theta) = \sqrt{\theta}$. In order to assess the relevance of this phenomenon, one can notice that a numerical approximation performed under the arbitrary value of $\theta = 2.5$ gives to this area a size such that about the 40% of the population has income (reservation prices) falling in it.[11]

[11] The value is not wholly arbitrary, since $\theta = 2.5$ yields a coefficient of variation $\kappa = 1.225$ very similar to the value of 1.237 recorded by Champernowne and Cowell (1998, p.78) for the distribution of labour income in the UK.