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DOUBLE FOURIER COSINE-JACOBI SERIES FOR FOX'S H-FUNCTION

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Abstract. *In this paper, we present a new class of double Fourier Cosine-Jacobi series for Fox's H-function.*

1. INTRODUCTION

The object of this paper is to introduce a new class of double Fourier Cosine-Jacobi series for Fox's H-function [5] and present one double Fourier series of this class. We also obtain a double Fourier Cosine-Jacobi series for Meijer's G-function [4,p. 207, (1)] as a particular case.

In what follows for sake of brevity:

$$\sum_{j=1}^{p} e_j - \sum_{j=1}^{q} f_j \equiv A, \sum_{j=1}^{n} e_j - \sum_{j=n+1}^{p} e_j + \sum_{j=1}^{m} f_j - \sum_{j=m+1}^{q} f_j \equiv B.$$

The following formulae are required in the proof:

The integral [2, p. 704, (2.2)]:

$$(1.1) \qquad \int_0^{\pi} \cos(ux) \left(\sin \frac{x}{2}\right)^{-2w_1} H_{p,q}^{m,n}\left[z\left(\sin\frac{x}{2}\right)^{-2h} \middle| \begin{matrix} (a_p, e_p) \\ (b_q, f_q) \end{matrix} \right] dx$$

$$= \sqrt{(\pi)}\, H_{p+2,q+2}^{m+1,n+1}\left[z \middle| \begin{matrix} (1-w_1-u, h), (a_p, e_p), (1-w_1+u, h) \\ (1/2-w_1, h), (b_q, f_q), (1-w_1, h) \end{matrix} \right],$$

where $h > 0, A \leq 0, B > 0, |argz| < 1/2B\pi, Re[1-2w_1+2h(1-a_j)/e_j] > 0$ $(j = 1, 2, \ldots, n)$.

The integral [1, p. 699, (2.3)]:

$$(1.2) \qquad \int_{-1}^{1} (1-y)^{w_2}(1+y)^b P_v^{(a,b)}(y) H_{p,q}^{m,n}\left[z(1-y)^k \middle| \begin{matrix} (a_p, e_p) \\ (b_q, f_q) \end{matrix} \right] dy$$

$$= \frac{2^{b+w_2+1}\Gamma(1+v+b)}{v!} \cdot$$

$$H_{p+2,q+2}^{m+1,n+1}\left[z2^k \middle| \begin{matrix} (-w_2, k), (a_p, e_p), (a-w_2, k) \\ (a-w_2+v, k), (b_q, f_q), (-1-b-w_2-v, k) \end{matrix} \right],$$

where $k > 0, A \leq 0, B > 0, |argz| < 1/2B\pi, Reb > -1, [Rew_2 + kb_j/f_j] > -1$ $(j = 1, \ldots, m)$.

The orthogonally property of the Jacobi polynomials [6, p. 285, (5) and (9)]:

$$(1.3)\qquad \int_{-1}^{1}(1-x)^{a}(1+x)^{b}P_{n}^{(a,b)}(x)P_{m}^{(a,b)}(x)dx$$
$$=0,\ \text{if } m\neq n,$$
$$=\frac{2^{a+b+1}\Gamma(a+n+1)\Gamma(b+n+1)}{n!(a+b+2n+1)\Gamma(a+b+n+1)},\ \text{if } m=n;$$

where $Re a>-1, Re b>-1$.

2. DOUBLE FOURIER COSINE-JACOBI SERIES

The double Fourier Cosine-Jacobi series to be established is

$$(2.1)\qquad \left(\sin\frac{x}{2}\right)^{-2x_1}(1-y)^{w_2}H_{p,q}^{m,n}\left[z\left(\sin\frac{x}{2}\right)^{-2h}(1-y)^{k}\left|\begin{matrix}(a_p,e_p)\\(b_q,f_q)\end{matrix}\right.\right]$$
$$=\frac{2^{w_2+1}}{\sqrt{(\pi)}}\sum_{r,t=0}^{\infty}\frac{(a+b+2t+1)\Gamma(a+b+t+1)}{\Gamma(a+t+1)}\cos(rx)P_{t}^{(a,b)}(y)H_{p+4,q+3}^{m+1,n+2}\cdot$$
$$\left[2^{k}z\left|\begin{matrix}(1-w_1-r,h),(-w_2-a,k),(a_p,e_p),(1-w_1+r,h),(-w_2,k)\\(1/2-w_1,h),(-w_2+t,k),(b_q,f_q),(1-w_1,h),(-1-a-b-w_2-t,k)\end{matrix}\right.\right],$$

where $h>0, k>0, |argz|<1/2B\pi, Rea>-1, Reb>-1, Re[1-2w_1+2h(1-a_j)/e_j]>0 (j=1,\dots,n), Re[w_2+kb_j/f_j]>-1 (j=1,\dots,m)$.

Proof. To establish (2.1), let

$$(2.2)\quad f(x,y)=\left(\sin\frac{x}{2}\right)^{-2w_1}(1-y)^{w_2}H_{p,q}^{m,n}\left[z\left(\sin\frac{x}{2}\right)^{-2h}(1-y)^{k}\left|\begin{matrix}(a_p,e_p)\\(b_q,f_q)\end{matrix}\right.\right]$$
$$=\sum_{r,t=0}^{\infty}A_{r,t}\cos(rx)P_{t}^{(a,b)}(y).$$

Equation (2.2) is valid, since $f(x,y)$ is defined in the region $0<x<\pi$, $-1<y<1$.

The problems concerning the possibility of expressing a function $f(x,y)$ as double Fourier series expansion are many and cumbersome. However, convergece of almost all double Fourier series expansions is covered by two-variables analogues of well known Dirichlet's conditions and the Jordan's theorem. In this respect, a brief discussion given by Casslaw and Jaeger [3, pp. 180-183] together with the references indicated in [3] provide a good coverage of the subject.

Multiplyng both sides of (2.2) by $(1-y)^a(1+y)^b P_v^{(a,b)}(y)$, integrating with respect to y from -1 to 1, and using (1.2) and (1.3), we obtain

$$(2.3)\quad 2^{w_2}\left(\sin\frac{x}{2}\right)^{-2w_1} H^{m+1,n_1}_{p+2,q+2}\cdot \cdot\left[2^k z\left(\sin\frac{x}{2}\right)^{-2h}\left|\begin{matrix}(-w_2-a,k),(a_p,e_p),(-w_2,k)\\(-w_2+v,k),(b_q,f_q),(-1-a-b-w_2-v,k)\end{matrix}\right.\right] =\sum_{r=0}^{\infty} A_{r,v}\frac{\Gamma(a+v+1)}{(a+b+2v+1)\Gamma(a+b+v+1)}\cos(rx).$$

Multiplyng both sides of (2.3) by $\cos(ux)$, integrating with respect to x from 0 to π, and using (1.1) and the orthogonality property of cosine functions; we get

$$(2.4)\quad A_{u,v}=\frac{2^{w_2+1}(a+b+2v+1)\Gamma(a+b+v+1)}{\sqrt{(\pi)}\Gamma(a+v+1)}\cdot H^{m+2,n+2}_{p+4,q+3}\left[2^k z\cdot \cdot\left|\begin{matrix}(1-w_1-u,h),(-w_2-a,k),(a_p,e_p),(1-w_1+u,h),(-w_2,k)\\(1/2-w_1,h),(-w_2+v,k),(b_q,f_q),(1-w_1,h),(-1-a-b-w_2-v,k)\end{matrix}\right.\right],$$

except that $A_{0,v}$ is one-half of the above value. From (2.2) and (2.4), the formula (2.1) is obtained.

Note. On applyng the same procedure as above, we can establish three other forms of two-dimensional expansions of this class with the help of alternative forms of (1.1) and (1.2).

3. PARTICULAR CASES

Since on specializing the parameters Fox's H-function yields almost all special functions appearing in applied mathematics and physical sciences. Therefore, the result presented in this paper is of a general character and hence may encompass several cases of interest. However, we present below one particular case of our Cosine-Jacobi series.

In (2.1), assuming h and k as positive integers, putting $e_j = f_i = 1 (j = 1,\dots,p; i = 1,\dots,q)$, using the formula [7, p. 18, (2.6.1)], then simplifying with the help of [7, p.10, (2.1.1)],[4, p. 4, (11)] and [4, p. 207, (1)], we obtain

$$(3.1)\quad \left(\sin\frac{x}{2}\right)^{-2w_1}(1-y)^{w_2} G^{m,n}_{p,q}\left[z\left(\sin\frac{x}{2}\right)^{-2h}(1-y)^k\left|\begin{matrix}a_p\\b_q\end{matrix}\right.\right] =\frac{2k^{-a-1}}{\sqrt{(\pi h)}}\sum_{r,t=0}^{\infty}\frac{(a+b+2t+1)\Gamma(a+b+t+1)}{\Gamma(a+t+1)}\cos(rx)P_t^{(a,b)}(y)G^{m+1,n+2}_{p+4,q+1}\cdot \cdot\left[2^k z\left|\begin{matrix}\Delta(h,1-w_1-r),\Delta(k,-w_2-a),a_p,\Delta(h,1-w_1+r),\Delta(k,-w_2)\\\Delta(h,1/2-w_1),\Delta(k,-w_2+t),b_q,\Delta(h,1-w_1),\Delta(k,1-a-b-w_2-t)\end{matrix}\right.\right],$$

where the conditions are analogous to (2.1) and the symbol $\Delta(h,\mu)$ represents the set of parameters

$$\frac{\mu}{h}, \frac{\mu+1}{h}, \ldots, \frac{\mu+h-1}{h}$$

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