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MODELING AND FORECASTING EXCHANGE RATE DYNAMICS IN PAKISTAN USING ARCH FAMILY OF MODELS

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Abstract: *The main objective of this paper is to provide an exclusive understanding about the theoretical and empirical working of the GARCH class of models as well as to exploit the potential gains in modeling conditional variance, once it is confirmed that conditional mean model errors present time varying volatility. Another objective is to search the best time series model among autoregressive moving average (ARMA), autoregressive conditional heteroscedasticity (ARCH), generalized autoregressive conditional heteroscedasticity (GARCH), and exponential generalized autoregressive conditional heteroscedasticity (EGARCH) to give best prediction of exchange rates. The data used in present study consists of monthly exchange rates of Pakistan for the period ranging from July 1981 to May 2010 obtained from State Bank of Pakistan. GARCH (1,2) is found to be best to remove the persistence in volatility while EGARCH(1,2) successfully overcome the leverage effect in the exchange rate returns under study.*

Keywords: *Conditional variance, Exchange rates, GARCH, EGARCH, Volatility modeling.*

1. Introduction

In the era of globalization and financial liberalization, exchange rates play an important role in international trade and finance for a small open economy like Pakistan. This is because movements in exchange rates affect the profitability of multinationals and increase exchange exposure to enterprises and financial institutions. A stable exchange rate may help venture and financial institutions in evaluating the performance of investments, financing and hedging, and

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thus reducing their operational risks. Fluctuations in the exchange rate may have a significant impact on the macroeconomic fundamentals such as interest rates, prices, wages, unemployment, and the level of output. The behavior of exchange rates is of crucial interest for the policy makers.

Recent developments in financial econometrics require the use of quantitative models that are able to explain the attitude of investors not only towards the expected returns and risks, but towards volatility as well. Hence, market participants should be aware of the need to manage risks associated with volatility, which requires models that are capable of dealing with the volatility of the market. Due to unexpected events like, the non-constant variance in the financial markets, and uncertainties in prices and returns, financial analysts started to model and explain the behavior of exchange rate returns and volatility using time series econometric models.

One of the most prominent tools for capturing such changing variance is the Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models. These models were developed by Engle [1] and extended by Baillie [2] and Nelson [3]. Two important characteristics within financial time series, the fat tails and volatility clustering (or volatility pooling), can be captured by the GARCH family models. Many studies have been done to model the exchange rates of a country. Corte et al [4] used Bayesian methods for estimation and ranking of a set of empirical exchange rate models, and constructed combined forecasts based on Bayesian model averaging. Sing [5] investigated the GARCH model for a comprehensive set of weighted (export and trade) and unweighted (official and black) real exchange rate series in India. He found evidence of dimensionally weak and statistically insignificant autoregressive conditional heteroscedasticity effects as compared to GARCH effects in almost all the exchange rate series. Other studies include Baillie and Bollerslev [2], Poon and Granger [6], Arize et al [7], Fidrmuc and Horváth [8] and Rapach and Strauss [9].

The economy of Pakistan is the 27th largest economy in the world in terms of purchasing and the 45th largest in absolute dollar terms. Pakistan has been categorized as an emerging market. There are three stock exchange markets in Pakistan. The Karachi Stock Exchange (KSE) being the most liquid and the biggest in terms of market capitalization and trading volume and been awarded the title of “best performing emerging stock market of the world” in 2002 by Business Week. Jalil and Feridun [10] explained the exchange rate movements in the Pakistani foreign exchange market using the market micro structure approach, which had not been applied to date due to the unavailability of high-frequency data on the order flow for Pakistan. Mohammad [11] empirically assess the effect of Euro-Dollar Exchange rate on chosen macroeconomic variables like real output, price level, and money supply for Pakistan. Mohammad [11] applied vector autoregessive (VAR) approach to find the relation between the above and the results are evident of no significant impact of Euro and US dollar depreciation on the macroeconomic variables of Pakistan. This paper offers insights on exchange rates in Pakistan and measures the sources of volatility by using Autoregressive Conditional Heteroscedasticity (ARCH), Generalized Autoregressive Conditional Heteroscedasticity (GARCH) and Exponential General Autoregressive Conditional Heteroscedasticity (EGARCH) techniques.

The rest of this paper is categorized as follows: Section 2 presents the description of the data used in this study and some econometric methodology used for analysis. Section 3 presents statistical features of the data and results of the analysis with some discussion. Finally last section concludes the text with some concluding remarks.

2. Material and Methods

2.1 Data and Econometric methodology

For conventional analysis and volatility modeling, the data used in the present study consists of monthly average Foreign Exchange rates of Pakistan (Pak rupees per US $). The data is obtained from State Bank of Pakistan (SBP) Karachi with sample period ranging from July 1981 to May 2010. Since the official exchange markets remain close on Sunday, there are total of 347 monthly observations. To model this data, different econometric time series models like ARMA, ARCH, GARCH, IGARCH and EGARCH are used. A short description of each is given in the following text.

2.2 ARMA (p,q)

In an ARMA (p, q) process, there are p autoregressive and q moving average terms. The algebraic representation of the statistical model is:

$$y_t = \mu + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \dots\dots\dots + \Phi_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots\dots + \theta_q \varepsilon_{t-q} \qquad (1)$$

where the intercept parameter μ is related to the mean of y_t and errors ε_t are assumed to be uncorrelated random variables, with $E[\varepsilon_t] = 0$ and $\mathrm{var}[\varepsilon_t] = \sigma_t^2$. If this process is stationary, then it must have a constant mean μ for all time periods.

2.3 Autoregressive Conditional Heteroscedasticity (ARCH) Model

The autoregressive conditional heteroscedasticity (ARCH) model is the first model of conditional heteroscedasticity. In econometrics, a model featuring autoregressive conditional heteroscedasticity considers the variance of the current error term or innovation to be a function of the actual sizes of the previous time periods' error terms. Often the variance is related to the squares of the previous innovations. Such models often called ARCH models were introduced by Engle [1] and is known as a first order autoregressive conditional heteroscedasticity ARCH(1) process.

Specifically, let ε_t denote the error terms of return residuals with respect to mean process and assume that $\varepsilon_t = \sigma_t z_t$ where $z_t \overset{iid}{\sim} N(0,1)$ and the series σ_t^2 are modeled by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \qquad (2)$$

Here $\alpha_0 > 0$ and $\alpha_i \geq o,\ i > o$. An ARCH(q) model can be estimated using ordinary least squares.

2.4 Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model

Unfortunately, volatility is not an easy phenomenon to predict or forecast. One class of models which have proved successful in forecasting volatility in many situations is the GARCH family of models. The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models were propounded by Engle [1] and Baillie [2]. The GARCH (p, q) model is formulated as:

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + ... + \alpha_q\varepsilon_{t-q}^2 + \beta_1\sigma_{t-1}^2 + ... + \beta_p\sigma_{t-p}^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \sum_{i=1}^{p}\beta_i\sigma_{t-i}^2 \quad (3)$$

where p is the order of the GARCH (lagged volatility) terms, and q is the order of the ARCH (lagged squared-error) terms.

2.5 Integrated Generalized Autoregressive Conditional Heteroscedasticity (IGARCH) Model

Integrated Generalized Autoregressive Conditional Heteroscedasticity IGARCH is a restricted version of the GARCH model, where the sum of the persistent parameters sum up to one, and therefore there is a unit root in the GARCH process. The condition for this is:

$$\sum_{i=1}^{p}\beta_i + \sum_{i=1}^{q}\alpha_i = 1 \quad (4)$$

2.6 Exponential General Autoregressive Conditional Heteroscedastic (EGARCH) Model

The Exponential General Autoregressive Conditional Heteroscedasticity (EGARCH) model is another popular GARCH model which is introduced by Nelson [3]. In EGARCH model there is no need for the parameters to follow nonnegative restriction and another important feature of EGARCH model is that it successfully capture the leverage effect while GARCH model is not to do so. EGARCH ensures that the variance is always positive even if the parameters are negative. The EGARCH (1, 1) could be given as:

$$\ln h_t^2 = \alpha_0 + \alpha_1\left|\frac{\varepsilon_{t-1}}{h_{t-1}}\right| + \phi\frac{\varepsilon_{t-1}}{h_{t-1}} + \beta_1 \ln h_{t-1}^2 \quad (5)$$

where ϕ captures the leverage effect (i.e. asymmetric effect).

2.7 Forecasting Performance

Commonly used measures for comparison of the forecasting performance are Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), and Theil-U inequality ([12]). The selected GARCH (1, 2) model in our study is evaluated by using all these measures for evaluating the forecasting performance. The mentioned forecast error statistics are defined as:

$$\text{Mean Absolute Error} = \frac{\sum_{t=T+1}^{T+k}\left|\hat{y}_t - y_t\right|}{h} \quad (6)$$

$$\text{Root Mean Squared Error} = \sqrt{\frac{\sum_{t=T+1}^{T+k}\left(\hat{y}_t - y_t\right)^2}{h}} \tag{7}$$

These two forecast error statistics depend on the scale of the dependent variable. These should be used as relative measures to compare forecasts for the same series across different models. The smaller the error, the better is the forecasting ability of that model according to a given criterion. The remaining two statistics are scale invariant. The Theil inequality coefficient always lies between zero and one, where zero indicates a perfect fit.

$$\text{Mean Absolute Percentage Error} = 100 \times \left[\frac{\sum_{t=T+1}^{T+k}\left|\frac{\hat{y}_t - y_t}{y_t}\right|}{h}\right] \tag{8}$$

$$\text{Theil Inequality Coefficient} = \frac{\sqrt{\frac{\sum_{t=T+1}^{T+k}\left(\hat{y}_t - y\right)^2}{h}}}{\sqrt{\frac{\sum_{t=T+1}^{T+k}\hat{y}_t^{\,2}}{h}} + \sqrt{\frac{\sum_{t=T+1}^{T+k}y_t^{\,2}}{h}}} \tag{9}$$

3. Results and Discussion

3.1 Descriptive statistics

To assess the distributional properties of the exchange rate return data, various descriptive statistics are reported in Table 1.

Table 1. Summary statistics.

Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Prob.*
0.27	0.57	2.12	8.48	691.07	<0.001

As expected for a time series of returns, the mean is close to zero. The standard deviation is also high which indicates high level of fluctuations of the exchange rate returns. There is also evidence of positive skewness, with long right tail indicating that exchange rate has non symmetric returns.

The histogram of the series in Figure 1 reports that the exchange rate returns are leptokurtic or fat tailed because of its large kurtosis value. The return series of exchange rate is non normal according to the Jarque-Bera test with p-value less than 0.001 (see Table 1). So the hypothesis of normality of the exchange rate returns is rejected.

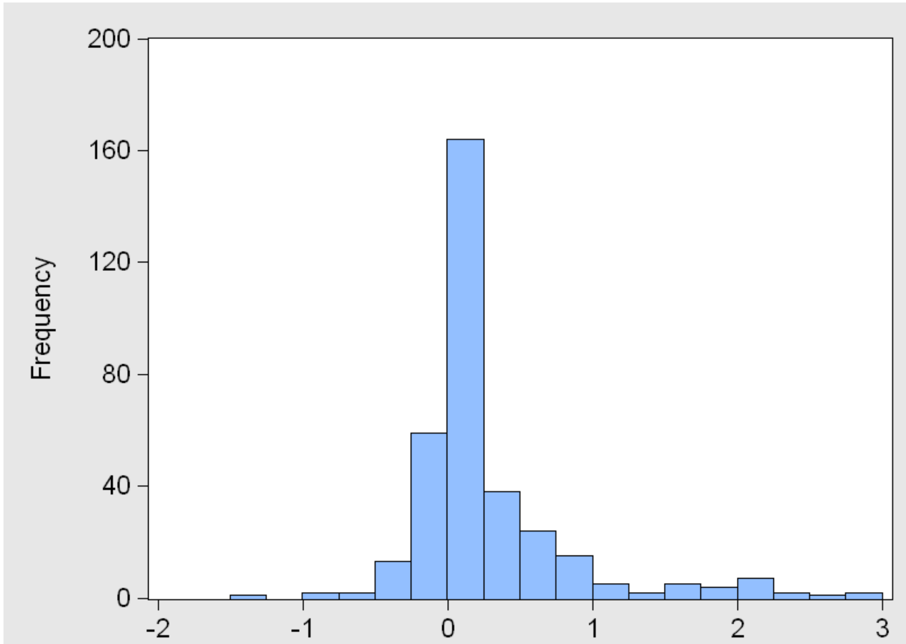


Figure 1. Histogram of exchange rate return series.

3.2 Tests for checking Stationarity

It is important to confirm whether the data is stationary before using the time series models like Autoregressive Integrated Moving Average (ARIMA), Autoregressive Conditional Heteroscedasticity (ARCH), Generalized Autoregressive Conditional Heteroscedasticity (GARCH) and Exponential General Autoregressive Conditional Heteroskedastic (EGARCH), which are being used in our study. Three important methods of checking stationarity of time series are graphical analysis, Correlogram and unit root test.

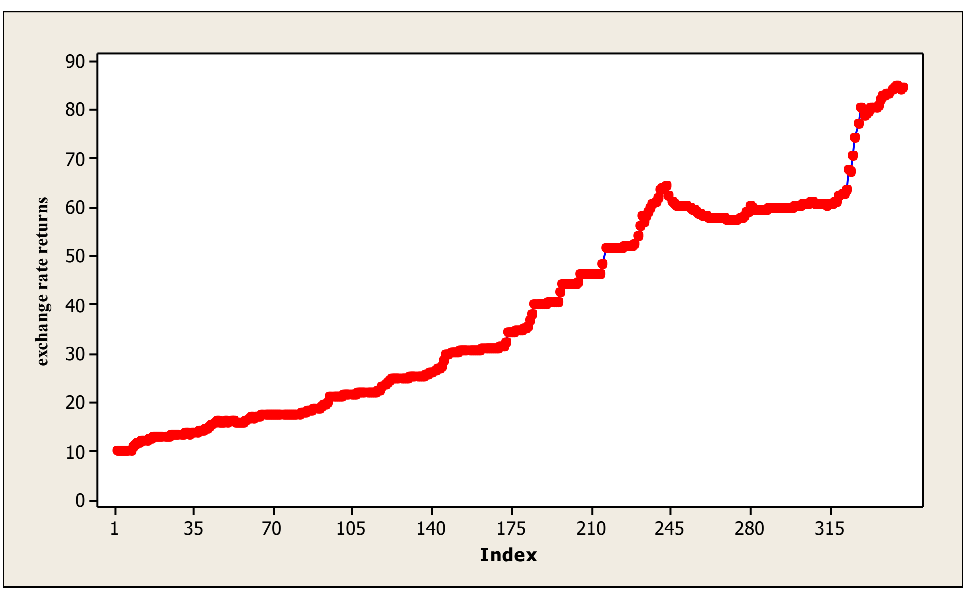


Figure 2. Time series plot of exchange rates series.

In Figure 2 the graphical analysis of exchange rate series shows an upward trend, suggesting that the mean of the exchange rates has been changing. This perhaps suggests that the exchange rate series is not stationary.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.990	0.990	343.12	0.000
		2	0.980	-0.023	680.15	0.000
		3	0.969	-0.026	1010.8	0.000
		4	0.958	-0.026	1334.9	0.000
		5	0.947	-0.007	1652.5	0.000
		6	0.936	-0.007	1963.4	0.000
		7	0.925	-0.001	2268.0	0.000
		8	0.914	-0.001	2566.2	0.000
		9	0.903	-0.003	2858.2	0.000
		10	0.892	-0.008	3143.9	0.000
		11	0.881	-0.002	3423.5	0.000
		12	0.870	0.003	3697.2	0.000
		13	0.859	-0.005	3965.0	0.000
		14	0.849	-0.007	4226.9	0.000
		15	0.838	-0.007	4483.0	0.000
		16	0.827	0.002	4733.5	0.000
		17	0.817	-0.006	4978.3	0.000
		18	0.806	-0.005	5217.6	0.000
		19	0.795	-0.024	5451.1	0.000
		20	0.784	-0.019	5678.9	0.000
		21	0.774	0.038	5901.4	0.000
		22	0.764	0.033	6119.1	0.000
		23	0.756	0.045	6332.6	0.000
		24	0.748	0.038	6542.5	0.000
		25	0.740	-0.019	6748.7	0.000

Figure 3. Correlogram of exchange rate series.

The Correlogram of the series is shown in Figure 3 which indicates a patter up to 30 lags. The autocorrelation coefficient starts with a very high value of 0.990 at lag 1 and declines very slowly towards zero. It seems that the exchange rate series is nonstationary. Table 2 presents the results of Augmented Dickey–Fuller (ADF) test and Phillips-Perron (PP) test on exchange rate data. Since the statistic value for both ADF and PP tests is greater than their corresponding critical values, so we do not reject the null hypothesis of the presence of unit root in the series and conclude that the exchange rate series is nonstationary.

To transform the nonstationary exchange rate series we calculate the exchange rate returns as

$$R_t = \ln(y_t / y_{t-1}) * 100 \tag{10}$$

The time series plot of the transformed data that is exchange rate returns series is shown in Figure 4. This plot shows that the mean of the series is now about constant. So we can assume that the series is stationary.

Table 2. ADF and Phillip-Perron test on exchange rate series.

Augmented Dickey-Fuller test statistic		**t-Statistic**	**Prob.***
		1.16	0.99
Test critical values:	1% level	-3.45	
	5% level	-2.87	
	10% level	-2.57	
Phillips-Perron test		**Adj. t-Stat**	**Prob.***
		1.20	0.99
Test critical values:	1% level	-3.45	
	5% level	-2.87	
	10% level	-2.57	

Furthermore, the Correlogram in Figure 5 shows no trend in exchange rate returns, hence suggesting that the exchange rate return series is stationary.

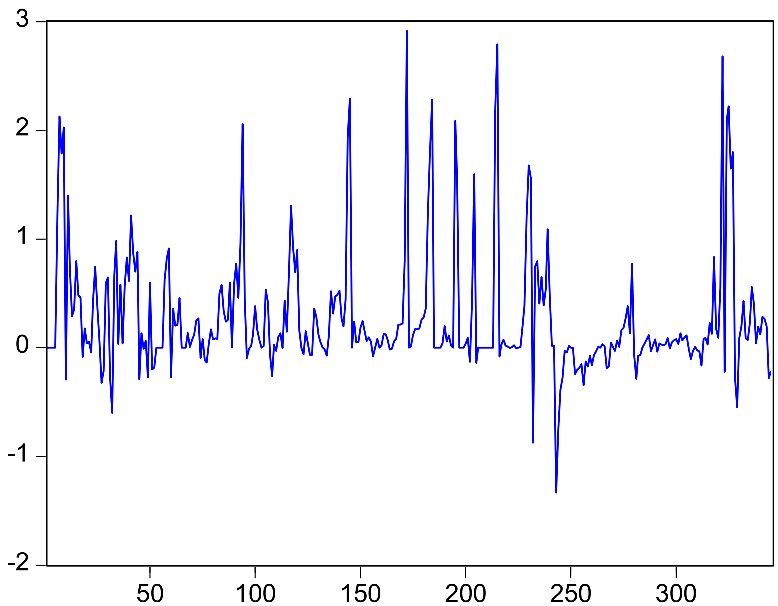
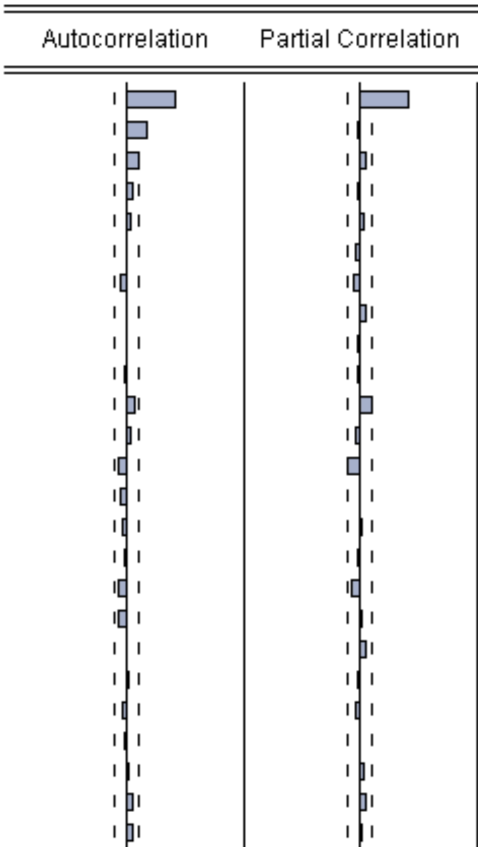


Figure 4. Time series plot of exchange rate returns



	AC	PAC	Q-Stat	Prob
1	0.427	0.427	63.714	0.000
2	0.175	-0.010	74.394	0.000
3	0.108	0.045	78.477	0.000
4	0.042	-0.023	79.099	0.000
5	0.035	0.025	79.533	0.000
6	-0.003	-0.033	79.535	0.000
7	-0.057	-0.057	80.695	0.000
8	0.003	0.060	80.699	0.000
9	-0.002	-0.020	80.700	0.000
10	-0.019	-0.013	80.829	0.000
11	0.070	0.100	82.597	0.000
12	0.028	-0.041	82.884	0.000
13	-0.069	-0.098	84.629	0.000
14	-0.055	0.001	85.717	0.000
15	-0.031	0.012	86.076	0.000
16	-0.023	-0.011	86.262	0.000
17	-0.070	-0.073	88.070	0.000
18	-0.066	0.013	89.684	0.000
19	0.005	0.047	89.693	0.000
20	0.012	-0.018	89.743	0.000
21	-0.032	-0.036	90.127	0.000
22	-0.023	-0.000	90.332	0.000
23	0.024	0.040	90.539	0.000
24	0.062	0.054	91.968	0.000
25	0.059	0.021	93.287	0.000

Figure 5. Correlogram of exchange rate returns.

In Table 3 the results of Augmented Dickey Fuller (ADF) test and Phillips Perron (PP) test show that the exchange rate return series is stationary. The variance is high that clearly exhibit volatility clustering, which allows us to carry on further to apply the ARCH family models.

Table 3. ADF and PP test on exchange rate return series.

Augmented Dickey-Fuller test statistic		**t-Statistic**	**Prob.***
		-11.74	<0.01
Test critical values:	-3.45	-3.45	
	-2.87	-2.87	
	-2.57	-2.57	
Phillips-Perron test		**Adj. t-Stat**	**Prob.***
		-11.73	<0.01
Test critical values:	-3.45	-3.45	
	-2.87	-2.87	
	-2.57	-2.57	

3.3 Model fitting

As the descriptive statistics given in Table 1 reflect that the distribution of the exchange rate return series does not follow a normal distribution it means volatility clustering is present. Suitable econometric modeling techniques are required for our exchange rate return series. To start with, we model the conditional mean process by autoregressive process AR(1) and moving average MA(1). Both of these processes demonstrate high correlation in residuals. Among different models applied to the data, ARMA(1, 2) appears to be relatively better fit on the basis of Akaike criterion, Schwarz criterion, and Durbin Watson statistic. The results of ARMA(1, 2) are shown in Table 4.

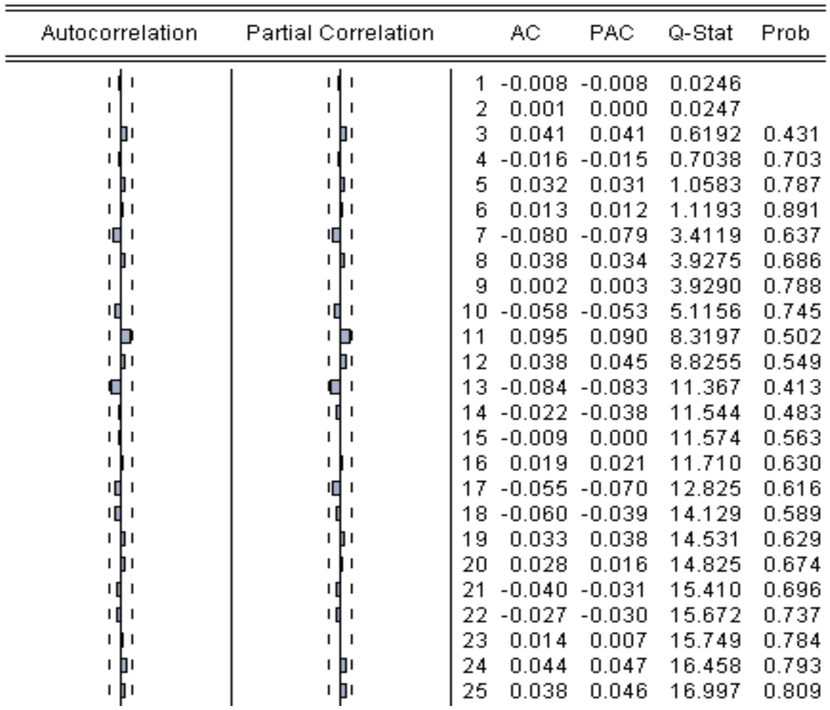
Table 4. Results of ARMA (1, 2) model.

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.27	0.05	5.66	<0.01
AR(1)	0.44	0.05	8.36	<0.01
MA(2)	-0.035	0.06	-0.59	0.55
R-squared	0.18	Mean dependent var	0.27	
Adjusted R-squared	0.18	S.D. dependent var	0.57	
S.E. of regression	0.51	Akaike info criterion	1.52	
Sum squared resid	90.35	Schwarz criterion	1.55	
Log likelihood	-258.41	Hannan-Quinn criter.	1.53	
F-statistic	38.42	Durbin-Watson stat	2.02	
Prob(F-statistic)	<0.01			

The correlogram of ARMA(1, 2) residuals in Figure 6 shows that none of the autocorrelation and partial autocorrelation is individually statistically significant. In other words the correlogram of both autocorrelation and partial autocorrelation give the impression that the estimated residuals are purely random. Hence, there is no need to search out for another ARMA model.

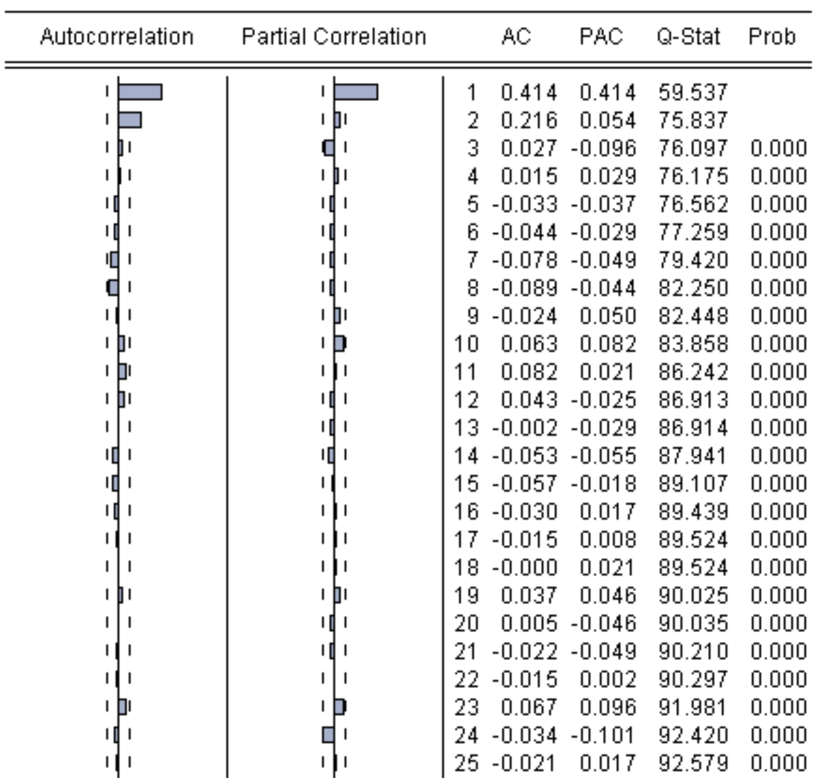
Once an appropriate ARMA model for conditional mean has been identified and estimated, the next step is to test whether the estimated errors are heteroscedastic or not. For this purpose the correlogram of squared residuals from ARMA(1, 2) is presented in Figure 7.

It shows high quantity of autocorrelation in residuals which suggest to carry on further to apply the ARCH family of models. The ARCH family of models requires the presence of 'ARCH effect' in the residuals. To test the presence of ARCH effect, we use the Lagrange Multiplier (LM) test for exchange rate returns series as suggested by Engle [1]. The results of Lagrange Multiplier test are presented in Table 5. The *p*-value indicates that there is evidence of remaining ARCH effect. So, we reject the null hypothesis of absence of ARCH effect even at 1% level of significance.



	AC	PAC	Q-Stat	Prob
1	-0.008	-0.008	0.0246	
2	0.001	0.000	0.0247	
3	0.041	0.041	0.6192	0.431
4	-0.016	-0.015	0.7038	0.703
5	0.032	0.031	1.0583	0.787
6	0.013	0.012	1.1193	0.891
7	-0.080	-0.079	3.4119	0.637
8	0.038	0.034	3.9275	0.686
9	0.002	0.003	3.9290	0.788
10	-0.058	-0.053	5.1156	0.745
11	0.095	0.090	8.3197	0.502
12	0.038	0.045	8.8255	0.549
13	-0.084	-0.083	11.367	0.413
14	-0.022	-0.038	11.544	0.483
15	-0.009	0.000	11.574	0.563
16	0.019	0.021	11.710	0.630
17	-0.055	-0.070	12.825	0.616
18	-0.060	-0.039	14.129	0.589
19	0.033	0.038	14.531	0.629
20	0.028	0.016	14.825	0.674
21	-0.040	-0.031	15.410	0.696
22	-0.027	-0.030	15.672	0.737
23	0.014	0.007	15.749	0.784
24	0.044	0.047	16.458	0.793
25	0.038	0.046	16.997	0.809

Figure 6. Correlogram of the residuals of ARMA(1, 2) model.



	AC	PAC	Q-Stat	Prob
1	0.414	0.414	59.537	
2	0.216	0.054	75.837	
3	0.027	-0.096	76.097	0.000
4	0.015	0.029	76.175	0.000
5	-0.033	-0.037	76.562	0.000
6	-0.044	-0.029	77.259	0.000
7	-0.078	-0.049	79.420	0.000
8	-0.089	-0.044	82.250	0.000
9	-0.024	0.050	82.448	0.000
10	0.063	0.082	83.858	0.000
11	0.082	0.021	86.242	0.000
12	0.043	-0.025	86.913	0.000
13	-0.002	-0.029	86.914	0.000
14	-0.053	-0.055	87.941	0.000
15	-0.057	-0.018	89.107	0.000
16	-0.030	0.017	89.439	0.000
17	-0.015	0.008	89.524	0.000
18	-0.000	0.021	89.524	0.000
19	0.037	0.046	90.025	0.000
20	0.005	-0.046	90.035	0.000
21	-0.022	-0.049	90.210	0.000
22	-0.015	0.002	90.297	0.000
23	0.067	0.096	91.981	0.000
24	-0.034	-0.101	92.420	0.000
25	-0.021	0.017	92.579	0.000

Figure 7. Correlogram of square residuals of ARMA (1, 2) model.

Table 5. ARCH LM test on ARMA(1, 2) residuals.

F-statistic	35.69	Prob. F(2,340)	<0.01
Obs*R-squared	59.51	Prob. Chi-Square(2)	<0.01

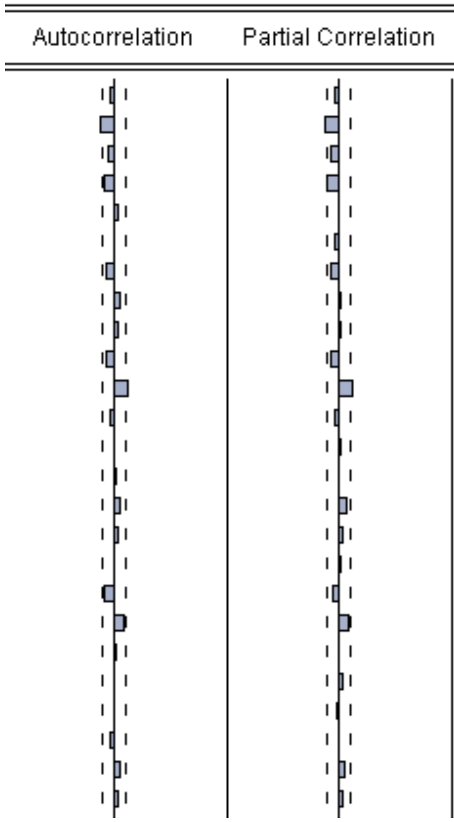
Therefore the conditional heteroscedasticity in estimated ARMA errors is modeled using GARCH(1, 2) specification. The results are shown in Table 6. The estimated coefficient of both conditional mean equation and conditional variance equation of GARCH(1, 2) model are highly significant. Therefore GARCH(1, 2) model can be cited as the suitable model on the basis of AIC and BIC criteria given in Table 6 and correlogram of the squared residuals from GARCH(1, 2) in Figure 8.

Table 6. Estimation Result of GARCH(1, 2) model.

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.0009	7.58E-05	12.0669	<0.001
AR(1)	-0.9511	0.0048	-197.6292	<0.001
MA(2)	-0.9028	0.0107	-84.0787	<0.001
Variance Equation				
C	0.0668	0.0155	4.3060	<0.001
RESID(-1)^2	2.1829	0.7702	2.8343	0.005
GARCH(-1)	0.1446	0.0270	5.3479	<0.001
GARCH(-2)	-0.0237	0.0072	-3.2997	0.001
GED PARAMETER	0.504927	0.040090	12.59483	0.0000

R-squared	0.5936	Mean dependent var	0.0002
Adjusted R-squared	0.5851	S.D. dependent var	0.9750
S.E. of regression	0.6280	Akaike info criterion	0.6541
Sum squared resid	131.7403	Schwarz criterion	0.7438
Log likelihood	-103.8500	Hannan-Quinn criter.	0.6898
F-statistic	69.6902	Durbin-Watson stat	2.5448
Prob(F-statistic)	<0.001		

Once the heteroscedasticity is modeled using appropriate GARCH model, next we check whether the GARCH(1, 2) has adequately captured the persistence in volatility and there is no ARCH effect left in the residuals from the selected models. The ARCH LM test is conducted for this purpose. The results of LM test given in Table 7 indicate that the residuals do not show any ARCH effect. Hence, GARCH(1, 2) is found to be reasonable to remove the persistence in volatility.



Lag	AC	PAC	Q-Stat	Prob
1	-0.048	-0.048	0.7942	
2	-0.126	-0.128	6.2526	
3	-0.060	-0.075	7.5169	0.006
4	-0.084	-0.111	9.9868	0.007
5	0.036	0.005	10.427	0.015
6	-0.004	-0.034	10.433	0.034
7	-0.066	-0.079	11.976	0.035
8	0.041	0.020	12.579	0.050
9	0.033	0.019	12.969	0.073
10	-0.072	-0.077	14.809	0.063
11	0.126	0.121	20.469	0.015
12	-0.045	-0.039	21.176	0.020
13	-0.008	0.013	21.198	0.031
14	0.011	0.002	21.243	0.047
15	0.047	0.079	22.038	0.055
16	0.039	0.038	22.574	0.068
17	-0.000	0.021	22.574	0.094
18	-0.095	-0.058	25.840	0.056
19	0.084	0.095	28.413	0.040
20	0.022	0.006	28.587	0.054
21	-0.007	0.034	28.607	0.072
22	-0.002	-0.013	28.609	0.096
23	-0.033	0.004	29.017	0.114
24	0.064	0.053	30.516	0.106
25	0.030	0.035	30.840	0.127

Figure 8. Correlogram of squared residuals from GARCH (1, 2) model.

Table 7. ARCH LM test on GARCH (1, 2) residuals.

F-statistic	0.4801	Prob. F(1,339)	0.4889
Obs*R-squared	0.4822	Prob. Chi-Square(1)	0.4874

Normality test of standardized squared residuals, however suggests that residuals are positively skewed due to the leverage effect. EGARCH is used to test the leverage effect that successfully captures the asymmetry. The results on asymmetric conditional variance for exchange rate return series are reported in Table 8. The GARCH(1, 2) parameter is significant, showing high degree of persistence. Coefficient of the asymmetric function is positive and significantly different from zero means there is no leverage effect left.

Table 8. Estimation Result of EGARCH (1, 2).

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.0007	0.0001	7.0329	<0.001
AR(1)	-0.9591	0.0049	-196.6726	<0.001
MA(2)	-0.9199	0.0088	-104.0558	<0.001
Variance Equation				
C(4)	0.0374	0.0266	1.4036	0.1604
C(5)	-0.0881	0.0480	-1.8373	0.0662
C(6)	0.3935	0.0805	4.8870	<0.001
C(7)	0.7996	0.2158	3.7054	<0.001
C(8)	0.2148	0.2141	1.0030	0.3159
GED PARAMETER	0.4692	0.0411	11.4232	<0.01

3.4 Forecast Analysis

Forecast performance of the fitted GARCH(1, 2) model of exchange rate returns is investigated through mean absolute error (MAE), root mean square error (RMSE), mean absolute percent error (MAPE), and Theil inequality coefficient. The results are shown in Table 9..

Table 9. Forecasting performance of exchange rate returns.

Root Mean Square Error	0.6921
Mean Absolute Error	0.3257
Mean Abs. Percent Error	390.29
Theil Inequality Coefficient	0.4801
Bias Proportion	0.0011
Variance Proportion	0.0019
Covariance Proportion	0.9971

The value of Theil inequality is 0.4801 indicating the model is a better fit. The bias proportion and variance proportion are close to zero. The value of covariance proportion is nearly one. We can say that this model is good for forecasting purpose along with capturing the volatility and the leverage effects.

4. Conclusion

This paper focuses on building a model for the exchange rate of Pakistan using time series methodology. As the financial time series like exchange rate may possess volatility, an attempt is made to model this volatility using ARCH and GARCH models. Monthly average foreign exchange rates of Pakistan for the period ranging from July 1981 to May 2010 are used for this purpose. First of all, the stationarity of the exchange rate series is examined using graphical analysis, correlogram and unit root test which showed the series as nonstationary. To make the exchange rate series stationary, the exchange rates are transformed to exchange rate returns. Then ARMA(1, 2) model is fitted to the data. To capture the volatility, GARCH (1, 2) model is used. This model is further having leverage effect. This leverage effect is captured using an EGARCH model. Finally, the forecast performance is measured using different measures like MAE, RMSE and MAPE etc. The GARCH family of models captures the volatility and leverage effect in the exchange rate returns and provides a model with fairly good forecasting performance.

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