ASPECTS OF THE UNIFORM $\lambda$-PROPERTY (*)
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Dedicated to the memory of Professor Gottfried Köthe

ABSTRACT. If $Z$ is a uniformly convex normed space, the quotient space $\ell_\infty(Z)/c_0(Z)$, which is not strictly convexifiable, is shown to have the uniform $\lambda$-property and its $\lambda$-function is calculated. An example is given of a Banach space $X$ with a closed linear subspace $Y$ such that $Y$ and $X/Y$ are strictly convex, yet $X$ fails to have the $\lambda$-property. Convex sequences which generate $B_{c_0}$ are characterized.

Every point in the closed unit ball of a strictly convex space is a convex combination of two extreme points. Thus, every strictly convex space has the uniform $\lambda$-property, a geometric property of normed spaces introduced in [1] and which represents one direction that can be followed in generalizing the notion of strict convexity. Because the strictly convex spaces represent such a fundamental class of normed spaces with the uniform $\lambda$-property, it is natural to ask whether these classes contain the same spaces, up to isomorphism. That is, does every normed space with the uniform $\lambda$-property possess an equivalent strictly convex norm? In this note, we show that the answer is no by proving that $\ell_\infty(Z)/c_0(Z)$ has the uniform $\lambda$-property whenever $Z$ is a uniformly convex space. Since $\ell_\infty(Z)/c_0(Z)$ is not strictly convexifiable, we obtain a negative answer to the preceding question. In particular, $\ell_\infty/c_0$ is an example of a much-studied classical Banach space which has the uniform $\lambda$-property but is not strictly convexifiable. We also show that the uniform $\lambda$-property is very far from being a three-space property. Namely, we give an example of a Banach space $X$ with a closed linear subspace $Y$ such that $Y$ and $X/Y$ are strictly convex, yet $X$ fails to have the $\lambda$-property. In the last section, we examine certain convex sequences, the so-called $B_X$-generating sequences, which naturally appear in the context of any discussion of a Banach space $X$ with the uniform $\lambda$-property. The $B_{\ell_\infty(R)}$-generating sequences are characterized.

0. PRELIMINARIES

Given a normed space $X$, $B_X$ denotes its closed unit ball and $S_X$ its closed unit sphere. If $x \in B_X$, a triple $(e, y, \lambda)$ is amenable to $x$, if $e \in \text{ext}(B_X), y \in B_X, 0 < \lambda \leq 1$ and

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\[ x = \lambda e + (1 - \lambda)y. \] In this case, we define

\[ \lambda(x) = \sup \{ \lambda : (e, y, \lambda) \text{ is amenable to } x \}. \]

\( X \) has the \( \lambda \)-property if each \( x \in B_X \) admits an amenable triple. If, in addition \( \lambda(X) \equiv \sup \{ \lambda(x) : x \in B_X \} > 0 \), then \( X \) is said to have the uniform \( \lambda \)-property. General facts concerning these properties appear in [1], [4] and [7]. Discussion of these properties for classical sequence and function spaces can be found in [1], [5], [8]-[10].

If \( Z \) is a normed space, \( \ell_\infty(Z) \) denotes the normed space of all bounded \( Z \)-valued sequences \( z = (z_n) \), where \( ||z|| = \sup_n ||z_n|| \). The closed linear subspace of \( \ell_\infty(Z) \) consisting of all zero \( z \) for which \( z_n \to 0 \) is denoted by \( c_0(Z) \). It is a well-known result of J. Bourgain [3] that \( \ell_\infty(Z)/c_0(Z) \) is not strictly convexifiable (i.e., does not admit an equivalent strictly convex norm) in the cases \( Z = \mathbb{R} \) or \( \mathbb{C} \). In these cases, the latter quotient space is denoted simply by \( \ell_\infty/c_0 \). For an arbitrary normed space \( Z \), fix \( z \in Z \). Then the mapping \( T : \ell_\infty/c_0 \to \ell_\infty(Z)/c_0(Z) \), defined by \( T((a_n) + c_0) = (a_nz) + c_0(Z) \), is easily seen to be a linear isometry of \( \ell_\infty/c_0 \) into \( \ell_\infty(Z)/c_0(Z) \). By Bourgain's result, it follows that \( \ell_\infty(Z)/c_0(Z) \) is not strictly convexifiable.

If \( Z \) is a uniformly convex normed space and \( (u_n), (v_n) \) are sequences in \( B_Z \) such that \( ||u_n + v_n|| \to 2 \), then \( ||u_n - v_n|| \to 0 \). If \( A \) is a subset of a normed space, \( \overline{\partial}(A) \) denotes the closed convex hull of \( A \).

1. \( \ell_\infty(Z)/c_0(Z) \) AND THE UNIFORM \( \lambda \)-PROPERTY

We write \( X = \ell_\infty(Z)/c_0(Z) \) and let \( Q : \ell_\infty(Z) \to X \) denote the canonical quotient mapping, defined by \( Q(z) = z + c_0(Z) \) for all \( z \in \ell_\infty(Z) \).

**Lemma 1.** \( Q(B_{\ell_\infty(Z)}) = B_X \) for any normed space \( Z \).

**Proof.** It suffices to show that \( B_X \subset Q(B_{\ell_\infty(Z)}) \). Let \( \hat{z} \in B_X \) and choose \( z = (z_n) \in \ell_\infty(Z) \) such that \( \hat{z} = Q(z) \). Since \( ||Q(z)|| \leq 1 \), the set \( N_k = \{ n : ||z_n|| \geq 1 + k^{-1} \} \) is finite for every \( k \in \mathbb{N} \). Define \( x = (x_n) \in B_{\ell_\infty(Z)} \) as follows:

\[ x_n = \begin{cases} z_n & \text{if } ||z_n|| \leq 1 \\ z_n/||z_n|| & \text{if } 1 < ||z_n|| \end{cases} \]

Observe that if \( n > \max N_k \) and \( 1 < ||z_n|| \), then \( 1 < ||z_n|| < 1 + k^{-1} \). It follows that \( ||x_n - z_n|| < k^{-1} \) if \( n > \max N_k \). Consequently, we have \( x - z \in c_0(Z) \) so that \( \hat{z} = Q(z) = Q(x) \). Since \( x \in B_{\ell_\infty(Z)} \), the proof is complete.
Lemma 2. (a) If \( Z \) is a strictly convex normed space, \( \operatorname{ext}(B_X) \subset Q(\operatorname{ext}(B_{\ell_1}(Z))) \). (b) If \( Z \) is a uniformly convex normed space, \( \operatorname{ext}(B_X) = Q(\operatorname{ext}(B_{\ell_1}(Z))) \).

Proof. (a) If \( \hat{e} \in \operatorname{ext}(B_X) \), then by Lemma 1, we can write \( \hat{e} = Q(x) \), where \( x = (x_n) \in B_{\ell_1}(Z) \). Then \( \|x_n\| \leq 1 \) for all \( n \) and we claim \( \|x_n\| \to 1 \). If not, there exists \( \varepsilon > 0 \) such that \( \|x_n\| \leq 1 - \varepsilon \) for an infinite subset \( \mathcal{N}_\varepsilon \) of \( \mathcal{N} \). Thus, for each \( n \in \mathcal{N}_\varepsilon \), there exist \( u_n, v_n \in B_Z \) such that \( 1 \geq \|u_n - x_n\|, \|v_n - x_n\| \geq \varepsilon \) and \( x_n = \frac{1}{2}(u_n + v_n) \). This implies that there exist \( u, v \in B_{\ell_1}(Z) \) such that \( u - x, v - x \not\in C_0(Z) \) and \( x = \frac{1}{2}(u + v) \). Hence, \( \hat{e} = \frac{1}{2}(Q(u) + Q(v)) \), where \( Q(u) \neq \hat{e} \neq Q(v) \). This contradiction establishes the claim. Since \( \|x_n\| \to 1 \), there is no loss of generality in assuming \( x_n \neq 0 \) for all \( n \). Then \( e = \frac{x_n}{\|x_n\|} \in \operatorname{ext}(B_{\ell_1}(Z)) \) and \( e - e \in c_0(Z) \), which implies \( Q(x) = Q(e) \) and completes the proof.

(b) Let \( e = (e_n) \in \operatorname{ext}(B_{\ell_1}(Z)) \). Then \( \|e_n\| = 1 \) for all \( n \) and \( \|Q(e_n)\| = 1 \). Suppose \( Q(e) = \frac{1}{2}((u + v) \), where \( u, v \in B_X \). Then \( \|u\| = \|v\| = 1 \) and, by Lemma 1, we can find \( u = (u_n), v = (v_n) \in B_{\ell_1}(Z) \) such that \( u = Q(u), v = Q(v) \). Since \( Q(e) = Q\left(\frac{u + v}{2}\right) \), it follows that \( e - \frac{1}{2}(u + v) \in c_0(Z) \); that is, \( \|e_n - \frac{1}{2}(u_n + v_n)\| \to 0 \). Uniform convexity of \( Z \) and the facts that \( \|u_n\|, \|v_n\| \leq 1, \|u_n + v_n\| \to 2 \) forces \( \|u_n - v_n\| \to 0 \). This implies \( u - v \in c_0(Z) \) which, in turn, implies \( u = Q(u) = Q(v) = v \). Therefore, \( Q(e) = u = v \), implying \( Q(e) \in \operatorname{ext}(B_X) \).

Theorem 3. Let \( Z \) be a uniformly convex normed space. The quotient space \( X = \ell_1(Z) / c_0(Z) \) has the uniform \( \lambda \)-property but is not strictly convexifiable. If \( \hat{z} \in B_X \), then

\[ (*) \quad \lambda(\hat{z}) = \sup \left\{ \frac{1}{2}(1 + \inf_n \|z_n\|) : z = (z_n) \in B_{\ell_1}(Z) \quad \text{and} \quad Q(z) = \hat{z} \right\}. \]

Proof. Let \( \hat{z} \in B_X \) and suppose \( \hat{z} = Q(z) \), where \( z = (z_n) \in B_{\ell_1}(Z) \). By Theorem 1.13 of [1], there exist \( e \in \operatorname{ext}(B_{\ell_1}(Z)), y \in B_{\ell_1}(Z) \) such that \( z = \lambda e + (1 - \lambda)y \), where \( \lambda = \frac{1}{2}(1 + \inf_n \|z_n\|) \). By Lemmas 1 and 2, \( (Q(e), Q(y), \lambda) \) is amenable to \( \hat{z} = Q(z) \). This proves \( X \) has the uniform \( \lambda \)-property and that \( \lambda(\hat{z}) \) is at least as large as the supremum indicated in (*).
On the other hand, given \( \varepsilon > 0 \), there exists a triple \((\hat{e}, \hat{y}, \lambda)\) amenable to \(\hat{z}\) such that 
\[
\lambda(\hat{z}) - \varepsilon < \lambda. 
\]
By Lemmas 1 and 2, we may assume \(\hat{e} = Q(e), \hat{y} = Q(y)\), where \(e \in \mathcal{E}(B_{\ell_\infty}, Y, B_{\ell_\infty}(Y))\). Let \(z = (z_n) = \lambda e + (1 - \lambda) y\) and observe that \(Q(z) = \hat{z}\).

By theorem 1.13 of [1], \(\lambda \leq \lambda(z) = \frac{1}{2} \left(1 + \inf_n \|z_n\|\right)\). Thus,
\[
\lambda(\hat{z}) \leq \frac{1}{2} \left(1 + \inf_n \|z_n\|\right) + \varepsilon,
\]
showing that the supremum indicated in (*) is at least as large as \(\lambda(\hat{z})\).

2. THREE-SPACE CONSIDERATIONS

The uniform \(\lambda\)-property is not a three-space property. For example, \(\ell_\infty\) has the uniform \(\lambda\)-property but contains a subspace \(Y(= c_0)\) without the \(\lambda\)-property. On the other hand, a classical example of V. Klee [5] shows that \(\ell_1\) can be given an equivalent strictly convex norm \(\|\cdot\|\) such that every separable Banach space is isometrically isomorphic to a quotient of \((\ell_1, \|\cdot\|)\). Thus, a quotient of a strictly convex space may fail to have the \(\lambda\)-property.

Our goal here is to show the existence of a Banach space \(X\) with a closed linear subspace \(Y\) such that \(Y\) and \(X/Y\) are both strictly convex, yet \(X\) fails to have the \(\lambda\)-property.

To this end, let \(Y\) be a real Banach space having two equivalent norms, \(\|\cdot\|_1\) and \(\|\cdot\|_2\), such that \((Y, \|\cdot\|_1)\) is strictly convex and the closed unit ball of \((Y, \|\cdot\|_2)\) fails to have an extreme point. For example \(Y = c_0\) is such a space. Let \(B_i\) denote the closed unit ball of \((Y, \|\cdot\|_i)\), \(i = 1, 2\). We may assume that \(2B_2 \subset B_1\). In \(X = Y \times \mathbb{R}\), let
\[
B = \overline{co}((B_1 \times \{0\}) \cup (B_2 \times \{-1, 1\})).
\]
If \(\|\cdot\|\) denotes the gauge functional of \(B\) in \(X\), then \(\|\cdot\|\) is a norm on \(X\), \((X, \|\cdot\|)\) is a Banach space and \(B\) is closed unit ball of \((X, \|\cdot\|)\). Routine calculations show that \((Y, \|\cdot\|_1)\) is isometrically isomorphic to the subspace \(Y \times \{0\}\) of \((X, \|\cdot\|)\), so that \(Y \times \{0\}\) is strictly convex. Obviously, the one-dimensional space \(X/(Y \times \{0\})\) is strictly convex.

Assume that \((X, \|\cdot\|)\) has the \(\lambda\)-property and define \(f \in X^*\) by \(f(x, t) = t\). Then \(f(0, 1) = \|f\| = 1\). Since \((X, \|\cdot\|)\) has the \(\lambda\)-property, \(f\) must attain its maximum on \(B\) at a member \((x, t)\) of \(\text{ext}(B)\) (see Theorem 3.3 of [1]). This forces \(t = 1\) and hence, by the definition of \(B\), we obtain \(x \in B_2\). Since \(B_2\) does not contain an extreme point, it follows that \((x, 1) \notin \text{ext}(B)\), a contradiction.

Remark 4. The preceding example also shows that a Banach space without the \(\lambda\)-property can contain a closed, one-codimensional subspace that is strictly convex.
3. $B_X$-GENERATING SEQUENCES

A sequence $(\lambda_k)$ of positive real numbers will be called a convex sequence in case $\sum_{k=1}^{\infty} \lambda_k = 1$. The following result has recently been shown in [2]:

**Theorem 5.** A Banach space $X$ has the $\lambda$-property if and only if $B_X$ has the convex series representation property; that is, for each $x \in B_X$, there exists a convex sequence $(\lambda_k)$ and a sequence $(e_k) \subset \text{ext } (B_X)$ such that $x = \sum_{k=1}^{\infty} \lambda_k e_k$.

On the other hand, it is well-known from [1] that

**Theorem 6.** A Banach space $X$ has the uniform $\lambda$-property if and only if there exists a convex sequence $(\lambda_k)$ such that for each $B_X$, there exists a sequence $(e_k) \subset \text{ext } (B_X)$ satisfying $x = \sum_{k=1}^{\infty} \lambda_k e_k$.

A sequence $(\lambda_k)$ satisfying the condition of Theorem 6 will be called a $B_X$-generating sequence. We see that Theorems 5 and 6 mark a clear distinction between Banach spaces with the $\lambda$-property and Banach spaces with the uniform $\lambda$-property. In the case of the $\lambda$-property, the sequence $(\lambda_k)$ of Theorem 5 depends on $x \in B_X$. In the case of the uniform $\lambda$-property, the sequence $(\lambda_k)$ is fixed and changing the extreme points $e_k$ in the sums $\sum_{k=1}^{\infty} \lambda_k e_k$ is sufficient to produce all the members of $B_X$. Hence, a Banach space $X$ has the uniform $\lambda$-property if and only if there is a $B_X$-generating sequence. For such a space $X$, it would be of interest to determine all the $B_X$-generating sequences. If this were possible, then given Banach spaces $X, Y$ with the uniform $\lambda$-property, one might be able to distinguish certain geometric or quantitative differences between $B_X$ and $B_Y$ in terms of differences between the collections of $B_X$-generating and $B_Y$-generating sequences.

The problem of characterizing the $B_X$-generating sequences for a Banach space $X$ with the uniform $\lambda$-property has only recently been considered. In this section, however, we settle this question for $X = \mathbb{R}$ and $X = \ell_\infty(\mathbb{R})$. Recall from [8] that a Banach space $X$ has the uniform $\lambda$-property if and only if $\ell_\infty(X)$ has the uniform $\lambda$-property.

**Lemma 7.** Let $X$ be a Banach space with the uniform $\lambda$-property. Then the families of $B_X$-generating sequences and $B_{\ell_\infty(X)}$-generating sequences are equal.

**Proof.** Let $(\lambda_k)$ be a $B_X$-generating sequence and let $x = (x_n) \in B_{\ell_\infty(X)}$. For each $n$,
there exist a sequence \((e_{k}^{(n)})_{k=1}^{\infty} \subset \text{ext} \ (B_X)\) such that
\[
x_n = \sum_{k=1}^{\infty} \lambda_k e_k^{(n)}, \quad n = 1, 2, \ldots
\]

If we define \(e_k = (e_k^{(n)})_{n=1}^{\infty}\), then \(e_k \in \text{ext} \ (B_{\ell_\infty}(X))\) and for each \(m\), we have
\[
\left\| x - \sum_{k=1}^{m} \lambda_k e_k \right\| = \sup_n \left\| x_n - \sum_{k=1}^{m} \lambda_k e_k^{(n)} \right\| \leq \sum_{k=m+1}^{\infty} \lambda_k.
\]

Consequently, \(x = \sum_{k=1}^{\infty} \lambda_k e_k\), proving that \((\lambda_k)\) is a \(B_{\ell_\infty}(X)\)-generating sequence. The proof of the converse is equally simple.

**Theorem 8.** Let \((\lambda_k)\) be a convex sequence. The following are equivalent:

(a) \((\lambda_k)\) is \(B_{\ell_\infty}(R)\)-generating.

(b) \((\lambda_k)\) is \(B_R\)-generating.

(c) For each \(t \in [0, 1]\), there is a sequence \((a_k)\) of 0's and 1's such that \(t = \sum_{k=1}^{\infty} \lambda_k a_k\).

(d) If \((\hat{\lambda}_k)\) is the nonincreasing rearrangement of \((\lambda_k)\), then for all \(n\) we have \(\hat{\lambda}_n \leq \sum_{k=n+1}^{\infty} \hat{\lambda}_k\).

**Proof.** That (a) \(\Leftrightarrow\) (b) follows from Lemma 7.

(b) \(\Rightarrow\) (c). Let \(t \in [0, 1]\). Since \(\text{ext}(B_R) = \{-1, 1\}\), there is a sequence \((b_k)\) of ± 1's such that \(2t - 1 = \sum_{k=1}^{\infty} \lambda_k b_k\). Then \(t = \sum_{k=1}^{\infty} \lambda_k (b_k + 1)/2\), and, for each \(k\), \((b_k + 1)/2\) equals 0 or 1.

(c) \(\Rightarrow\) (b). If \(-1 \leq x \leq 1\), write \(x = 2t - 1\), where \(0 \leq t \leq 1\). By hypothesis, there is a sequence \((a_k)\) of 0's and 1's such that \(t = \sum_{k=1}^{\infty} \lambda_k a_k\). It follows that \(x = \sum_{k=1}^{\infty} \lambda_k (2a_k - 1)\) and, for each \(k\), \(2a_k - 1\) is either \(-1\) or 1. Consequently \((\lambda_k)\) is a \(B_R\)-generating sequence.

(c) \(\Rightarrow\) (d). Since \((\hat{\lambda}_k)\) also satisfies statement (c), there is no loss of generality in assuming \((\lambda_k)\) is nonincreasing. Assume, to the contrary, that \(\lambda_n > \sum_{k=n+1}^{\infty} \lambda_k\) for some \(n\). Given a sequence \((a_k)\) of 0's and 1's, write \(t = \sum_{k=1}^{\infty} \lambda_k a_k\). If \(a_k = 0\) for \(1 \leq k \leq n\), then \(t \leq \sum_{k=n+1}^{\infty} \lambda_k a_k = \sum_{k=n+1}^{\infty} \lambda_k (2a_k - 1)\).
\[ \leq \sum_{k=n+1}^{\infty} \lambda_k. \] On the other hand, if \( a_{k_0} = 1 \) for some \( k_0 \) between 1 and \( n \), then \( t \geq \lambda_{k_0} \geq \lambda_n \).

This shows that \( t \notin \left( \sum_{k=n+1}^{\infty} \lambda_k, \lambda_n \right) \) and contradicts the fact that \((\lambda_k)\) satisfies (c).

(d) \( \Rightarrow \) (c). Without loss of generality, \((\lambda_k)\) is nonincreasing. Define \( \lambda_0 = 1 \) and let \( 0 < t < 1 \). Let \( n_i \) be the smallest nonnegative integer such that \( \lambda_{n_i+1} \leq t \). If \( \sum_{k=n_i+1}^{m} \lambda_k \leq t \) for all \( m \geq n_i + 1 \), then we obtain \( \lambda_{n_i} \leq \sum_{k=n_i+1}^{\infty} \lambda_k \leq t \), which is impossible if \( n_i = 0 \) and contradicts the definition of \( n_i \) if \( n_i \geq 1 \). Consequently, there is an integer \( m_1 \geq n_i + 1 \) such that

\[ \sum_{k=n_i+1}^{m_1} \lambda_k \leq t < \sum_{k=n_i+1}^{m_1+1} \lambda_k \] (1)

If \( t = \sum_{k=n_i+1}^{m_1} \lambda_k \), then \( t = \sum_{k=1}^{\infty} \lambda_k a_k \) for the obvious sequence \((a_k)\) of 0's and 1's. Thus, we may assume strict inequality holds in (1).

Assume that \( 0 \leq n_i < m_1 < \ldots < n_j < m_j \) have been chosen so that

\[ \sum_{i=1}^{j} \left( \sum_{k=n_i+1}^{m_i} \lambda_k \right) < t < \sum_{i=1}^{j-1} \left( \sum_{k=n_i+1}^{m_i} \lambda_k \right) + \sum_{k=n_j+1}^{m_{j+1}} \lambda_k \] (2)

If \( s_j \) denotes the left-hand side of (2), then (2) becomes

\[ s_j < t < s_j + \lambda_{m_{j+1}}, \]

which implies \( 0 < t - s_j < \lambda_{m_{j+1}} \). There is a largest integer \( n_{j+1} \geq m_j + 1 \) such that

\[ t < s_j + \lambda_{n_j}, \] (3)

This implies

\[ s_j + \lambda_{n_{j+1}} \leq t \] (4)
If \( s_j + \sum_{k=n_{j+1}+1}^{m} \lambda_k \leq t \) for all \( m \geq n_{j+1} + 1 \), we obtain

\[
s_j + \lambda_{n_{j+1}} \leq s_j + \sum_{k=n_{j+1}+1}^{\infty} \lambda_k \leq t,
\]

which contradicts (3). Therefore, there exists \( m \geq n_{j+1} + 1 \) such that \( t < s_j + \sum_{k=n_{j+1}+1}^{m} \lambda_k \).

This fact, together with (4), implies that there is an integer \( m_{j+1} \geq n_{j+1} + 1 \) such that

\[
(5) \quad s_j + \sum_{k=n_{j+1}+1}^{m_{j+1}} \lambda_k \leq t < s_j + \sum_{k=n_{j+1}+1}^{m_{j+1}+1} \lambda_k
\]

If equality holds in the left-hand side of (5), then \( t = s_{j+1} \). If the left-hand side of (5) is a strict inequality, then \( 0 < t - s_{j+1} < \lambda_{m_{j+1}+1} \). By induction, either \( t = s_j \) for some \( j \) or \( t = \lim_{j \to \infty} s_j \). In either case, \( t = \sum_{h=1}^{\infty} \lambda_h a_k \) for a sequence \((a_k)\) of 0's and 1's.

**Remark 9.** In order to illustrate an application of these result, let \( 0 < \lambda < 1 \) and define \( \lambda_k = \lambda(1 - \lambda)^{k-1} \). Then \((\lambda_k)\) a nonincreasing sequence of positive numbers satisfying

\[
\sum_{k=1}^{\infty} \lambda_k = 1.
\]

For each positive integer \( n \), we have \( \sum_{k=n+1}^{\infty} \lambda(1 - \lambda)^{k-1} = (1 - \lambda)^n \). It follows that \((\lambda_k)\) is a \( B_{\ell_\infty(R)} \)-generating sequence if and only if

\[
\lambda(1 - \lambda)^{n-1} \leq (1 - \lambda)^n
\]

for each \( n \). This occurs if and only if \( \lambda \leq \frac{1}{2} \).
REFERENCES


[5] A.S. Granero, The $\lambda$-function in the spaces $\left(\bigoplus_{i \in I} X_i\right)_p$ and $L_p(\mu, X)$, $1 \leq p \leq \infty$, preprint.


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