CONTACT *CR*-PRODUCT OF A TRANS-SASAKIAN MANIFOLD

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Abstract. *We obtain a necessary and sufficient condition for a CR-submanifold of a trans-Sasakian manifold to be contact CR-product in terms of fundamental tensor of Weingarten with respect to the normal section as well as to the canonical structure. We have also obtained some results on CR-submanifolds of α-Sasakian and β-Kenmotsu manifolds.*

INTRODUCTION

In 1978, Bejancu introduced the notion of *CR*-submanifold of a Kaehler manifold [1]. Since then several papers on *CR*-submanifolds of Kaehler manifolds have been published. On the other hand, *CR*-submanifolds of Sasakian manifold have been studied by Kobayashi [14], J.S. Pak [16], Yano & Kon [17] and the present authors [9]. Moreover, *CR*-submanifolds of Kenmotsu manifold have been studied by Bejancu and Papaghuic [3]. One has the notion of α-Sasakian and β-Kenmotsu structure also [12]. In 1985, Oubina introduced a new class of almost contact Riemannian manifold known as trans-Sasakian manifold [15].

A trans-Sasakian manifold is a generalization of both α-Sasakian and β-Kenmotsu manifolds. One of the present authors has studied *CR*-submanifolds of trans-Sasakian manifolds ([10],[11]). In this paper we study contact *CR*-product of trans-Sasakian manifolds.

1. PRELIMINARIES

Let $\bar{M}$ be an m-dimensional almost contact metric manifold with structure tensors (ϕ, ξ, η, g). Then they satisfy [4]

$$\phi^2 = -1 + \eta(x)\xi,\ \eta(\xi) = 1,\ \phi\xi = 0,\ \eta(\xi) = 1 \tag{1.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{1.2}$$

where X, Y are vector fields on $\bar{M}$.

An almost contact metric structure (ϕ, ξ, η, g) on $\bar{M}$ is called trans-Sasakian if [5]

$$(\bar{\nabla}_X\phi)(Y) = \alpha\{g(X, Y)\xi - \eta(Y)X\} + \beta\{g(\phi X, Y)\xi - \eta(Y)\phi X\} \tag{1.3}$$

where α and β are non zero constants, $\bar{\nabla}$ is a Riemannian connection of g and we say that the trans-Sasakian structure is of type (α, β). In particular, a trans-Sasakian manifold is normal. From the above formula, one easily obtains

$$\bar{\nabla}_X\xi = -\alpha\phi X + \beta\{X - \eta(X)\xi\} \tag{1.4}$$

Let M be an n-dimensional isometrically immersed submanifold of $\bar{M}$ and tangent to ξ. Let g be the metric tensor field on $\bar{M}$ as well as the induced metric on M.

Definition. An n-dimensional Riemannian submanifold M of a trans-Sasakian manifold $\bar{M}$ is called a CR-submanifold if ξ is tangent to M and there exists on M a differentiable distribution $D : x \longrightarrow D_x \subset T_xM$ satisfying the following conditions:

(i) D_x is invariant under ϕ i.e. $\phi\, D_x \subset D_x$ for each $x \in M$

(ii) the complementary orthogonal distribution $D^{\perp} : x \longrightarrow D_x^{\perp} \subset T_xM$ is totally real under ϕ i.e. $\phi D_x^{\perp} T_x^{\perp} M$ for each $x \in M$ where T_xM and $T_x^{\perp}M$ are the tangent space and the normal space at $x \in M$, respectively.

If dim. $D_x^{\perp} = 0$ (resp. dim. $D_x = 0$) then the CR-submanifold is called an invariant (resp. totally real) submanifold. The pair $(D, D^{\perp})$ is called ξ-horizontal (ξ-vertical) if $\xi_x \in D_x$ (resp. $\xi_x \in D_x^{\perp}$) for each $x \in M$ [14].

We call a CR-submanifold M proper if it is neither invariant nor anti-invariant.

For a vector field X tangent to M, we put

$$\phi X = PX + FX \tag{1.5}$$

where PX and FX are the tangential and the normal components of ϕX respectively. Then P is an endomorphism of the tangent bundle TM and F is a normal-bundle valued 1-form on TM and they satisfy the condition

$$P^2 = P, F^2 = F \text{ and } PF = FP = 0$$

Also, for a vector field N normal to M, we put

$$\phi N = tN + fN \tag{1.6}$$

where tN (resp. fN) denotes the tangential (resp. normal) component of ϕN. Then f is an endomorphism of $T^{\perp}M$ and t is a tangent-bundle-valued 1-form on $T^{\perp}M$.

The Gauss-Weingarten formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y), \ \bar{\nabla}_X N = -A_N X + \nabla_X^{\perp} N, \tag{1.7}$$

for $X, Y \in T(M)$, $N \in T^{\perp}(M)$ and where ∇ is the Riemannian connection on M, $\bar{\nabla}^{\perp}$ is the connection in the normal bundle, h is the second fundamental form of M, and A_N is the Weingarten endomorphism associated with N satisfying

$$g(A_N X, Y) = g(h(X, Y), N) \tag{1.8}$$

If we denote the orthogonal component of $\phi D^{\perp}$ in $T^{\perp}M$ by μ, then we have

$$T^{\perp}M = \phi D^{\perp} \oplus \mu.$$

It is obvious that $\phi\mu = \mu$.

2. BASIC LEMMAS

In this section we deduce some results on CR-submanifold of α-Sasakian and β-Kenmotsu manifolds. We also prepare certain lemmas without proof for later use.

In [10] we obtained

Lemma *Let M be a CR-submanifold of a trans-Sasakian manifold* $\bar{M}$. *Then we have*

$$P\nabla_X\phi PY - PA_{\phi FY}X = \phi P\nabla_X Y + \alpha g(X,Y)P\xi + \beta g(\phi PX, Y)P\xi$$

$$-\alpha\eta(Y)PX - \beta\eta(Y)\phi PX \tag{2.1}$$

$$Q\nabla_X\phi PY - QA_{\phi FY}X = Bh(X,Y) + \{\beta g(\phi FX, Y) + \alpha g(X,Y)\} F\xi - \alpha\eta(Y)FX \tag{2.2}$$

$$h(X,\phi PY) + \nabla_X^{\perp}\phi FY = \phi F\nabla_X Y + Ch(X,Y) - \beta\eta(Y)\phi FX \tag{2.3}$$

for any $X, Y \in TM$.

From the above Lemma, we have

Lemma 2.1. *Let M be a CR-submanifold of a α-Sasakian manifold* $\bar{M}$. *Then we have*

$$P\nabla_X\phi PY - PA_{\phi FY}X = \phi P\nabla_X Y + \alpha\{g(X,Y)P\xi - \eta(Y)PX\} \tag{2.4}$$

$$F\nabla_X\phi PY - QA_{\phi FY}X = Bh(X,Y) + \alpha\{g(X,Y)F\xi - \eta(Y)FX\} \tag{2.5}$$

$$h(X,\phi PY) + \nabla_X^{\perp}\phi FY = \phi F\nabla_X Y + Ch(X,Y) \tag{2.6}$$

for any $X, Y \in TM$.

Lemma 2.2. *Let M be a CR-submanifold of a β-Kenmotsu manifold* $\bar{M}$. *Then we have*

$$P\nabla_X\phi PY - PA_{\phi FY}X = \phi P\nabla_X Y + \beta\{g(\phi PX, Y)P\xi - \eta(Y)\phi PX\} \tag{2.7}$$

$$F\nabla_X\phi PY - FA_{\phi QY}X = Bh(X,Y) + \beta g(\phi FX, Y)Q\xi \tag{2.8}$$

$$h(X,\phi PY) + \nabla_X^{\perp}\phi FY = \phi F\nabla_X Y + Ch(X,Y) - \beta\eta(Y)\phi FX \tag{2.9}$$

for any $X, Y \in TM$.

Let us define the covariant differentiations of P and F as follows:

$$(\bar{\nabla}_X P)(Y) = \nabla_X PY - P\nabla_X Y, \tag{2.10}$$

$$(\bar{\nabla}_X F)(Y) = \nabla_X^{\perp}(FY) - F\nabla_X Y, \tag{2.11}$$

for any vector fields X and Y tangent to M and any vector field N normal to M.

The endomorphism P (resp. the 1-form F) is parallel if $\bar{\nabla}P = 0$ (resp. $\bar{\nabla}F = 0$).

Now from (1.3) and (1.5)∼(1.8), one can easily prove the following:

Proposition 2.3. *Let M be a CR-submanifold of a trans-Sasakian manifold* $\bar{M}$. *Then we have*

$$(\bar{\nabla}_X P)(Y) = A_{FY}X + th(X,Y) + \alpha\{g(X,Y)\xi - \eta(Y)X\} + \beta\{g(PX,Y)\xi - \eta(Y)PX\}, \tag{2.12}$$

$$(\bar{\nabla}_X F)(Y) = fh(X,Y) - h(X,PY) - \beta\eta(Y)FX \tag{2.13}$$

for all $X, Y \in TM$.

Thus using (2.10) and (2.11), equations (2.12) and (2.13) give

$$P\nabla_X Z = -A_{FZ}X - th(X,Z) + \alpha\{\eta(Z)X - g(X,Z)\xi\} + \beta\{\eta(Z)PX - g(PX,Z)\xi\} \quad (2.14)$$

and

$$F\nabla_X Z = \nabla_X^{\perp} FZ - fh(X,Z) - \beta\eta(Z)FZX \quad (2.15)$$

for any X tangent to M and $Z \in D^{\perp}$.

From proposition 2.3, we have

Lemma 2.4. *Let M be a CR-submanifold of a α-Sasakian manifold $\bar{M}$. Then we have*

$$(\bar{\nabla}_X P)(Y) = A_{FY}X + th(X,Y) + \alpha\{g(X,Y)\xi - \eta(Y)X\} \quad (2.16)$$

$$(\bar{\nabla}_X F)(Y) = fh(X,Y) - h(X,PY) \quad (2.17)$$

for any $X, Y \in TM$.

Lemma 2.5. *Let M be a CR-submanifold of a β-Kenmotsu manifold $\bar{M}$. Then we have*

$$(\bar{\nabla}_X P)(Y) = A_{FY}X + th(X,Y) + \beta\{g(PX,Y)\xi - \eta(Y)PX\} \quad (2.18)$$

$$(\bar{\nabla}_X F)(Y) = fh(X,Y) - h(X,PY) - \beta\eta(Y)FX \quad (2.19)$$

for any $X, Y \in TM$.

Lemma [10] *Let M be a ξ-horizontal CR-submanifold of a trans-Sasakian manifold $\bar{M}$. Then the horizontal distribution D is integrable if and only if*

$$g(h(X,PY) - h(Y,PX), FZ) = 0 \quad (2.20)$$

for any $X, Y \in D$ and $Z \in D^{\perp}$.

Lemma [11] *Let M be a CR-submanifolds of a trans-Sasakian manifold $\bar{M}$. Then the leaf $M^{\perp}$ of $D^{\perp}$ is totally geodesic in M if and only if*

$$g(h(X,W), FZ) + \alpha\eta(Y)g(W,Z) = 0 \quad (2.21)$$

for any $Y \in D$, and $W, Z \in D^{\perp}$.

Thus we obtain

Lemma (2.6) *Let M be a CR-submanifold of a β-Kenmotsu manifold $\bar{M}$. The leaf $M^{\perp}$ of $D^{\perp}$ is totally geodesic in M if and only if*

$$g(h(Y,W), FZ) = 0$$

for $Y \in D, W, Z \in D^{\perp}$.

Corollary (2.7) *Let M be a ξ-vertical CR-submanifold of a trans-Sasakian manifold $\bar{M}$. Then the leaf $M^{\perp}$ of $D^{\perp}$ is totally geodesic in M if and only if*

$$g(h(D, D^{\perp}), \phi D^{\perp}) = 0$$

Lemma [11] *Let M be a CR-submanifold of a trans-Sasakian manifold $\bar{M}$. Then P is parallel if and only if*

$$A_{FX}Y - A_{FY}X = \alpha\{\eta(X)Y - \eta(Y)X\} + \beta\{\eta(Y)PX - \eta(X)PY\} \qquad (2.22)$$

for any X, Y tangent to M.

From this, we have:

Lemma (2.8) *Let M be a CR-submanifold of a α-Sasakian manifold $\bar{M}$. Then P is parallel if and only if*

$$A_{FX}Y - A_{FY}X = \alpha\{\eta(X)Y - \eta(Y)X\}$$

for any X, Y tangent to M.

Lemma (2.9) *Let M be a CR-submanifold of a β-Kenmotsu manifold $\bar{M}$. Then P is parallel if and only if*

$$A_{FX}Y - A_{FY}X = \beta\{\eta(Y)PX - \eta(X)PY\}$$

for any X, Y tangent to M.

4. CONTACT CR-PRODUCT

Definition. A submanifold M of a trans-Sasakian manifold $\bar{M}$ is called a contact CR-product if it is locally a Riemannian product of M^T and $M^{\perp}$, where M^T, $M^{\perp}$ denote the leaves of the distributions D and $D^{\perp}$ respectively.

We now prove the following:

Theorem 3.1. *Let M be a ξ-horizontal CR-submanifold of a trans-Sasakian manifold $\bar{M}$. Then M is a contact CR-product if and only if*

$$A_{\phi W}X + \alpha\eta(X)W = 0 \qquad (3.1)$$

for any $X \in D$ and $W \in D^{\perp}$.

Proof. If a CR-submanifold M of a trans-Sasakian manifold $\bar{M}$ is contact CR-product then from (2.21) we have

$$g(A_{\phi W}X, Z) + \alpha\eta(X)g(W, Z) = 0$$

for $X \in D$, $W, Z \in D^{\perp}$. From this we get

$$A_{\phi W}X + \alpha\eta(X)W \in D \qquad (3.2)$$

for any $X \in D$, $W \in D^{\perp}$.

As D is totally geodesic in M, we have for $Y \in D$

$$\begin{aligned} g(A_{\phi W}X + \alpha\eta(X)W, Y) &= g(h(X,Y), \phi W) = -g(\phi h(X,Y), W) \\ &= -g(\phi\bar{\nabla}_X Y - \phi\nabla_X Y, W) \\ &= -g(\phi\bar{\nabla}_X Y, W) \\ &= g(\nabla_X \phi Y, W) \\ &= 0 \end{aligned}$$

for any $X, Y \in D$ and $W \in D^{\perp}$.

This means

$$A_{\phi W}X + \alpha\eta(X)W \in D^{\perp} \tag{3.3}$$

Thus (3.1) follows form (3.2) & (3.3).

Conversely, equation (3.1) gives

$$g(h(X,Z) + \alpha\eta(X)\phi Z, \phi W) = 0$$

for any $X \in D$, $W, Z \in D^{\perp}$, which means that the leaf $M^{\perp}$ of $D^{\perp}$ is totally geodesic in M.

Next, suppose $M^{\perp}$ be the leaf of D. Then from (1.3) and (3.1), we have

$$\begin{aligned} g(\nabla_X Y, Z) = g(\bar{\nabla}_X Y, Z) &= g(\phi\bar{\nabla}_X Y, \phi Z) = g(\bar{\nabla}_X \phi Y, \phi Z) \\ &= g(h(X, \phi Y), \phi Z) = g(A_{\phi Z}X, \phi Y) \\ &= -\alpha\eta(X)g(\phi Y, Z) = 0 \end{aligned}$$

for any $X, Y \in D$ and $Z \in D^{\perp}$, i.e. the leaf $M^{\perp}$ of D is totally geodesic in M. Thus the submanifold M is a contact CR-product.

Next we have

Theorem 3.2. *A ξ-horizontal CR-submanifold of a trans-Sasakian manifold $\bar{M}$ is a contact CR-product if and only if P is parallel.*

Proof. Let M be a CR-submanifold of a trans-Sasakian manifold $\bar{M}$ and P be parallel. Then from (2.12) we get

$$\begin{aligned} A_{FY}X + th(X,Y) + \alpha\{h(X,Y)\xi - \eta(Y)X\} \\ +\beta\{g(PX,Y)\xi - \eta(Y)PX\} = 0 \end{aligned} \tag{3.4}$$

for any vector fields X, Y tangent to M.

In (3.4), if the vector field Y is in D, then using the fact that $FY = 0$, (3.4) can be written as

$$th(X,Y) + \alpha\{g(X,Y)\xi - \eta(Y)X\} + \beta\{g(PX,Y)\xi - \eta(Y)PX\} = 0.$$

From this, we obtain

$$g(th(X,Y), Z) - \alpha\eta(Y)g(X,Z) - \beta\eta(Y)g(PX,Z) = 0$$

for any vector field X tangent to M, $Y \in D$ and $Z \in D^{\perp}$.

This equation means that

$$g(A_{\phi Z}, X) + \alpha\eta(Y)g(X, Z) = 0$$

which gives

$$A_{\phi Z}Y + \alpha\eta(Y)Z = 0$$

for any $Y \in D, Z \in D^{\perp}$.

Thus by virtue of Theorem 3.1 the above equation tells us that the submanifold M is a contact CR-product.

Conversely, in a contact CR-product of a trans-Sasakian manifold, it is trivial that the endomorphism P is parallel.

Theorem 3.3. *Let M be a ξ-horizontal CR-submanifold of a trans-Sasakian manifold $\bar{M}$. Then the following statements are equivalent:*

(i)M is a contact CR-product

(ii)$A_{\phi D^{\perp}}\phi D + \alpha\eta(0)D^{\perp} = \{0\}$,

(iii)$(\bar{\nabla}_U P)D \subset D^{\perp}, U \in TM$,

(iv)$(\bar{\nabla}_U P)D^{\perp} \subset D^{\perp}, U \in TM$.

Proof. The proof of *(i)* $\Longleftrightarrow$ *(ii)* is given in Theorem 3.1. Here we give an alternative proof of the same. Assume that *(ii)* holds. Then making an inner product of this with a tangent vector U we have for $X \in D, Z \in D^{\perp}$.

$$g(h(X, U), FZ) + \alpha\eta(X)g(U, Z) = 0 \tag{3.5}$$

which shows that the leaf $M^{\perp}$ of $D^{\perp}$ is totally geodesic in M by virtue of lemma 2.6. Similarly, putting $U = \phi Y (Y \in D)$ in the above equation, we get

$$g(h(X, \phi Y), FZ) = 0$$

for $X, Y \in D, Z \in D^{\perp}$.

Thus (2.20) holds and consequently D is integrable. Let M^T be a leaf of D. Then for $X, Y \in D, Z \in D^{\perp}$ and using (1.3), and (2.4) we have

$$\begin{aligned} g(Z, \nabla_Y \phi X) &= -g(\nabla_Y Z, \phi X) = g(\phi \nabla_Y Z, X) \\ &= g(P\nabla_Y Z, X) = g(A_{FZ}Y, X) \\ &= g(h(X, Y), FZ) = 0 \end{aligned}$$

which shows that M^T is totally geodesic in M and hence M is a contact CR-product. Then we have $\nabla_U X \in D$ for $X \in D$ and $\nabla_U X \in D^{\perp}$ where $U \in TM$.

Now from (2.14) we obtain

$$P\nabla_U Z = -A_{FZ}U - th(U, Z) - \alpha g(U, Z)\xi,$$

for any U tangent to M and $Z \in D^{\perp}$.

As $\phi X = PX$ for $X \in D$, using (3.6) we have

$$\begin{aligned} g(\nabla_U Z, X) &= g(\phi\nabla_U Z, \phi X) + \eta(\nabla_U Z)\eta(X) \\ &= g(P\nabla_U Z, PX) + \eta(\nabla_U Z)\eta(X) \\ &= -g(A_{FZ}U, PX) - g(th(U,Z), PX) \\ &- \alpha g(U,Z)g(\xi, PX) + \eta(\nabla_U Z)\eta(X) \\ &= g(\phi A_{FZ}U, X) + \eta(\nabla_U Z)\eta(X) \\ &= g(\phi A_{FZ}U, X) = g(PA_{FZ}U, X) \end{aligned}$$

where we have used the fact that $\nabla_U Z \in D^{\perp}$ for $Z \in D^{\perp}$ and $\xi \in D$.
Thus using the above equation for $X, Y \in D, Z \in D^{\perp}$ and $U \in TM$.

$$\begin{aligned} g(\phi A_{\phi Z}U + \alpha\eta(X)Z, Y) &= g(\phi A_{\phi Z}U, Y) + \alpha\eta(X)g(Y,Z) \\ &= g(\phi A_{\phi Z}U, Y) \\ &= g(PA_{FZ}U, Y) \\ &= g(\nabla_U Z, Y) = 0 \end{aligned}$$

Hence (*ii*) holds.
Next, from (3.6), we have

$$(\bar{\nabla}_U P)(Y) = th(U,Y) + \alpha\,\{g(U,Y)\xi - \eta(Y)U\}$$
$$+\beta\,\{g(PU,Y)\xi - \eta(Y)PU\}$$

for any $U \in TM, Y \in D$. Then, if $Z \in D^{\perp}$ we have

$$\begin{aligned} g((\bar{\nabla}_U P)(Y), Z) &= g(th(U,Y), Z) - \alpha\eta(Y)g(U,Z) \\ &= -g(h(U,Y), FZ) - \alpha\eta(Y)g(U,Z). \end{aligned}$$

Thus (*ii*) holds if and only if (*iii*) holds. The equivalence of (*iii*) and (*iv*) follows directly as $\nabla_U P$ is the skew symmetric operator, which completes the proof of the theorem.

Definition [18]: A submanifold M is called contact totally umbilical if

$$h(U,V) = (g(U,V) - \eta(U)\eta(V))N_0 + \eta(U)FV + \eta(V)FU$$

N_0 being some normal vector field.
Then from corollary (2.7), we have

Proposition 3.4. *If M is totally contact umbilical ξ-vertical CR-submanifold of a trans-Sasakian manifold $\bar{M}$, then the leaf $M^{\perp}$ of $D^{\perp}$ is totally geodesic in M.*
We now prove

Proposition 3.5. *Let M be a CR-submanifold of a trans-Sasakian manifold $\bar{M}$. Then P is parallel if and only if M is an anti-invariant submanifold.*

Proof. Suppose P is parallel. Then taking $Y = \xi$ in (2.2) and using (1.5) we get

$$0 = (\bar{\nabla}_X P)(\xi) = \nabla_X P\xi - P\nabla_X \xi$$

$$= A_{F\xi}X + th(X, \xi) + \alpha\{g(X, \xi)\xi - \eta(\xi)X\} + \beta\{g(PX, \xi)\xi - \eta(\xi)PX\}$$

Applying P on the above equation, we get

$$PA_{F\xi}X + Pth(X, \xi) + \alpha\{g(X, \xi)P\xi - PX\} + \beta\{g(PX, \xi)P\xi - P^2X\} = 0$$

i.e., $Pth(X, \xi) - \alpha PX - \beta P^2X = 0$

Now using $h(X, \xi) = -\alpha\phi FX, P_0F = F_0P = 0$ and $P^2 = P, F^2 = F$ ([10]), we get

$$(\alpha + \beta)PX = 0,$$

which implies that $PX = 0$. Hence M is an anti-invariant submanifold. The converse is trivial.

Finally, we have

Proposition 3.6. *Let M be ξ-horizontal contact CR-product of a trans-Sasakian manifold $\bar{M}$. Then for unit vector $X \in D$ with $\eta(X) = 0$ and $Z \in D^\perp$, we have*

(*a*) $g(h(\nabla_X\phi X, Z), \phi Z) = -\alpha^2$

(*b*) $g(h(\nabla_{\phi X}X, Z), \phi Z) = \alpha^2$

(*c*) $g(h(\phi X, \nabla_X Z), \phi Z) = 0$

(*d*) $g(h(X, \nabla_{\phi X}Z), \phi Z) = 0$

Proof. As M is a ξ-horizontal contact CR-product so the leaf $M^\perp$ of $D^\perp$ is totally geodesic in M. Thus using (3.5), by virtue of (1.4), we have

$$\begin{aligned} g(h(\nabla_X\phi X, Z), \phi Z) &= -\alpha\eta(\nabla_X\phi X)g(Z, Z) \\ &= -\alpha g(\nabla_X\phi X, \xi) \\ &= \alpha g(\nabla_X\xi, \phi X) \\ &= \alpha g(\bar{\nabla}_X\xi, \phi X) \\ &= \alpha g(-\alpha\phi X + \beta X, \phi X) \\ &= -\alpha^2 g(\phi X, \phi X) \\ &= -\alpha^2 \end{aligned}$$

Similarly we can prove $(b), (c)$ and (d).

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