BOUNDED, MONTEL AND COMPACT OPERATORS ON SPACES OF MOSCATELLI TYPE

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Abstract. *We characterize the pairs (E, F) of Moscatelli Fréchet and LB spaces such that: 1) Every continuous linear map from E into F sends bounded sets in E into precompact ones in F. 2) Every bounded map from E into F sends some 0-neighbourhood in E into a precompact set in F.*

1. INTRODUCTION, NOTATION AND DEFINITIONS

The Fréchet and *LB* spaces of Moscatelli type have been used in the literature as a ready source of examples and counterexamples in the theory of Fréchet and *DF* spaces (in fact they were introduced by Moscatelli with the aim to find a twisted quojection). In this aspect, it is significative the Fréchet Moscatelli space constructed by Taskinen in order to give a counterexample to the Grothendieck's problem of topologies. We refer the reader to [2] and [3].

The theory of operator ideals in the class of Banach spaces has been successfully systematized by Pietsch and his school. Given an operator ideal $\mathcal{A}$ in the class of Banach spaces, Pietsch [12] considers two maximal ways to extend it to the class of locally convex spaces (in short l.c.s.): the largest or weak extension $\mathcal{A}^W$ and the smallest or strong one $\mathcal{A}^S$. If $\mathcal{A} = \mathcal{L}$ (the ideal of all operators), then $\mathcal{L}^S = \mathcal{LB}$, the ideal of bounded operators, i. e. $T \in \mathcal{LB}(E, F)$ if $T(U)$ is bounded in F for some zero neighbourhood U in E. If $\mathcal{A} = \mathcal{K}$ (the ideal of compact operators), it holds that:

1) $\mathcal{K}^w$ is the ideal $\mathcal{PM}$ of pre-Montel operators, i.e. $T \in \mathcal{K}^W(E, F) = \mathcal{PM}(E, F)$ if for each bounded set B in E, $T(B)$ is precompact in F. It is clear that if F is quasicomplete, $\mathcal{K}^W(E, F)$ coincides with the component $\mathcal{M}(E, F)$ of the ideal $\mathcal{M}$ of Montel operators, i.e. $T \in \mathcal{M}(E, F)$ if $T(B)$ is relatively compact in F for every bounded set B in E.

2) If the precompact sets of F are metrizable, $\mathcal{K}^S(E, F)$ is the component $\mathcal{PK}(E, F)$ of the ideal $\mathcal{PK}$ of precompact operators, i.e. $T \in \mathcal{PK}(E, F)$ if for some 0-neighbourhood U of E, $T(U)$ is precompact in F. Moreover, if F is quasicomplete, $\mathcal{K}^S(E, F) = \mathcal{K}(E, F)$. All the spaces we shall use in this paper will have metrizable precompact sets as consequence of the results of Cascales and Orihuela in [5], but they will not be quasicomplete in general. Then we shall need to use $\mathcal{PM}$ and $\mathcal{PK}$ instead of $\mathcal{M}$ and $\mathcal{K}$.

If a pair (E, F) of l. c. s. verifies $\mathcal{A}(E, F) = \mathcal{L}(E, F)$, we shall write $(E, F) \in \mathcal{A}$. A classical problem in the class of Banach spaces is to find pairs (X, Y) such that $(X, Y) \in \mathcal{K}$. In the class of l. c. s. the problem is translated to the search of pairs (E, F) of l. c. s. for which $(E, F) \in \mathcal{A}$ or $\mathcal{A}(E, F) = \mathcal{B}(E, F)$, where $\mathcal{A}$ and $\mathcal{B}$ are some of the mentioned ideals. These questions have been treated in several papers for various classes of l. c. s., see for example [1], [4], [6], [8], [10] and [13]. In this note, *our purpose is to characterize the pairs of Moscatelli spaces such that* $(E, F) \in \mathcal{PM}$ *and* $\mathcal{LB}(E, F) = \mathcal{PK}(E, F)$.

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The notation is standard. In all the paper, p and q will be numbers in $[1,\infty[\cup\{0\}$. The symbol ℓ_0 will represent the classical space c_0, and $(e^n)_{n=1}^{\infty}$ will be the canonical Schauder basis of ℓ_p, and e_i^n the i-component of e^n.

1. Definition. *(Banach spaces of vector valued sequences).* If $(X_n)_{n=1}^{\infty}$ is a sequence of Banach spaces, the space

$$\ell_p((X_n)) = \left\{ (x_n) \in \prod_{n=1}^{\infty} X_n \quad | \quad (\|x_n\|) \in \ell_p \right\}$$

(we denote by the same symbol $\|.\|$ the norm on every space X_n), is a Banach space with the norm induced by those one of ℓ_p (see proposition 1.2 of [3]).

2. Definition. *(Moscatelli Fréchet spaces).* For every p, let $(X_n^p)_{n=1}^{\infty}$ and $(Y_n^p)_{n=1}^{\infty}$ be two sequences of Banach spaces and $(f_n^p)_{n=1}^{\infty} \subset \mathcal{L}(Y_n^p, X_n^p)$ a sequence of operators such that $\|f_n^p\| \leq 1$, $n \in N$. We consider the Banach spaces

$$F_k^p = \left\{ (w_n) \in \left(\prod_{n<k} Y_n^p \right) \times \left(\prod_{n \geq k} X_n^p \right) \quad | \quad (\|w_n\|) \in \ell_p \right\} \qquad \forall k \in N$$

and the linking maps $J_{k,k+1} \in \mathcal{L}(F_{k+1}^p, F_k^p)$ defined by $J_{k,k+1}((x_n)) = ((x_n)_{n<k}, f_k^p(x_k), (x_n)_{n>k})$. The Fréchet space of Moscatelli type associated to $(Y_n^p), (X_n^p)$ and (f_n^p) is the space $\mathcal{F}_p = proj_{k \in N}(F_k^p, J_{k,k+1})$.

Without loss of generality, $\mathcal{F}_p$ can be supposed always to be a reduced projective limit taking the closure of $f_k^p(Y_k^p)$ in X_k^p instead of X_k^p, $k \in N$. Every Y_m^p, $m \in N$, is complemented in $\mathcal{F}_p$, see [3].

3. Definition. *(Moscatelli LB spaces).* For every p, let $(X_n^p)_{n=1}^{\infty}$ and $(Y_n^p)_{n=1}^{\infty}$ be two sequences of Banach spaces and $(f_n^p)_{n=1}^{\infty} \subset \mathcal{L}(Y_n^p, X_n^p)$ a sequence of injective maps such that $\|f_n^p\| \leq 1$, $n \in N$. We consider the Banach spaces

$$G_k^p = \left\{ (w_n) \in \left(\prod_{n<k} X_n^p \right) \times \left(\prod_{n \geq k} Y_n^p \right) \quad | \quad (\|w_n\|) \in \ell_p \right\}, \qquad \forall k \in N.$$

The (LB)-space of Moscatelli type associated to $(X_n^p)_{n=1}^{\infty}$ and $(Y_n^p)_{n=1}^{\infty}$ is the inductive limit $G_p = ind_{k \in N} G_k^p$.

By proposition 2.1 of [3], for every $s = (\lambda, (\mu_k)) \in S := R^+ \times (R^+)^N$, there is a norm h_{λ,μ_k} on Y_k^p, $k \in N$, equivalent to the original one such that F_p is the regular inductive limit $F_p = ind_{s \in S} H_s^p$, where $H_s^p = \ell_p((Y_k^p, h_{\lambda,\mu_k}))$. On the other hand, G_p is regular if and only if it is complete (see [2]). In this case, G_p is a *reduced* projective limit $G_p = proj_{s \in S} L_s^p$ where $L_s^p = \ell_p((X_k^p, h_{\lambda,\mu_k}))$ and h_{λ,μ_k} is a certain norm on X_k^p equivalent to the original one. In the general case, G_p is a dense subspace of a projective limit of this kind (see lemma 2.4 of [3]). It follows that every X_k^p, $k \in N$, is complemented in G_p. The regularity of G_p is characterized in [3] in terms of spaces X_k^p.

2. THE RELATIONS $(E, F) \in M$ AND $LB(E, F) = K(E, F)$ IN THE CLASS OF MOSCATELLI SPACES

The Fréchet and *LB* spaces of Moscatelli type have some common features in the sense that they can be described as dense subspaces of countable and uncountable projective limits respectively or by uncountable or countable regular inductive limits of spaces isomorphic to spaces of $\ell_p((Z_n))$ type. Hence, we shall use later on the results of the following theorems, which have been proved in [11]:

4. Theorem. *Let $F = proj_{t \in T} F_t$ be a reduced projective limit of l. c. s. F_t, and G a dense subspace of F. Let $E = ind_{s \in S} E_s$ be a regular inductive limit of l. c. s. E_s.*

1) If $(E_s, F_t) \in PM$ *for each s and t, then* $(E, G) \in PM$.

2) If $(F_t, E_s) \in PK$ *for each s and t, then* $LB(G, E) = PK(G, E)$.

5. Theorem. *1) Let $(X_n)_{n=1}^{\infty}$ be Banach spaces and let G be an LF space. Then $(\ell_p((X_n)), G) \in PK$ if and only if $(\ell_p, G) \in PK$ and $(X_n, G) \in PK$ for every $n \in N$. Moreover, $(G, \ell_p((X_n))) \in M$ if and only if $(G, \ell_p) \in M$ and $(G, X_n) \in M$ for all $n \in N$.*

2) Let $(X_n)_{n=1}^{\infty}$ be Banach spaces and let G be an LF space. Then $LB(G, \ell_p((X_n))) = K(G, \ell_p((X_n)))$ if and only if $LB(G, \ell_p) = K(G, \ell_p)$ and $LB(G, X_n) = K(G, X_n)$ for every $n \in N$.

3) Let $(X_n)_{n=1}^{\infty}$ and $(Y_n)_{n=1}^{\infty}$ be two sequences of Banach spaces. Then $(\ell_p((X_n)), \ell_q((Y_n))) \in K$ if and only if $p > q \leq q$ or $p = 0$ and $1 \geq 1 < \infty$ and $(\ell_p, Y_n) \in K$, $(X_n, \ell_q) \in K$ and $(X_n, Y_m) \in K$ for every $n, m \in N$.

Now we will study Montel maps between Moscatelli spaces. Since $(E, F) \in M$ if E or F is a Montel space, we only need to consider non-Montel Moscatelli spaces. We recall that F_p (resp. G_p) is a Montel space if and only if $F_p = \prod_{n=1}^{\infty} Y_n^p$ (resp. $G_p = \bigoplus_{n \in N} X_n^p$) holds algebraically and topologically, where all Y_n^p (resp. all X_n^p) are finite dimensional Banach spaces, (see [3]). First, we need the following lemma.

6. Lemma. *For every p, suppose that F_p is neither a Montel space nor a product of Banach spaces, and that G_p is neither Montel nor a direct sum of Banach spaces. Then*

1) If Y is a l. c. s. and $(\ell_p, Y) \notin PK$, then $LB(F_p, Y) \neq PM(F_p, Y)$ and $LB(G_p, Y) \neq PM(G_p, Y)$.

2) If Y is a l. c. s. such that $(Y, \ell_p) \notin M$, then $LB(Y, F_p) \neq M(F_p, Y)$ and $LB(Y, G_p) \neq PM(Y, G_p)$.

3) If $1 \leq p \leq q$ or $q = 0, LB(F_p, F_q) \neq M(F_p, F_q), LB(F_p, G_q) \neq PM(F_p, G_q)$, $LB(G_p, F_q) \neq M(G_p, F_q)$ and $LB(G_p, G_q) \neq PM(G_p, G_q)$.

Proof. By proposition 2.7 of [3], in the case of F_p, for every $k \in N$ there are $n_k \in N$, $y_{n_k} \in Y_{n_k}^p$ and $y'_{n_k} \in (X_{n_k}^p)'$ such that $n_k < n_{k+1}$, $\|f_{n_k}^p(y_{n_k})\| = 1$, $\|y'_{n_k}\| \leq 1$ and $< y'_{n_k}, f_{n_k}^p(y_{n_k}) > = 1$. In the case of G_p we can suppose furthermore that $(y_{n_k})_{k=1}^{\infty}$ are the non null components of $(y_i) \in G_p$ which lies in some $G_r^p = \ell_p\ (X_1^p, \ldots, X_{r-1}^p, Y_r^p, Y_{r+1}^p \ldots) \subset G_p$ with $r \leq n_1$ and $< y'_{n_k}, y_{n_k} > = \|y_{n_k}\|$.

1) We first consider the particular case $Y = \ell_q$ with $p \leq q$. Define $T : F_p \longrightarrow \ell_q$ by $T((z_n)) = (\eta_i)$ where $\eta_{n_k} = < y'_{n_k}, f_{n_k}^p\ (z_{n_k}) >$ if $i = n_k$, $k \in N$ and $\eta_i = 0$ if $i \neq n_k$ for all $k \in N$. Since $p \leq q$, T is *bounded*. The sequence $\left(e_i^{n_k} y_{n_k}\right)_{k=1}^{\infty}$ is bounded in F_p because $\|f_{n_k}^p(y_{n_k})\| = 1$. But $T((e_i^{n_k} y_{n_k})) = e^{n_k}$, and $(e^{n_k})_{k=1}^{\infty}$ has no convergent subsequence in ℓ_q. Then T is not Montel.

In the general case, choose a non precompact map $A \in L(\ell_p, Y)$ and a bounded sequence $((\alpha_i^n))_{n=1}^{\infty}$ in ℓ_p such that $A((\alpha_i^n))$ has no convergent subnet in $\hat{Y}$. We define $S \in L(\ell_p, \ell_p)$ by $S((\beta_i)) = (\beta_{n_k})_{k=1}^{\infty}$. For every $m \in N$, let $(z_i^m)_{i=1}^{\infty} \in F_p$ be such that $z_{n_k}^m = \alpha_k^m\, y_{n_k}$, $k \in N$, and $z_i^m = 0$ if $i \neq n_k$ $\forall k \in N$. It is clear that $(z_i^m)_{m=1}^{\infty}$ is a bounded sequence in F_p. Consider the map $T \in L(F_p, \ell_p)$ of above. We have $ST((z_n^m)) = (\alpha_n^m)$ and hence $AST \notin PM(F_p, Y)$.

In the case of G_p the proof is the same with sligth variants: take $\eta_{n_k} = < y'_{n_k}, z_{n_k} >$, $k \in N$ and $(e_i^{n_k} \|y_{n_k}\|^{-1} y_{n_k})_{k=1}^{\infty}$ to show that T is not Montel and $z_{n_k}^m = \alpha_k^m \, \|y_{n_k}\|^{-1} \, y_{n_k}$.

2) We define $S \in L(\ell_p, F_p)$ by $S((\alpha_i)) = (z_i)$ where $z_{n_k} = \alpha_k \, y_{n_k}$, $k \in N$ and $z_i = 0$ if $i \neq n_k$ $\forall k \in N$. If $A \in L(Y, \ell_p)$ sends some bounded sequence in Y into a sequence $(\alpha_i^n)_{n=1}^{\infty}$ of ℓ_p without convergent subsequences, it is easy to check that $T = SA \in L(Y, F_p)$ is not Montel. In the case of G_p, the proof is analogous but defining $z_{n_k} = \alpha_k \, \|y_{n_k}\|^{-1} \, y_{n_k}$ for $k \in N$.

3) We consider the map $T \in L(F_p, \ell_q)$ defined in 1). By proposition 2.7 of [3], for every $k \in N$, we choose $m_k \in N$, and $z_{m_k} \in Y_{m_k}^q$ such that $m_k < m_{k+1}$ and $\|f_{m_k}^q\, (z_{m_k})\| = 1$. Now we define $S \in L(\ell_q, F_q)$ by $S((\alpha_1)) = (w_i)$ where $w_{m_k} = \alpha_{n_k}\, z_{m_k}$ for each $k \in N$ and $w_i = 0$ if $i \neq m_k$ $\forall k \in N$. It is easy to see that the *bounded* map $ST \in L(F_p, F_q)$ is not Montel. In the case of G_q we shall take $r \in N$ and $(z_i) \in G_r^q = \ell_q(X_1^q, \dots, X_{r-1}^q, Y_r^q, Y_{r+1}^q \dots) \subset G_q$ with infinitely many components $z_{h_k} \neq 0$. Define $S : \ell_q \longrightarrow G_r^q \subset G_q$ by $S((\alpha_i)) = (w_i)$ with $w_{h_k} = \alpha_{n_k} \, \|z_{h_k}\|^{-1} \, z_{h_k}$ and $w_i = 0$ when $i \neq h_k$ $\forall k \in N$. It is easy to see that $S \in L(\ell_q, G_r^q)$ but $ST \notin PM\,(F_p, G_q)$ using the method of 1). In the cases (G_p, F_q) and (G_p, G_q), the proofs are respectively the same ones of above, since the map T of 1) is continuous also from G_p into ℓ_q.

In the next theorems, we characterize completely the pairs $(E, F) \in PM$ when E and F are Fréchet or *LB* spaces of Moscatelli type. The proofs in all cases are straightforward, having in mind the structure of Moscatelli spaces (they can be represented as dense subspaces of projective or inductive limits of spaces of $\ell_p((Z_n))$ type) and two results of Grothendieck: the first one is the locally convex version of the Schauder theorem (see §42,1. (8) of [9]); the second one asserts that $LB(E, F) = L(E, F)$ if E is a *DF* space and F is a Fréchet space (see proposition 6.2.1 of [8] for instance). The sufficient conditions are consequences of theorems 4 and 5 (in some cases, we must take the whole space G_p as E_s); the necessity follows from lemma 6 and the facts that Y_n^p is complemented in F_p and X_n^p in complemented in G_p.

7. Theorem. *Suppose that F_p and G_p are not Montel spaces and $F_q = \prod_{n=1}^{\infty} Y_n^q$, with some Y_n^q infinite dimensional.*

1) If $F_p = \prod_{n=1}^{\infty} Y_n^p$, then $(F_p, F_q) \in M$ if and only if $(Y_n^p, Y_m^p) \in K$ $\forall\, m, n \in N$.

2) If F_p is not a product of Banach spaces, then $(F_p, F_q) \in M$ if and only if $(\ell_p, Y_n^q) \in K$ and $(Y_n^p, Y_m^q) \in K$ for all $n, m \in N$.

3) If $G_p = \bigoplus_{n=1}^{\infty} X_n^p$, then $(G_p, F_q) \in M$ if and only if $(X_m^p, Y_n^q) \in K$ $\forall\, n, m \in N$.

4) If G_p is not a direct sum of Banach spaces, then $(G_p, F_q) \in M$ if and only if $(\ell_p, Y_n^q) \in K$ and $(X_m^p, Y_n^q) \in K$ $\forall n, m \in N$.

8. Theorem. *Suppose that F_p, G_p and F_q are not Montel spaces and F_q is not a product of Banach spaces.*

1) If $F_p = \prod_{n=1}^{\infty} Y_n^p$ with some Y_k^p infinite dimensional, then $(F_p, F_q) \in M$ if and only if $(Y_n^p, Y_m^q) \in K$ and $(Y_n^p, \ell_q) \in K$ $\forall\, m, n \in N$.

2) If F_p is not a product of Banach spaces and $1 \leq p \leq q$ or $p \leq 1$ and $q = 0$, then $(F_p, F_q) \notin M$. However, if $p > q \geq 1$ or $p = 0$ and $q \geq 1$, then $(F_p, F_q) \in M$ if and only if (ℓ_p, Y_m^q)

$\in K$, $(Y_n^p, \ell_q) \in K$ *and* $(Y_n^p, Y_m^q) \in K - n, m \in N$.

3) If $G_p = \bigoplus_{n=1}^{\infty} X_n^p$ *with some* X_n^p *infinite dimensional, then* $(G_p, F_q) \in M$ *if and only if* $(X_n^p, \ell_q) \in K$ *and* $(X_n^p, Y_m^q) \in K$.

4) If G_p *is not a direct sum of Banach spaces, and* $1 \leq p \leq q$ *or* $p \geq 1$ *and* $q = 0$*, then* $(G_p, F_q) \notin M$*. However, if* $p > q \geq 1$ *or* $p = 0$ *and* $q \geq 1$*,* $(G_p, F_q) \in M$ *if and only if* $(\ell_p, Y_n^q) \in K$, $(X_n^p, \ell_q) \in K$ *and* $(X_n^p, Y_m^q) \in K$ $\forall\, n, m \in N$.

9. Theorem. *Suppose that* G_q *is not a Montel space and* $F_p = \prod_{n=1}^{\infty} Y_n^p$*, with some* Y_n^p *infinite dimensional.*

1) If $G_q = \bigoplus_{n=1}^{\infty} X_n^q$ *with some* X_n^q *infinite dimensional,* $(F_p, G_q) \in M$ *if and only if* $(Y_m^p, X_n^q) \in K$ *for all* $n, m \in N$.

2) If G_q *is not a direct sum of Banach spaces,* $(F_p, G_q) \in PM$ *if and only if* $(Y_m^p, \ell_q) \in K$ *and* $(Y_m^p, X_n^q) \in K$ *for all* $n, m \in N$.

10. Theorem. *Suppose that* F_p *and* G_q *are not Montel spaces and* F_p *is not a product of Banach spaces.*

1) If $G_q = \bigoplus_{n=1}^{\infty} X_n^q$ *with some* X_n^q *infinite dimensional, then* $(F_p, G_q) \in M$ *if and only if* $(\ell_p, X_n^q) \in K$ *and* $(Y_m^p, X_n^q) \in K$ *for all* $n, m \in N$.

2) If G_q *is not a direct sum of Banach spaces and* $1 \leq p \leq q$ *or* $p \geq 1$ *and* $q = 0$*, then* $(F_p, G_q) \notin PM$*. However, if* $p > q \geq 1$ *or* $p = 0$ *and* $q \geq 1$*, then* $(F_p, G_q) \in PM$ *if and only if* $(Y_n^p, \ell_q) \in K$, $(Y_n^p, X_m^q) \in K$ *and* $(\ell_p, X_n^q) \in K$ $\forall n, m \in N$.

11. Theorem. *Suppose that* G_p *is not a Montel space and* $G_q = \bigoplus_{n=1}^{\infty} X_n^q$ *with some* X_n^q *infinite dimensional.*

1) If $G_p = \bigoplus_{n=1}^{\infty} X_n^p$ *with some* X_n^p *infinite dimensional, then* $(G_p, G_q) \in M$ *if and only if* $(X_m^p, X_n^q) \in K$ $\forall n, m \in N$.

2) If G_p *is not a direct sum of Banach spaces,* $G_p, G_q) \in M$ *if and only if* $(\ell_p, X_n^q) \in K$*, and* $(X_m^p, X_n^q) \in K$ $\forall n, m \in N$.

12. Theorem. *Suppose that* G_p *and* G_q *are not Montel spaces and* G_q *is not a direct sum of Banach spaces.*

1) If $G_p = \bigoplus_{n=1}^{\infty} X_n^p$ *with some* X_n^p *infinite dimensional, then* $(G_p, G_q) \in PM$ *if and only if* $(X_m^p, \ell_q) \in K$ *and* $(X_m^p, X_n^q) \in K$ $\forall n, m \in N$.

2) If G_p *is not a direct sum of Banach spaces and* $1 \leq p \leq q$ *or* $p \geq 1$ *and* $q = 0$*, then* $(G_p, G_q) \notin PM$*. However, if* $p > q \geq 1$ *or* $p = 0$ *and* $q \geq 1$ *then* $(G_p, G_q) \in PM$ *if and only if* $(\ell_p, X_n^q) \in K$, $(X_n^p, \ell_q) \in K$ *and* $(X_n^p, X_m^q) \in K$ $\forall n, m \in N$.

With the same spirit we characterize the pairs of Moscatelli spaces such that $LB(E, F) = PK(E, F)$ through the following theorem. Remark that a bounded linear map on a topological product must vanish on all but a finite number of factor spaces. Then the proof follows the same lines of argumentation of the previous theorems and uses also its results.

13. Theorem.

1) $LB(F_p, F_q) = K(F_p, F_q)$ *if and only if* $(F_p, F_q) \in M$.

2) $LB(G_p, F_q) = K(G_p, F_q)$ *if and only if* $(G_p, F_q) \in M$.

3) $LB(F_p, G_q) = PK(F_p, G_q)$ *if and only if* $(F_p, G_q) \in PM$.

4) $LB(G_p, G_q) = PK(G_p, G_q)$ *if and only if* $(G_p, G_q) \in PM$.

The last application concerns to the existence of copies of c_0.

14. Proposition. *If $p \geq 1$ and c_0 is not a subspace of any Y_n^p (resp. X_n^p), then F_p (resp. G_p) does not contains c_0.*

Proof. From chapter V, Theorem 8 of [7], a Banach space Z does not contains c_0 if and only if $(c_0, Z) \in K$. From the hypothesis and theorem 5, we have $(c_0, F_p) \in K$ (resp. $(c_0, G_p) \in PK$) and the result follows.

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