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ON A THEOREM OF EXISTENCE AND UNIQUENESS FOR THE ELASTOSTATICS OF A NON LINEAR DIELECTRIC (°)

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Summary. *In this paper we shall prove a theorem of existence and uniqueness of the solution for the equilibrium equations of a non linear elastic dielectric, provided that for the connected linear problem an existence and uniqueness theorem is valid* (°°°).

To this purpose we shall apply a functional analysis method which has been previously used by Van Buren [1] and Stoppelli [2] for a non linear elastic system. The underneath scheme will be the continuum-elastic one.

1. STATEMENT OF THE PROBLEM.

Let $\mathscr{B}$ be an elastic dielectric and $\mathscr{C}$ an equilibrium configuration for it, under the action of given forces and electric fields.

If $\mathscr{C}_*$ is a reference configuration, the deformation that $\mathscr{B}$ will

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(°°°) Such a theorem is proved in [7].

experience from $\mathscr{C}_*$ to $\mathscr{C}$, is expressed by the displacement:

$$\underset{\sim}{u} = \chi(\underset{\sim}{X}) \in C^2(^\bullet\mathscr{C}_*)$$

where $\underset{\sim}{X} + \underset{\sim}{u} \in \mathscr{C}$, $\underset{\sim}{X} \in \mathscr{C}_*$; that is, if (u^i) are the Eulerian coordinates and X^L the Lagrangian coordinates of any point $\underset{\sim}{X} \in \mathscr{C}_*$ the deformation will be expressed by the scalar functions

$$u^i = \chi^i(X^L) \in C^2 \qquad i,L = 1,2,3.$$

Let $\underset{\sim}{T}_*$ be the Piola-Kirchhooff tensor, $\underset{\sim}{\mathscr{D}}_*$ the Lagrangian induction vector with respect to the reference configuration, (1) ζ the electric potential, ρ_* the reference mass density, $\underset{\sim}{b}$ the specific force, ζ the specific enthalpy, $\varepsilon_L = \phi_{,L}$ the Lagrangian electric field.

The fundamental system of the problem for which we are searching a theorem of existence and uniqueness is:

$$(1.1) \qquad \begin{cases} \operatorname{Div} \underset{\sim}{T}_*(u^i_{,L};\phi_{,L}) + \rho_* \varepsilon \underset{\sim}{b} = 0 \\ \operatorname{Div} \underset{\sim}{\mathscr{D}}_*(u^i_{,L};\phi_{,L}) = 0 \end{cases}$$

with the boundary conditions

$$\begin{cases} \underset{\sim}{u} = 0 & \text{on } \partial\mathscr{C}'_* \\ \underset{\sim}{T}_*\cdot\underset{\sim}{n} = \varepsilon \underset{\sim}{t}_* & \text{on } \partial\mathscr{C}''_* = \partial\mathscr{C}_* - \partial\mathscr{C}'_* \\ \phi = \mu\bar{\phi} & \text{on } \partial\mathscr{C}_* \end{cases}$$

(1) Concerning the quantities which appear in Maxwell's equations we refer the reader to [6].

where $\underset{\sim}{n}_*$ is the unity exterior vector of the normal to the surface $\partial \mathscr{C}''_*$, $\mu\bar{\phi}$ is the electric boundary potential which we assume sectionally constant on $\partial \mathscr{C}_*$ and the real parameters ε and μ are supposed to controll the boundary conditions. Finally we shall suppose that $\mathscr{C}_*$ is a *natural* configuration. Moreover, we shall use the following thermodynamic relations:

$$T_*^{iL} = \rho_* \frac{\partial \zeta}{\partial u^i} = T_*^{iL}(u^i{}_{,L};\phi_{,L})$$

$$\mathscr{D}_*^L = -\rho_* \frac{\partial \zeta}{\partial \phi_{,L}} = \mathscr{D}_*^L(u^i{}_{,L};\phi_{,L})$$

which are estabished in [6].

Let us set

$$\underset{\sim}{u} = (\underset{\sim}{u}, \mathscr{S}').$$

The boundary problem (1.1) (1.2) becomes:

$$(1.3) \qquad \begin{cases} \mathrm{Div}\ T_*(u^\alpha{}_{,L}) + \rho_* \varepsilon \underset{\sim}{b} = \underset{\sim}{0} & \alpha = 1,2,3,4 \\ \mathrm{Div}\ \mathscr{D}_*(u^\alpha{}_{,L}) = 0 & \\ \underset{\sim}{u} = (\underset{\sim}{0}, \mu\bar{\phi}) \text{ on } \partial \mathscr{C}' & \\ \underset{\sim}{T}_* \cdot \underset{\sim}{n}_* = \varepsilon \underset{\sim}{t}_* \text{ on } \partial \mathscr{C}'' & \end{cases}$$

and so we are led to search for the two parameters family $\underset{\sim}{u} = \underset{\sim}{u}(X,\varepsilon,\mu)$ of solutions of (1.3), provided that the corresponding problem for the infinitesimal elasticity has one and only one differentiable solution.

2. FUNCTIONAL SPACES AND HYPOTHESES OF THE THEOREM..

Let $\mathscr{S}$ be a normed vectorial space whose elements are fourdimensional fiedls $\underset{\sim}{u}(\underset{\sim}{X}) = (\underset{\sim}{u}(\underset{\sim}{X}), \phi(\underset{\sim}{X}))$, where $\underset{\sim}{u}(\underset{\sim}{X})$ vanishes on $\partial \mathscr{C}_*^!$ and $\phi(X)$ is sectionally constant on $\partial \mathscr{C}_*$, moreover we assume that $\underset{\sim}{u}$ is regular and having norm $\| u \|$ to be later suitably chosen. Similarly, let $\mathscr{S}'$ be a normed vectorial space, to be later defined, whose elements are the couples $(\underset{\sim}{h},\underset{\sim}{g})$ such that:

$$\underset{\sim}{h} = [\mathrm{Div}\, \underset{\sim}{T}_*(u^\alpha{}_{,L}) + \rho_* \,\varepsilon\, \underset{\sim}{b} \;,\; \mathrm{Div}\mathscr{D}(u^\alpha{}_{,L})]$$

$$\underset{\sim}{g} = [- \underset{\sim}{T}_*(\underset{\sim}{u}^\alpha{}_{,L} \cdot \underset{\sim}{n}_* + \varepsilon \underset{\sim}{t}_* \;,\; -\phi + \mu \bar{\phi}] .$$

The equations system (1.3) may be interpreted as a non linear functional

$$\mathscr{F} : \mathscr{S} \to \mathscr{S}'$$

such that

$$\mathscr{F}(\underset{\sim}{u}) = (\underset{\sim}{h} \;;\; \underset{\sim}{g}) \;. \tag{2.1}$$

The field $\underset{\sim}{u} = \underset{\sim}{0}$ certainly is an element of $\mathscr{S}$ to which the couple $[(\rho_*\varepsilon \underset{\sim}{b},0);\; \varepsilon \underset{\sim}{t},\mu\phi_*)]$ corresponds; moreover $\mathscr{F} \in C^1$ is a neighborhood I_0 of $\underset{\sim}{0}$ and if the Frèchet differential $D_0\mathscr{F} : \mathscr{S} \to \mathscr{S}'$ evaluated at $(\underset{\sim}{0})$ is an isomorphism, by means of the theorem on the inverse mapping, a neighborhood $\mathscr{N}$ of $\underset{\sim}{0}$ will exist such that the correspondence between $\mathscr{N}$ and $\mathscr{F}(\mathscr{N})$ is one to one.

Therefore if one proves that the element $[(\underset{\sim}{0},0);(\underset{\sim}{0},0)] \in \mathscr{F}(\mathscr{N})$, one and only one displacement $\underset{\sim}{u}_*$ will exist such that $\mathscr{F}(\underset{\sim}{u}_*) = [(\underset{\sim}{0},0);(\underset{\sim}{0},0)]$.

Taking into account the given definition for the couple $(\underset{\sim}{h},\underset{\sim}{g})$ the displacemente $\underset{\sim}{u}$ will be the solution of (1.3), and consequently a theorem of local existence and uniqueness of the solution of system (1.3) will remain proved.

After all, in order to prove by means of the inverse mapping theorem, a theorem of existence and uniqueness for the solution of the system (1.3) we need define the spaces $\mathscr{S}$, $\mathscr{S}'$ and to prove that:

1) $\mathscr{F}$ as defined in (2.1) belongs to C^1;

2) $D_0\mathscr{F}$ is an isomorphism;

3) $[(\underset{\sim}{0},0);(\underset{\sim}{0},0)] \in \mathscr{F}(\mathscr{N})$.

We shall assume that the following regularity conditions hold:

i) the dielectric $\mathscr{B}$ is homogeneous and the reference coonfiguration $\mathscr{C}_*$ is a homogeneous natural state of $\mathscr{B}$;

ii) the region $\mathscr{C}_*$ is bounded;

iii) the boundary $\partial\mathscr{C}_*$ is a $C^{2+\lambda}$ differentiable surface, that is, can be covered with an atlas of local coordinates (v^1,v^2,v^3) such that the displacement $X^L = X^L(v^i) \in C^2$ and has second derivatives Hölder-continuous with index λ; moreover $(v^1,v^2,v^3) \in \mathscr{C}_*$ if and only if $v^3 > 0$ otherwise $(v^1,v^2,v^3) \in \partial\mathscr{C}_*$ if and only if $v^3 = 0$ and in this case (v^1,v^2) becomes a local coordinates system of $\partial\mathscr{C}_*$;

iv) the function $[\underset{\sim}{T}_*(u^\alpha{}_{,L}),\ \underset{\sim}{\mathscr{D}}_*(u^\alpha{}_{,L})] \in C^3$ is a 12-dimensional space region of the matrices $(u^\alpha{}_{,L})$ which exhibits the property

$$\| u^\alpha{}_{,L} \| \leq \gamma \quad (\gamma > 0)$$

v) $\underset{\sim}{b} \in C^{0+\lambda}$ in $\mathscr{C}_*$;

vi) the elasticities of $\mathscr{B}$ in $\mathscr{C}_*$ are such that a solution of class $C^{2+\lambda}$ of the problem with boundary conditions of positions in linear elasticity, exists. When all these hypotheses are fulfilled, the Banach space $\mathscr{S}$ is defined as the set of all the vectorial fields $\underset{\sim}{u}(\underset{\sim}{X}) \in C^{2+\lambda}$ in $\mathscr{C}_*$ with respect to the norm:

$$(2.2)\quad \| \underset{\sim}{u} \| = \Sigma_\alpha \max_{\underset{\sim}{X}\in\mathscr{C}_*} |u^\alpha(\underset{\sim}{X})| + \Sigma_{\alpha,L} \max_{\underset{\sim}{X}\in\mathscr{C}_*} |u^\alpha{}_{,L}(\underset{\sim}{X})| +$$

$$+ \Sigma_{\alpha,L,M} \left(\max_{\underset{\sim}{X}\in\mathscr{C}_*} |u^\alpha{}_{,LM}(\underset{\sim}{X})| + B^\alpha_{LM}\right),$$

where B^α_{LM} is the coefficient of Hölder of $u^\alpha{}_{,LM}$ in $\mathscr{C}_* \cup \partial\mathscr{C}_*$.

Analogously $\mathscr{S}'$ is defined as the Banach space of all the couples $(\underset{\sim}{h},\underset{\sim}{g})$, where $\underset{\sim}{h} \in C^{o+\lambda}$ in $\mathscr{C}_*$ and $\underset{\sim}{g} \in C^{2+\lambda}$ on $\partial\mathscr{C}_*$ and vanishing on $\partial\mathscr{C}_*^{!}$, with norm

$$(2.3)\quad \|\underset{\sim}{h},\underset{\sim}{g}\| = \|\underset{\sim}{h}\|_{C^{o+\lambda}(\mathscr{C}_*)} + \|\underset{\sim}{g}\|_{C^{2+\lambda}(\partial\mathscr{C}_*)}$$

where the norms in the right side of (2.3) are defined as in (2.2). We shall call I the open set in $\mathscr{S}$, whose elements are such that $\| u^\alpha{}_{,L} \| < \gamma$ when $\underset{\sim}{X} \in \mathscr{C}_*$.

3. PROOF OF THE THEOREM.

In order to prove that $\mathscr{F}$ is a local diffeomorphism of class C^1 in I, let us first observe that:

$$(3.1)\ \Delta\mathscr{F} = \mathscr{F}(\underset{\sim}{u} + \Delta\underset{\sim}{u}) - \mathscr{F}(\underset{\sim}{u}) = (\Delta\underset{\sim}{h};\Delta g) =$$

$$= \{[\mathrm{Div}\ \underset{\sim}{T}_*(u^\alpha{}_{,L} + \Delta u_{,L}) + \rho_*\varepsilon\underset{\sim}{b},\ \mathrm{Div}\ \mathscr{D}_*(u^\alpha{}_{,L} + \Delta u^\alpha{}_{,L})];$$

$$[(-T_*(u^\alpha_{,L}+\Delta u^\alpha_{,L})\cdot \underset{\sim}{n}_*+\varepsilon \underset{\sim}{t}_*),(-\phi-\Delta\phi+\mu\bar{\phi})]\}-\{[\mathrm{Div}\ \underset{\sim}{T}_*(u^\alpha_{,L})+$$

$$+\rho_*\varepsilon \underset{\sim}{b},\ \mathrm{Div}\ \underset{\sim}{\mathcal{D}}_*(u^\alpha_{,L})];[(-\underset{\sim}{T}_*(u^\alpha_{L})\cdot \underset{\sim}{n}_*+\varepsilon \underset{\sim}{t}_*),\ (-\phi+\mu\bar{\phi})]\}=$$

$$=\{[\mathrm{Div}(\underset{\sim}{T}_*(u^\alpha_{,L}+\Delta u^\alpha_{,L})-\underset{\sim}{T}_*(u^\alpha_{,L})),\ \mathrm{Div}(\underset{\sim}{\mathcal{D}}_*(u^\alpha_{,L}+\Delta u^\alpha_{,L}-$$

$$-\underset{\sim}{\mathcal{D}}_*(u^\alpha_{,L})];[(-\underset{\sim}{T}_*(u^\alpha_{,L}+\Delta u^\alpha_{,L})\cdot \underset{\sim}{n}_*+\underset{\sim}{T}_*(u^\alpha_{,L})\cdot \underset{\sim}{n}_*),(-\Delta\phi)]\}=$$

$$=\{[\mathrm{Div}(\frac{\partial \underset{\sim}{T}_*}{\partial u^\alpha_{,L}}\Delta u^\alpha_{,L}),\ \mathrm{Div}(\frac{\partial \mathcal{D}_*}{\partial u^\alpha_{,L}}\Delta u^\alpha_{,L})];[(-\frac{\partial \underset{\sim}{T}_*}{\partial u^\alpha_{,L}}\Delta u^\alpha_{,L}),(-\Delta\phi)]\}+0(\| u^\alpha_{,L}\|)$$

where $0(\|\Delta u^\alpha_{,L}\|)$ is an higher order infinitesimal as compared with the norm of $\underset{\sim}{u}$.

We observe that:

$$(3.2)\qquad \frac{\partial}{\partial X^M}\left(\frac{\partial T_*^{iM}}{u^\alpha_{,L}}\Delta u^\alpha_{,L}\right)=$$

$$=\frac{\partial}{\partial X^M}\quad\frac{\partial T_*^{iM}}{\partial u^\alpha_{,L}}\Delta u^\alpha_{,L}+\frac{\partial T_*^{iM}}{\partial u^\alpha_{,L}}\Delta u^\alpha_{,LM}=$$

$$=\frac{\partial^2 T_*^{iM}}{\partial u^\alpha_{,L}\ \partial u^\beta_{,N}}\ u^\beta_{NM}\ \Delta u^\alpha_{,L}+\frac{\partial T_*^{iM}}{\partial u^\alpha_{,L}}\Delta u^\alpha_{,LM}=$$

$$=\left(\frac{\partial T_*^{iM}}{\partial u^\alpha_{,L}}\ \frac{\partial^2}{\partial X^L\ \partial X^M}+\frac{\partial^2 T_*}{\partial u^\alpha_{,L}\ \partial u^\beta_{,N}}\ u^\beta_{NM}\ \frac{\partial}{\partial X^L}\right)\Delta u^\alpha.$$

If we define:

$$(3.2)_1 \quad A_\alpha^{iML} \equiv \frac{\partial T_*^{iM}}{\partial u^\alpha_{,L}} = (L_j^{iML}, P^{iML})$$

where

$$(3.2)_2 \quad L_j^{iML} = \frac{\partial T_*^{iM}}{\partial u^j_{,L}}$$

is the elasticity tensor and

$$(3.2)_3 \quad P^{iML} = \frac{\partial T_*^{iM}}{\partial \phi_{,L}}$$

is the piezoelectric tensor; if we set

$$B_{\alpha\beta}^{iL\ MN} = \frac{\partial^2 T_*^{iM}}{\partial u^\alpha_{,L}\, \partial u^\beta_{,N}} ,$$

(3.2) becomes

$$(3.3) \quad (A_\alpha^{iLM} \frac{\partial^2}{\partial X^L\, \partial X^M} + B_{\alpha\beta}^{iLMN}\, u^\beta_{NM} \frac{\partial}{\partial X^L})\, \Delta u^\alpha .$$

Moreover we observe that:

$$(3.4) \quad \frac{\partial}{\partial X^M}\left(\frac{\partial \mathscr{D}_*^M}{\partial u^\alpha_{,L}} \Delta u^\alpha_{,L}\right) = \left(\frac{\partial}{\partial X^M} \frac{\partial \mathscr{D}_*^M}{\partial u_{,L}}\right) \Delta u^\alpha_{,L} +$$

$$+ \frac{\partial \mathscr{D}^M_*}{\partial u^\alpha_{,L}} \Delta u^\alpha_{,LM} = \frac{\partial^2 \mathscr{D}^M_*}{\partial u^\alpha_L \, \partial u^\beta_{,N}} u^\beta_{,NM} \, \Delta u^\alpha_{,L} +$$

$$+ \frac{\partial \mathscr{D}^M_*}{\partial u^\alpha_{,L}} \Delta u^\alpha_{,LM} = \left(\frac{\partial \mathscr{D}^M_*}{\partial u^\alpha_{,L}} \frac{\partial^2}{\partial X^L \, \partial X^M} + \frac{\partial^2 \mathscr{D}^M_*}{\partial u^\alpha_{,L} \, \partial u^\beta_{,M}} u^\beta_{,NM} \frac{\partial}{\partial X^M} \right) \Delta u^\alpha_{,L}$$

If we set:

$$(3.5)_1 \quad \Phi^{ML}_\alpha \equiv \frac{\partial \mathscr{D}^M_*}{\partial u^\alpha} = (\Omega^{ML}_j, \ \varepsilon^{ML})$$

where

$$(3.5)_2 \quad \Omega^{ML}_j \equiv \frac{\partial \mathscr{D}^M_*}{\partial u^j_{,L}}$$

$$(3.5)_3 \quad \varepsilon^{ML} \equiv \frac{\partial \mathscr{D}^M_*}{\partial \phi_{,L}}$$

is the piezoelectric tensor, and we use the notation:

$$(3.5)_4 \quad \Sigma^{MLN}_{\alpha\beta} \quad \frac{\partial^2 \mathscr{D}^M_*}{\partial u^\alpha_{,L} \, \partial u^\beta_{,N}}$$

equation (3.4) becomes:

$$(3.6) \quad \left(\Phi^{ML}_\alpha \frac{\partial^2}{\partial X^L \, \partial X^M} + \Sigma^{MLN}_{\alpha\beta} \, u^\beta_{,NM} \frac{\partial}{\partial X^L} \right) \Delta u^\alpha .$$

Taking into account (3.4) and (3.6), (3.1) becomes:

$$\Delta\mathscr{F} = \{[(A_\alpha^{iML}\frac{\partial^2}{\partial X^L\,\partial X^M} + B_{\alpha\beta}^{iLMN}\cdot u^\beta_{,NM}\frac{\partial}{\partial X^L})\Delta u^\alpha,$$

$$(\phi_\alpha^{ML}\frac{\partial^2}{\partial X^L\,\partial X^M} + \Sigma_{\alpha\beta}^{MLN}u^\beta_{,NM}\frac{\partial}{\partial X^L})\Delta u]; [(-A_\alpha^{iML}\Delta u^\alpha_{,L}n_{*L}),(-\Delta\phi)]\}+0(\|\Delta u_{,L}\|)$$

We observe that the operator $\mathscr{F}$ is Fréchet derivable for every $\underset{\sim}{u} \in \mathscr{S}$, in fact the operator

$$D_{\underset{\sim}{u}}\mathscr{F} = [(A_\alpha^{iML}\frac{\partial^2}{\partial X^L\,\partial X^M} + B_{\alpha\beta}^{iLMN}u^\beta_{,NM}\frac{\partial}{\partial X^L}),(\phi_\alpha^{NL}\frac{\partial^2}{\partial X^L\,\partial X^M} +$$

$$+\Sigma_{\alpha\beta}^{MLN}u^{\beta}_{,NM}\frac{\partial}{\partial X^L})] ; [(-A_\alpha^{iML}n_{*L}\frac{\partial}{\partial X^L}),(-\delta_{\alpha 4})]$$

is bounded and it concides with the Fréchet derivative of $\mathscr{F}$ at the point $\underset{\sim}{u}$; if we calculate $D_{\underset{\sim}{u}}\mathscr{F}$ in $\underset{\sim}{u} = \underset{\sim}{0}$, we shall obtain a system where only the second order derivatives of $\Delta\underset{\sim}{u}$ will appear, since

$$\frac{\partial^2 u}{\partial X^L\,\partial X^M} = 0 .$$

Therefore if we recall $(3.2)_2,(3.2)_3,(3.3)_2,(3.3)_3$ and denote by Grad $\underset{\sim}{u}$ the gradient of $\underset{\sim}{u}$ with respect to X^L, the application $D_0\mathscr{F}\Delta\underset{\sim}{u} = (\Delta\underset{\sim}{h};\ \Delta\underset{\sim}{g}) \equiv [(\Delta\underset{\sim}{h}_1,\ h_2);(\Delta\underset{\sim}{g}_1,\Delta g_2)]$ where Δg_2 is sectionally constant, can be written as follows:

$$
(3.7)\begin{cases}
\mathrm{Div}[\underset{\sim}{L} \cdot \mathrm{Grad}\Delta \underset{\sim}{u} + \underset{\sim}{P} \cdot \mathrm{Grad}\,\Delta\,\phi] = \Delta \underset{\sim}{h}_1 & \\
\mathrm{Div}[\underset{\sim}{\Omega} \cdot \mathrm{Grad}\Delta \underset{\sim}{u} + \varepsilon\cdot \mathrm{Grad}\,\Delta\,\phi\,] = \Delta h_2 & \text{in } \mathscr{C}_* \\
\Delta \underset{\sim}{u} = 0 \quad \text{on } \partial \mathscr{C}'_* \quad , -\Delta\phi = \Delta g_2 & \text{on } \partial \mathscr{C}_* \\
-[L\cdot \mathrm{Grad}\ \Delta \underset{\sim}{u} + \underset{\sim}{P} \cdot \mathrm{Grad}\ \Delta\phi] \cdot \underset{\sim}{n}_{*L} = \Delta \underset{\sim}{g}_1 & \text{on } \partial \mathscr{C}''_*
\end{cases}
$$

which expresses the equilibrium mixed boundary value problem for a linear elastic dielectric subject to a specific force $-\Delta\underset{\sim}{h}_1$, surface traction $-\Delta\underset{\sim}{g}_1$, charge density Δh_2 and with potential $-\Delta g_2$ sectionally constant on $\partial\mathscr{C}_*$. Since there is a theorem (3) on the existence and uniqueness of the solution for (3.7), then $D_0\mathscr{F}$ is an isomorphism.

By means of the inverse mapping theorem, $\mathscr{F}'$ is a local diffeomorphism of class C^1 in the neighborhood of $\underset{\sim}{0}$.

This means that $\mathscr{F}$ generates a one to one mapping between a neighborhood $\mathscr{N}(\underset{\sim}{0})$ and the neighborhood $\mathscr{F}(\mathscr{N})$ of

$$\mathscr{F}(\underset{\sim}{0}) = [(\rho_*,\varepsilon\,\underset{\sim}{b},0);(\varepsilon\,\underset{\sim}{t}_+,-\mu\bar{\phi})] \ .$$

Therefore if $|\varepsilon|$ is sufficiently small, $[(\underset{\sim}{0},0);(\underset{\sim}{0},0)]\in\mathscr{F}(\mathscr{N})$ and correspondly only one poit $\underset{\sim}{u}_*\in\mathscr{N}$ exists such that $\mathscr{F}(\underset{\sim}{u}_*) =$ $= [(\underset{\sim}{0},0);(\underset{\sim}{0},0)]$. We may then state the following theorem for system (1.3) with mixed boundary conditions for an elastic dielectric.

THEOREM. *If conditions i,ii,...,iv) hold, then it is possible to find two positive numbers ξ, ζ such that for every $\varepsilon > 0$ for which $\varepsilon < \xi$ system (1.3) exhibits one and only one solution*

(3) Cfr. [7]

$u_* \in \mathcal{U}$ satisfing condition $\| u_* \| < \zeta$.

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