Abstract

This dissertation is devoted, mainly, to a specific class of Combinatorial Optimization problems: the Multicast problem and some related variants. Specifically, given a graph $G = (V, E)$ and a subset R of elements of the set of the nodes V, the Multicast problem consists in determining a connected subset T of the set of the edges, whose elements connect all the nodes belonging to R (using possibly some nodes not in R) in such a way that an objective function representing the cost of the connection is minimized.

The major part of the presented results is devoted to a particular type of network, the Ad-Hoc wireless network. The nodes of these networks are electronic devices (sensors, computers, radio transmitters etc.) which transmit radio signals without using a fix infrastructure and without a centralized administration. The Multicast problem, in this case, consists in assigning a power to the devices of the network in such a way that the elements belonging to the set R receive the messages originated from a particular node of the network, called source, and the total amount of assigned power is minimized. One of the peculiarity of a radio transmission is that every signal forwarded by a node can be received by all the nodes placed in the transmission range of the communication and, thus, contrary to the wired network case, performing only one transmission and so paying the cost of a single arc, it is possible to connect several nodes at the same time.

In particular, we can summarize the main contributions of in this dissertation as follows:

- We propose a Set Covering formulation for the Minimum Power Multicast problem in wireless Ad-Hoc networks, which results to be stronger than certain formulations presented in literature and we propose two exact methods for solving the problem making use of a possible reduction of the size of the problem which is based on the properties of the Set Covering problem.

- We present also two heuristics for generating valid inequalities of the first Chvátal closure of the Set Covering polytope and, thus, for strengthening the linear relaxation of the formulation of the Minimum Power Multicast problem. In the case of wireless networks with a small number of nodes, we compare the optimum value and the computational time for solving the linear relaxation of the problems with the addition of the constraints generated by the heuristics with the optimum value and the computational time occuring for solving the linear relaxation of the problems with all the cuts belonging to the first Chvátal closure of the Set Covering polytope.

- Moreover, an innovative variant of the Multicast problem is considered, in which to the devices of a wireless network is assigned a probability of failure in the reception and transmission of the messages. Indeed, we present here three mixed integer programming formulations for the problem of connecting the source with all the other nodes of the network (R is the set of all the nodes of the network except the source) with a reliability threshold. The solution, hence, not only guarantees a connection, but in fact gives a robust connection of the elements of the network with the source.

- Finally, another variant of the Multicast problem, considered in the dissertation, is the problem of finding not only a connection of a subset R with a source with the minimum total cost (or weight) but, assigned to each arc a delay, we deal with the problem of finding a minimum cost arborescence connecting the source with the elements of R with additional time limit constraints. For this problem, in case of wired networks, four mixed integer programming formulations are proposed together with a preprocessing procedure for reducing the size of the problem. The four formulations with the preprocessing procedure have been tested on some Steiner Tree problems proposed in the SteinLib library [48], where the delay on the arcs have been randomly generated in a correlated and non-correlated way with respect to the costs of the arcs.

Mathematics Subject Classification 2000: *Primary:* 90C27, 90C11 *Secondary:* 90C35, 90B10, 68R10.

Keywords: Combinatorial Optimization; Multicast problem; Mixed integer programming formulations; Set Covering problem; Wireless Ad-Hoc networks; Probabilistic Broadcast problem; Delay-constrained Steiner Tree problem; Preprocessing.