Chapter 3

Chvátal-Gomory cuts for the Multicast polytope

In this chapter, we want to highlight some properties of the polytope of the Set Covering formulation (see Proposition 1.2.2) for the Multicasting problem in the wireless Ad-Hoc networks. The inequalities in section 2.5 can be added to the problem to reduce the feasible region of the MPM problem, but in general they are not able to cut off any optimal fractional solution of the linear relaxation of the problem. The purpose here is to propose heuristics that generate valid inequalities for the Set Covering polytope that cut off fractional optimal solutions of the linear relaxation of the MPM problem. In particular, in section 3.2 we propose two heuristics that find violated inequalities with right hand side two belonging to the first Chvátal closure of the MPM problem's polytope. The optimal value of the linear relaxation of the problems with the cuts generated with the heuristics is compared in section 3.4 with the optimal value obtained by solving the problems over the first Chvátal closure polytope (see section 3.3).

3.1 Introduction

First of all, we give here the definition of a Chvátal-Gomory cut and of the first Chvátal closure polyhedron for a general IP problem. Given the Integer Programming problem:

$$\begin{aligned} &\min c^T x \\ &s.t. \\ &\quad Ax \geq b \\ &\quad x \geq 0, \\ &\quad x \text{ integer} \end{aligned} \tag{3.1}$$

where A is a $m \times n$ real matrix, c and b are a n-dimensional and a m-dimensional vectors respectively and x is a n-dimensional vector of variables that take integer values, a Chvátal-Gomory cut, indicated by CG cut, is defined as follows ([19], [35]):

Definition 3.1.1 (Chvátal-Gomory cut). A *Chvátal-Gomory cut* is a valid inequality for $P_I(A)$ of the form:

$$\lceil u^T A \rceil x \geq \lceil u^T b \rceil$$

where $u \in R_+^m$ is the CG multiplier vector and $\lceil .. \rceil$ is the upper integer part.

The first Chvátal closure polyhedron is the polyhedron obtained by intersecting the relaxed polyhedron P(A) with all the CG cuts.

Definition 3.1.2 (First Chvátal closure). The *first Chvátal closure* of $P(A)$ is the polyhedron $P_1(A)$ defined as follows [19]:

$$P_1(A) := \{x \in \mathbb{R}_+^n : \; Ax \geq b, \lceil u^T A \rceil x \geq \lceil u^T b \rceil \quad \forall u \in \mathbb{R}_+^m\}.$$

The three polyhedrons are related by the relations

$$P_I(A) \subseteq P_1(A) \subseteq P(A)$$

therefore, $P_1(A)$ is a better approximation of $P_I(A)$ than $P(A)$.

For this reason, we try to find violated CG cuts to cut off fractional solution of the linear relaxation of the Set Covering formulation for the MPM problem.

The Minimum Power Multicast problem can be expressed in a general form:

$$\begin{aligned} &\min p^T x \\ &s.t. \\ &\qquad Bx \geq \mathbf{1} \\ &\qquad x \in \{0,1\}^{|A|} \end{aligned} \tag{3.2}$$

where $B = (b_{ij})_{i \in M, j \in N}$ is a $0-1$ matrix, $p \in R^{|A|}$ is the array of the powers and A is the set of the arcs of the network and M and N are the index sets of the rows and the columns respectively of the matrix B. The Set Covering polytope is denoted by $P_I(B)$ and the relaxed polytope by $P(B)$. We denote once more by n the number of nodes of the wireless network and m the number of destinations.

For the results ([7], [8], [22]) reported in the first introductive chapter (see Proposition 1.2.2), we can make here some remarks about the polytope of the Minimum Power Multicasting problem.

Remark 3.1.1. The polytope $P_I(B)$ is always nonempty (if $n \geq 2$ and $m \geq 1$, then $|N_i| \geq 1$ for all $i \in M$) and it is full-dimensional if $n \geq 3$.

Indeed, in this case, $|M| \geq 2$ and for each $i \in M$ the cardinality of N^i is at least equal to two.

Remark 3.1.2. If $n \geq 4$, then for each $j \in N$ the inequality $x_j \geq 0$ is a facet of $P_I(B)$. In fact, for each $i \in M$ and $j \in N$ the cardinality of $N^i \setminus \{j\}$ is at least equal to two. Furthermore all the inequalities $x_j \leq 1$ with $j \in N$ are facets of $P_I(B)$.

The heuristics that we propose, generate valid inequalities with right hand side equal to two and the principle of construction of these inequalities is the following method proposed in ([7], [8]).

Chvátal Gomory cuts can be generated considering positive linear combination of the rows of the matrix and rounding up to the nearest integer all the coefficients of the obtained inequality. In particular, positive linear combinations can be built selecting a subset U of the set of the row indices M, adding all the inequalities of the problem with index in U, then dividing all the coefficients by $|U|-\epsilon$ for a certain positive small enough ϵ and finally rounding all the coefficients up.

Remark 3.1.3. The CG cut relative to a selected $U \subseteq M$ can be obtained by adding all the inequalities $b_i^T x \geq 1$ with $i \in U$ and dividing the resulting inequality by $|U| - \epsilon$:

$$\frac{1}{|U| - \epsilon} \sum_{i \in S} b_i^T x \geq \frac{|U|}{|U| - \epsilon}$$

and finally rounding both members of the inequality up:

$$\left\lceil \frac{1}{|U| - \epsilon} \sum_{i \in S} b_i^T \right\rceil x \geq 2$$

for $0 < \epsilon < 1$.

Looking at the columns of the submatrix of B constituted by all the rows whose index belong to U, it is easy to give a value to the coefficients of the new inequality, indeed, we have ([7], [8]):

Remark 3.1.4. For each $U \subseteq M$ the coefficients of a CG cut can be obtained in this manner:

$$\pi_j^U = \begin{cases} 0 & \text{if } b_{ij} = 0 \text{ for all } i \in U, \\ 2 & \text{if } b_{ij} = 1 \text{ for all } i \in U, \\ 1 & \text{otherwise,} \end{cases} \tag{3.3}$$

so that the inequality $\pi^U x \geq 2$ is the CG cut relative to the choice of U.

Remark 3.1.5.

(i) If $U = \{i\}$, then the inequality $\pi^U x \geq 2$ reduces to the original row $b_i^T x \geq 1$.

(ii) If $U = M$ and the Multicast problem is a Broadcast problem ($m = n-1$), then the inequality generated by the previous method becomes:

$$\sum_{(i,j)\in A\setminus\{(s,v_n^s)\}} x_{ij} + 2x_{sv_n^s} \geq 2.$$

This inequality means that either the source communicates with its most distant node v_n^s or, in order to satisfy the "wireless" connection with all the other destinations, there must be at least another transmitting node in the network in addition to the source.

(iii) If $U = M$ and $m < n-1$ and k is the position of the most distant destination with respect to the source in the array v^s, then the inequality generated by the previous method becomes:

$$\sum_{(i,j)\in A\setminus\{(s,v_j^s):\, 1\leq j<k\}} x_{ij} + 2\sum_{j=k}^{n} x_{sv_j^s} \geq 2$$

that means that either the source is assigned the power to reach v_k^s or at least there are two hops in the network.

Before going on, we want to insert here two valid inequalities, one for the Broadcast problem and one for the more general Multicast problem in wireless networks. These inequalities have both right hand side equal to two.

The first inequality is for the Broadcast problem. We recall that v_2^s and v_n^s represent respectively the closest and the most distant node with respect to the source and that $v_n^{v_2^s}$ is the most distant node with respect to the node which is the closest to the source. In this section, we indicate with w the node v_2^s. Two sets $\mathcal{A}$ and $\mathcal{B}$ must be introduced. $\mathcal{A}$ is the set of all the arcs of A outgoing from a node i, different from the source s and the node w and incoming in a node j which is different from w and furthermore, which is more distant with respect to i than the node v_n^w, i.e.

$$\mathcal{A} := \{(i,j) \in A :\ i \in V \setminus \{s,w\},\, j \in V \setminus \{w\},\ d_{ij} \geq d_{i\,v_n^w}\}.$$

Analogously $\mathcal{B}$ is the set of all the arcs of A outgoing from a node i which is different from the source s and the node w and incoming in a node j which is more distant with respect to i than the node v_n^s, i.e.

$$\mathcal{B} := \{(i,j) \in A :\ i \in V \setminus \{s,w\},\ j \in V,\ d_{ij} \geq d_{iv_n^s}\}.$$

Proposition 3.1.1. *The following inequality:*

$$\sum_{i \in V \setminus \{v_1^s, v_2^s, v_n^s\}} x_{si} + 2x_{s\,v_n^s} + \sum_{i \in V \setminus \{v_1^w, v_n^w\}} x_{w\,i} + 2x_{w\,v_n^w} +$$

$$+ \sum_{(i,j) \in \mathcal{A}} x_{ij} + \sum_{(i,j) \in \mathcal{B} \setminus \mathcal{A}} x_{ij} \geq 2 \tag{3.4}$$

is a valid inequality for $P_I(B)$.

In the multicasting case, denoting by v_k^s the most distant destination from the source and by v_h^w the most distant destination with respect to w,

$\mathcal{A}$ is the set of all the arcs of A outgoing from a node i which is different from the source s and the node w and incoming in a node j, different from w, which is more distant with respect to i than the node v_h^w, i.e.

$$\mathcal{A} := \{(i,j) \in A : \ i \in V \setminus \{s,w\}, \, j \in V \setminus \{w\}, \ d_{ij} \geq d_{iv_h^w}\},$$

and $\mathcal{B}$ is the set of all the arcs of A outgoing from a node i which is different from the source s and the node w and incoming in a node j, which is more distant with respect to i than the node v_k^s, i.e.

$$\mathcal{B} := \{(i,j) \in A : \ i \in V \setminus \{s,w\}, \ j \in V \setminus \{s\}, \ d_{ij} \geq d_{iv_k^s}\}.$$

Proposition 3.1.2. *The inequality:*

$$\sum_{2<i<k} x_{s\ v_i^s} + 2\sum_{i=k}^{n} x_{s\,v_i^s} + \sum_{2<i<h} x_{w\ v_i^w} + 2\sum_{i=h}^{n} x_{w\ v_i^w} +$$
$$\sum_{(i,j)\in\mathcal{A}} x_{ij} + \sum_{(i,j)\in\mathcal{B}\setminus\mathcal{A}} x_{ij} \geq 2 \qquad (3.5)$$

is a valid inequality for $P_I(B)$.

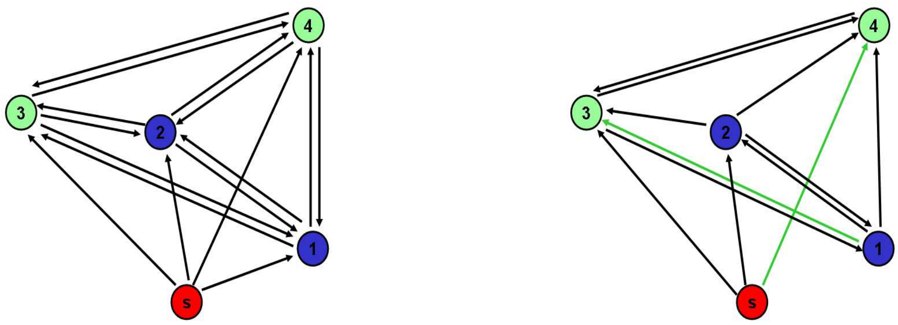

Figure 3.1: An inequality with right hand side two

Naturally, inequality (3.4) is a particular case of inequality (3.5); we give here a simple example for explaining how to construct inequality (3.5).

Example 3.1.1. For the network in Figure 3.1 the distance arrays are the following: $v^s = (s, 1, 2, 3, 4)$, $v^1 = (1, s, 2, 4, 3)$, $v^2 = (2, 3, s, 4, 1)$, $v^3 = (3, 2, s, 4, 1)$, $v^4 = (4, 2, 1, 3, s)$, hence, the set $\mathcal{A} =: \{(2,3), (2,4), (2,1), (4,3)\}$ and $\mathcal{B} := \{(2,4), (2,1), (3,4), (3,1)\}$, and the inequality (3.5) is:

$$x_{s2} + x_{s3} + 2x_{s4} + x_{12} + x_{14} + 2x_{13} + x_{23} + x_{24} + x_{21} + x_{43} + x_{34} + x_{31} \geq 2$$

In fact inequality (3.5) forces the source either to reach directly its most distant destination 4 (the green arc in Figure 3.1) or to communicate with a node placed between 1 and 4 and at this point, it is required another transmission to cover node 4. If the source transmits toward its closest node 1, the latter is forced to reach directly its most distant destination 3 (the green arc in Figure 3.1) or to communicate with another node and, in this case, the constraint forces another communication to cover node 3.

3.2 Heuristics for generating a CG cut with right hand side two

The aim of the heuristics is to find CG cuts with right hand side equal to two that cut off fractional optimal solutions of the linear relaxation of the Multicasting problem. Starting with the support of the optimal solution for the LP problem two propositions can be useful. According to Definition 1.1.9, if x^* is an optimal solution of the linear relaxation of the Multicasting problem, its support is the set $Supp := \{j \in N : \; x^*_j > 0\}$, moreover, the set of the column indices j such that $x^*_j = 1$ can be denoted by I , i.e. $I := \{j \in N : \; x^*_j = 1\}$.

Proposition 3.2.1. *([7], [8]) Let $\pi^T x \geq 2$ be an inequality that cuts off the fractional optimal solution x^*, then $\pi^U_j = 0$ for all $j \in I$.*

The above proposition suggests a first criterion for selecting the subset U of M, indeed, we have:

Remark 3.2.1. The set U does not contain any row i of the matrix B such that exists at least a $j \in I$ with $b_{ij} = 1$.

The second proposition is the following:

Proposition 3.2.2. *Let $\pi^U x \geq 2$ be an inequality that cuts off x^*, then for all $i \in U$ it holds that $b_i^T x^* < 2$.*

Hence another rule for selecting the subset U is:

Remark 3.2.2. The set U does not contain any row i of the matrix B such that $\sum_{j \in M} b_{ij}\, x_j^* \geq 2$.

The inputs of the heuristics are a current fractional solution x^* of the linear relaxation of the problem (see 3.2) and the constraint matrix B. The goal is to find a subset $U \subset M$ such that $\pi^U\, x^* < 2$ and, initially, U is set to be equal to M. Using Propositions 3.2.2 and 3.2.1, the heuristics eliminate from U, first of all, all the row indices i such that $b_i^T\, x^* \geq 2$ and then all the row indices i such that $b_{ij} = 1$ and $x_j^* = 1$.

Given a subset U of M, we denote by *value* the quantity:

$$value(U) := \sum_{j \in Supp} \pi_j^U\, x_j^*,$$

where the coefficients π^U are computed using the definition (3.3).

3.2.1 Row-criterion

The elements of the support are ordered in an increasing way with respect to the x_j^*'s value, and, then, if there exist j and k in $Supp$ such that $x_j^* = x_k^*$ the elements of the support are ordered in an increasing way with respect to the number of ones present in the corresponding column in the submatrix whose row indices are in U.

Until a cut is found or all the rows whose indices are in U have been explored,

Step 0: We select a row $i \in U$ and we set $W := \emptyset$;

Step 1: While $value(U \setminus W) \geq 2$ and $|W| < |U| - 1$, iteratively we select a column j in the ordered support such that $b_{ij} = 0$ and we update W, $W := W \cup \{k \in U : \ b_{kj} = 1\}$;

Step 2: If $value(U \setminus W) < 2$ we have found a cut that cuts off the current fractional solution x^* and we add it to the MPM formulation, if, otherwise, $|W| = |U| - 1$ we select a new row $h \in U$ setting again $W := \emptyset$ and we come back to Step 1.

3.2.2 Greedy-criterion

The column j corresponding to the greatest value of x_j^* is selected and the element j is eliminated from the set $Supp$ (that is $Supp := Supp \setminus \{j\}$). All the indices i of the current U such that $b_{ij} = 1$ are eliminated from U, U is updated ($U = U \setminus \{i \in U : \ b_{ij} = 1\}$) and $value(U)$ is computed. While $value(U) \geq 2$ and $|U| > 1$, we choose the column $k \in Supp$ such that the coefficients π^U relative to $U \setminus \{i \in U : \ b_{ik} = 1\}$ give the smallest value of *value* among all the possible choices of an element in the current set *Supp*. We updated *Supp* and U, $Supp := Supp \setminus \{k\}$ and $U := U \setminus \{i \in U : \ b_{ik} = 1\}$ respectively and we check again the value of *value* and the cardinality of U. If $value(U) < 2$, the cut whose coefficients are π^U has been found and we add it to the Set Covering formulation for the MPM problem; if $value(U) \geq 2$ and $|U| \leq 1$ with this heuristic no more cuts can be added.

3.3 Most violated inequality over the first Chvátal closure

The heurists find a violated inequality with right hand side equal to two. If one wants to find the most violated inequality over the first Chvátal closure, then a MIP problem which has been proved to be an $NP-$ hard problem [30] must be solved.

Formally the Multicasting problem is:

$$\begin{aligned}
&\min p^T x \\
&s.t. \\
&\quad Bx \geq \mathbf{1} \\
&\quad -Ix \geq -\mathbf{1} \\
&\quad x \geq 0, \ x \text{ integer.}
\end{aligned} \tag{3.6}$$

If x^* is the optimal solution for the linear relaxation of this problem, then the separation problem, is the problem of finding $u \in \mathbb{R}_+^{|M|}$ and $v \in \mathbb{R}_+^{|N|}$ such that $\lceil u^T B - v^T I \rceil x < \lceil u^T \mathbf{1} - v^T \mathbf{1} \rceil$ or proving that no cut is violated, that is, no such u and v exist. If a cut can be found, minimizing the difference: $\lceil u^T B - v^T I \rceil x - \lceil u^T \mathbf{1} - v^T \mathbf{1} \rceil$ produces the most violated CG cut.

Remark 3.3.1. The vectors u and v can be assumed to have each component less than one [31] as each coefficient of the problem is integer. In fact, suppose for axample that $u_i \geq 1$ for an $i \in M$. The CG cut associated with u_i is dominated, since it can be obtained as the sum of $\lfloor u_i \rfloor$ times the constraint $b_i^T x \geq 1$ and the CG cut associated with the fractional part of u_i.

Denoted by $\pi := \lceil u^T B - v^T I \rceil$ and by $\pi_0 := \lceil u^T \mathbf{1} - v^T \mathbf{1} \rceil$ for a certain $u \in \mathbb{R}_+^{|M|}$ and $v \in \mathbb{R}_+^{|N|}$, the separation model [31] can be formulated as follows:

$$\begin{aligned}
&\min \pi^T x^* - \pi_0 \\
&s.t. \\
&\qquad \pi_j \geq u^T B_j - v_j && \forall j \in \{1, .., |N|\} \\
&\qquad \pi_0 < u^T \mathbf{1} - v^T \mathbf{1} + 1 && \\
&\qquad 0 \leq u_i \leq 1 - \epsilon && \forall i \in \{1, .., |M|\} \\
&\qquad 0 \leq v_k \leq 1 - \epsilon && \forall k \in \{1, .., |N|\} \\
&\qquad \pi, \pi_0 \text{ integer}
\end{aligned} \tag{3.7}$$

Naturally, even in this case ϵ is a positive, but small enough, real number that has been set to 0.01 as recommended in [31]. To reduce the number of integer variables π one can observe that all the variables x_i with $x_i^* = 0$ do not give any contribution to the objective function value of the separation problem and so, the separation problem itself can be constructed only on the support of the solution x^*. Indeed, for any $j \in N \setminus Supp$ the value of the corresponding π_j can be computed using the optimal value of the variables u and v, that is $\pi_j = \lceil u^T B_j - v_j \rceil$.

The separation problem can be, thus, reduced to the following MIP problem [31]:

$$
\begin{aligned}
\min & \sum_{j \in Supp} \pi_j x_j^* - \pi_0 \\
s.t. & \\
& s_j + \pi_j - u^T B_j + v_j = 0 \quad && \forall j \in Supp \\
& s_0 + \pi_0 - u^T \mathbf{1} + v^T \mathbf{1} = 0 && \\
& 0 \le u_i \le 1 - \epsilon && \forall i \in \{1, .., |M|\} \\
& 0 \le v_k \le 1 - \epsilon && \forall k \in \{1, .., |N|\} \\
& 0 \le s_j \le 1 - \epsilon && j \in Supp \cup \{0\} \\
& \pi_j \text{ integer} && j \in Supp \cup \{0\}
\end{aligned}
\tag{3.8}
$$

where the variables $s_j = \lceil u^T B_j - v_j \rceil - u^T B_j + v_j$ are slack variables.

3.4 Preliminary computational results

The two heuristics and the exact separation problem have been implemented in C and the codes have run on a Opteron 246 machine with 2 GB RAM memory using the version 9.1 of Cplex as solver.

The experiments have been performed on the set of test problems with increasing number of nodes and of possible destinations generated in chapter 2, whose linear relaxation do not provide an integer solution. While the linear relaxation of the MPM problem provides a fractional solution and a CG cut can be found using the heuristic processes in sections 3.2.1 or 3.2.2 or solving the separation problem (3.8) it is added to the current formulation and the problem is solved again. In the Table 3.1, we want to present the preliminary results obtained with networks with up to 15 nodes. We report there the number of nodes n, the number of destinations m and the seed from which the problem has been generated *seed*.

Table 3.1: Heuristics-Exact problem of generating CG cuts

				3.2.1			*3.2.2*			*3.8*		
n	m	$seed$	$\frac{OPT-LP}{LP}$	CG	Gap	T	CG	Gap	T	CG	Gap	T
10	7	1	0.007	3	0	0.01	1	0.007	0.01	5	0	4
10	8	1	0.007	3	0	0.03	1	0.007	0	5	0	3
10	9	1	0.012	4	0	0.01	2	0.005	0.02	2	0	0.8
15	5	14	0.002	1	0	0.03	1	0	0.04	1	0	0.03
15	6	14	0.002	1	0	0.05	1	0	0.04	1	0	0.03
15	7	14	0.002	1	0	0.03	1	0	0.04	1	0	0.04
15	8	20	0.000	1	0	0.08	1	0	0.06	1	0	0.03
15	9	10	0.139	23	0.024	1.76	9	0.093	0.84	-	-	> 600
15	9	20	0.000	1	0	0.09	1	0	0.07	1	0	0.04
15	10	2	0.005	4	0	1.11	1	0.005	0.15	2	0	123
15	10	10	0.139	23	0.024	1.77	9	0.093	0.83	-	-	> 600
15	10	20	0.033	16	0	0.75	1	0.033	0.19	-	-	> 600
15	11	2	0.005	4	0	1.12	1	0.005	0.13	2	0	125
15	11	10	0.139	33	0.006	3.01	1	0.139	0.17	-	-	> 600
15	11	20	0.034	19	0	0.96	1	0.034	0.19	-	-	> 600
15	12	2	0.005	4	0	1.12	1	0.005	0.14	2	0	425.18
15	12	10	0.152	31	0.001	3.04	1	0.152	0.18	-	-	> 600
15	12	20	0.032	11	0	0.6	1	0.032	0.24	-	-	> 600
15	13	2	0.005	4	0	1.12	1	0.005	0.14	2	0	425.91
15	13	3	0.019	4	0	0.35	1	0.019	0.08	6	0	146.03
15	13	10	0.152	32	0.036	4.86	4	0.128	0.32	-	-	> 600
15	13	18	0.010	7	0	0.42	1	0.010	0.07	-	-	> 600
15	13	20	0.032	11	0	0.61	1	0.032	0.24	-	-	> 600
15	14	2	0.005	4	0	1.11	1	0.005	0.13	2	0	442.75
15	14	3	0.019	6	0	0.36	1	0.019	0.08	6	0	145.65
15	14	10	0.152	34	0.020	2.05	4	0.128	0.31	-	-	> 600
15	14	18	0.004	2	0	0.28	1	0.004	0.11	4	0	86.10
15	14	20	0.032	11	0	0.61	1	0.032	0.23	-	-	> 600

The column $(OPT - LP)/LP$ reports the gap between the optimal solution OPT of the integer problems and the optimal value of their linear relaxations LP. *Gap* is the ratio $(OPT - LP)/LP$ where OPT is the optimal value of the integer problem (3.6) while LP is the optimal value of the linear relaxation of the problem with the addition of the CG cuts that can be generated with the different methods. For each problem, we report the number CG of the CG cut generated, the *Gap* and the computational time T. The computational time T does not include the preprocessing time of the matrix but only the time for solving the linear relaxations of the problems and the time for generating the cuts. If T is greater than 600 seconds, then the computation is interrupted.

Obviously, finding the most violated inequalities in the first Chvátal closure on the basis of the current fractional solutions and inserting them to the formulation, gives the best value of the lower bounds but it is also true that it is too time consuming even for small networks (15 nodes); there are several cases in which the whole problem is not solved within the time limit.

The heuristic of section 3.2.1 provides cuts that reduce strongly the gap and, in most of the considered cases, the optimal solution of the linear relaxation of the problems with the generated CG cuts is integer. However, it generates more cuts than the other approaches and it is not as fast as the procedure with the heuristic in section 3.2.2.

The heuristic in section 3.2.2 is the fastest and it provides few cuts that reduce the gap but not so strongly as for the cuts found with the procedure in section 3.2.1 or solving the problem (3.8); in many cases, also with graphs with 10 nodes, inserting the CG cuts of heuristic in 3.2.2 to the linear relaxation of the problems does not reduce to zero the value *Gap*.

3.5 Concluding remarks

The row-based heuristic is able to generate CG cuts that improve the lower bounds of the linear relaxation of the Set Covering polytope for the Multicasting problems in wireless networks in a reasonable time, but two steps can still be done: the first is to find facet defining inequalities not necessarily belonging to the first closure and the second is to generate facet defining inequalities without scanning, in the worst case, all the rows of the current matrix U (as in the heuristic procedure in section 3.2.1).

The programs Porta [17] (POlyhedron representation transformation algorithm) and cdd [34] have been run on the randomly generated MPM problem in order to obtain an explicit description of the Set Covering polytope. Unfortunately, it is not possible to terminate the programs for the problems whose linear relaxations have a fractional optimal solution, because neither of them is able to provide (in days of computation) the description of the polytope for networks with more than 5 nodes. At present, all the generated graphs with 5 nodes (more than 500 problems have been generated) can be solved just with the linear relaxation of the problem and no more constraints than those that are in the formulations are required in the description of their polytopes.

The effectiveness of the Set Covering formulation (2.25)-(2.26) for the Minimum Power Multicast problems, has been also checked using the tool in [5]. No coefficient of the constraint matrix is strengthened by the code that Andersen *et al.* propose.