Electronic Journal of Applied Statistical Analysis
EJASA, Electron. J. App. Stat. Anal.
http://siba-ese.unisalento.it/index.php/ejasa/index
e-ISSN: 2070-5948
DOI: 10.1285/i20705948v15n2p463

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Published: 20 June 2022

Electronic Journal of Applied Statistical Analysis
Vol. 15, Issue 02, June 2022, 463-478
DOI: 10.1285/i20705948v15n2p463

New double stage ranked set sampling for estimating the population mean

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Published: 20 June 2022

In environmental and many other areas, the main focus of survey is to measure elements using an efficient and cost-effective sampling technique. One way to reach that is by using Ranked set sampling (RSS). RSS is an alternative sampling technique that can be advantageous when measuring the variable of interest is either costly or time-consuming but ranking small sets of units according to the character under investigation by eye or other methods not requiring actual quantifications. The purpose of this article is to introduce a new modification of RSS to estimate the mean of the target population. This proposed technique is a double-stage approach that combines median RSS (MRSS) and MiniMax RSS (MMRSS). The performance of the empirical mean and variance estimators based on the proposed technique are compared with their counterparts in Double RSS (DRSS), Extreme RSS (ERSS), Double Extreme RSS (DERSS), MMRSS, RSS, and simple random sampling (SRS) via Monte Carlo simulation. Simulation results revealed that this new modification is almost always more efficient than their counterparts using MMRSS and SRS, while it is more efficient than RSS in many cases especially when the distribution is asymmetric.

keywords: Double stage, Efficiency, Mean Estimation, Median ranked set sampling, Minimax ranked set sampling, Monte Carlo simulation.

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ISSN: 2070-5948
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1 Introduction

A sample is a subset of individuals chosen using a specific sampling technique to represent a study population in order to allow researchers to draw statistical inferences about that population without the need of investigating every element. Several sampling techniques are presented in the literature and the most common is known as simple random sampling (SRS) where each individual in the population has an equal chance of being selected as a subject. Cost-effective sampling technique is one of the crucial concerns in many fields such as ecological experiments, agricultural studies, environmental surveys, and human populations. Nonetheless, quantifying the elements in such fields can be more expensive than ranking them physically according to a variable of interest. For example, the age of fish in a lake can be ranked visually by observing their size, specifically their length; since both length and age are highly correlated, i.e., as the length of a fish gets larger its age will be older. Therefore, it becomes necessary to come up with estimators which use the visual ranking in sampling technique to reduce the number of elements we need to quantify and to improve the efficiency of these estimators.

To estimate the average pasture and forage yields, McIntyre (1952) first introduced a proficient sampling technique, which is later known as ranked set sampling (RSS), to collect data by employing ranking on elements in a way that improved the parametric estimation. The specialty of RSS over SRS is that it improves the efficiency of the estimator of the population mean using smaller number of sample measurements by making use of other resources such as expert knowledge, auxiliary information, or personal judgment which are easy to accomplish and relatively cheap. Takahasi and Wakimoto (1968) derived the theoretical underpinning results of RSS. They also showed that RSS produced unbiased estimators that are more efficient than the ones produced from SRS in estimating population mean of the same sample size.

Research on RSS technique remains very active field in recent years in statistical community and continues to attract attention in many environmental, agricultural, medical and biological studies. Many authors have focused on developing and modifying the basic RSS to make it more efficient, for some recent bibliography and detailed discussion on the theory and applications of RSS see for instance: Dell and Clutter (1972) demonstrated that the mean based on RSS is unbiased and more efficient than the mean based on SRS even when errors in ranking are present, Al-Omari (2012) suggested two modified ratio estimators of the population mean under SRS and median RSS techniques, Hanandeh and Al-Saleh (2013) estimated the parameters of Downton's bivariate exponential distribution based on moving extreme RSS, Haq et al. (2014) explored mixed RSS technique, Al-Omari (2015) considered L RSS technique to estimate the distribution function, Haq et al. (2015) investigated varied L RSS, Al-Omari (2016) obtained a measure of sample entropy for continuous random variable based on RSS, Al-Nasser and Al-Omari (2018) developed a new modification of RSS called MiniMax RSS, Al-Omari and Al-Nasser (2018) used multistage median RSS to estimate ratio, Al-Omari and Haq (2019) proposed new RSS methods to estimate the population mean, Obeidat et al. (2020) studied the parameters of generalized Gompertz distribution using RSS, Samuh et al. (2020) applied RSS techniques to estimate the new Weibull-Pareto distribution parameters, Al-Nasser et al. (2020) proposed extreme ranked repetitive sampling control charts, Hanandeh and Al-Nasser (2020) introduced an improved Shewhart control chart based on MiniMax RSS, Hanandeh and Al-Nasser (2021) modified the MiniMax RSS, Samuh et al. (2021) proposed a mixed double RSS, Hanandeh et al. (2022) suggested some new mixed RSS, Al-Omari and Bouza (2014) discussed a detailed review of RSS and its variations.

The RSS scheme can be described as follows:

1. A SRS of size r^2 elements is drawn randomly from the study population, where r is the set size.

2. The chosen r^2 elements are allocated randomly into r sets each of size r.

3. The r elements within each set are ranked by visual inspection or via any certain costless mechanism from the lowest to the largest with respect to a variable of interest.

4. Then the r measurements are chosen by quantifying the lowest ranked element from the 1^{th} set, the 2^{nd} lowest ranked element from the 2^{nd} set, and this procedure is continued until the unit with the highest rank is quantified from the r^{th} set.

The whole process (1)-(4) is referred to as one cycle of RSS. Note that although r^2 elements are selected from the study population, only r elements are actually measured and used in the final sample for inferential purposes.

The cycle can be replicated a number of times, say m, if needed, to generate a sample of size rm observations.

A one cycle of this process resulted in r ranked set samples is displayed in Table 1.

Table 1: Illustration of r RSS observations in the k^{th} set cycle

Sample	*Lowest*	2^{nd}*Lowest*		*Largest*	*RSS*
1	$X_{(1:1)k}$	$X_{(1:2)k}$	.....	$X_{(1:r)k}$	$\boldsymbol{X_{(1:1)k}}$
2	$X_{(2:1)k}$	$X_{(2:2)k}$	.....	$X_{(2:r)k}$	$\boldsymbol{X_{(2:2)k}}$
r	$X_{(r:1)k}$	$X_{(r:2)k}$	.....	$X_{(r:r)k}$	$\boldsymbol{X_{(r:r)k}}$

where $X_{(i:r)j}$, the i^{th} order statistic in a set of size r in the j^{th} cycle, where $i = 1, 2, \ldots, r$ and $j = 1, 2, \ldots, m$. Clearly, we measure only r $(X_{(1:1)k}, X_{(2:2)k}, \ldots, X_{(r:r)k})$ diagonal elements, and they constitute RSS in the k^{th} set cycle. The resulted observations from repeating the above cycle m times form a RSS sample of size rm denoted by $\boldsymbol{X_{(1:1)1}}, \boldsymbol{X_{(2:2)1}}, \ldots, \boldsymbol{X_{(r:r)m}}$. These quantified elements are independent but not identically distributed. Note that, in practice, the set size r should be small so that the ranking can be easily achieved without errors, while the replication index m is as large as possible.

In this article, new double-stage RSS schemes are developed, estimators based on the proposed sampling technique are shown to be more efficient than the estimators based on SRS with same set size and than RSS and some of its variations in many cases. The remainder of this article proceeds as follows. Section 2 provides a brief overview on some RSS variations. Section 3 introduces RSS(MMRSS) and MRSS(MMRSS) schemes with mean estimators. Section 4 offers detailed simulation experiments to compare mean estimators in terms of relative efficiency (RE) when sampling from symmetric and asymmetric distributions. Finally, Section 5 summarizes the article with a conclusion and an eye toward future work.

2 Brief review of some RSS variations

2.1 MiniMax RSS (MMRSS)

Al-Nasser and Al-Omari (2018) proposed MMRSS scheme. To obtain a sample of size r based on MMRSS scheme, the following steps are required:

1. Select r random samples of size $i = 1, 2, 3, \dots, r$ elements from the study population and rank the elements within each sample in ascending order by a visual way with respect to a variable of interest.

2. If the sample size is even, select the largest ranked element. Otherwise, if the sample size is odd, select the lowest ranked element.

3. The cycle may be repeated m times if needed to get rm elements. These rm elements form the MMRSS data.

2.2 Median RSS (MRSS)

Muttlak (1997) introduced MRSS scheme for estimation of population mean. MRSS scheme can be elaborated as follows:

1. Identify r^2 elements from the study population and allocate them into r sets each of size r.

2. Rank the elements within each set increasingly using any cheap method.

3. If the set size r is even, select the $\left(\frac{r}{2}\right)^{th}$ ranked element and $\left(\frac{r+2}{2}\right)^{th}$ ranked element from first and last $\frac{r}{2}$ sets, respectively. While if the set size r is odd, select $\left(\frac{r+1}{2}\right)^{th}$ ranked element from all sets.

4. This completes one cycle and resulted in MRSS of size r. The whole procedure can be repeated m times to obtain an MRSS of size mr.

2.3 Extreme RSS (ERSS)

Samawi et al. (1996) investigated ERSS to estimate the population mean only using the minimum or maximum RSS from each set. The procedure for ERSS can be implemented as follows:

1. Randomly identify r sets each of size r elements from the study population.
2. Rank the elements within each set in ascending order by a human expert.
3. If the set size r is even, the lowest ranked elements of each set are chosen from the first $\frac{r}{2}$ sets, and the largest ranked elements of each set are chosen from the other $\frac{r}{2}$sets. While if the set size is odd, the lowest ranked elements from the first $\frac{r-1}{2}$ sets, the largest ranked elements from the other $\frac{r-1}{2}$ sets and median element from the remaining last set are selected.
4. This completes one cycle and resulted in ERSS of size r. The whole procedure can be repeated m times to increase the sample size to obtain ERSS of size mr elements.

2.4 Double RSS (DRSS)

Al-Saleh and Al-Kadiri (2000) suggested DRSS scheme for estimation of population mean. DRSS scheme can be executed as follows.

1. Identify r^3 elements from the study population.
2. Randomly allocate these elements to r sets each of size r^2 elements.
3. Apply RSS method to the resulted r sets to get r RSS each of size r.
4. Again, apply RSS method to these r RSS to get a DRSS of size r.
5. This completes one cycle and resulted in a DRSS of size r. The whole process can be repeated m times to obtain a DRSS of size mr.

Note that Double ERSS can be executed similarly but by using ERSS instead of RSS (Samawi et al. (2002)).

3 Demonstration of the RSS(MMRSS) and MRSS(MMRSS) and their mathematical setup

In this section, we suggest a new double stage ranked set variations, namely, RSS(MMRSS) and MRSS(MMRSS), for estimating the population mean. These techniques can be used as an alternative to DRSS when it is destructive, time-consuming, or expensive to identify m^3 elements from the study population, especially when there is a shortage of experimental elements. Now, assume that X_{ijk}, $i, j= 1, 2, 3, \ldots, r$, is the j^{th} element in the i^{th} sample in the k^{th} cycle and that $X_{(i:j)k}$, $i, j= 1, 2, 3, \ldots, r$, is the j^{th} order statistic in the i^{th} sample in the k^{th} cycle.

3.1 Mixed MMRSS and RSS scheme

Step 1. Select r samples at random each of size $1, 2, ..., r$ elements respectively from the study population and rank the elements in each sample in an increasing magnitude based on any cost free method.

$X_{(1:1)k}$				
$X_{(2:1)k}$	$X_{(2:2)k}$			
$X_{(r:1)k}$	$X_{(r:2)k}$	.....	$X_{(r:(r-1))k}$	$X_{(r:r)k}$

Step 2. Utilize MMRSS on each of the choosen samples to select an MMRSS of size r elements (assuming that r is even). This resulted in r MiniMax ranked sets

					MMRSS
$X_{(1:1)k}$					$\boldsymbol{Z_1 = X_{(1:1)k}}$
$X_{(2:1)k}$	$X_{(2:2)k}$				$\boldsymbol{Z_2 = X_{(2:2)k}}$
$X_{(r:1)k}$	$X_{(r:2)k}$	.....	$X_{(r:(r-1))k}$	$X_{(r:r)k}$	$\boldsymbol{Z_r = X_{(r:r)k}}$

Step 3. Conduct RSS on the selected elements produced from step 2 to obtain RSS(MMRSS) of size r.

					RSS(MMRSS)
$Z_{(1:1)k}$	$Z_{(1:2)k}$	.....	$Z_{(1:(r-1))k}$	$Z_{(1:r)k}$	$\boldsymbol{W_1 = Z_{(1:1)k}}$
$Z_{(2:1)k}$	$Z_{(2:2)k}$	.....	$Z_{(2:(r-1))k}$	$Z_{(2:r)k}$	$\boldsymbol{W_2 = Z_{(2:2)k}}$
$Z_{(r:1)k}$	$Z_{(r:2)k}$	.....	$Z_{(r:(r-1))k}$	$Z_{(r:r)k}$	$\boldsymbol{W_r = Z_{(r:r)k}}$

Step 4. The above sampling procedure (Steps 1 through 3) can be replicated m times to get an RSS(MMRSS) sample of size rm,

3.2 Mixed MMRSS and MRSS scheme

Step 1. Draw r samples at random each of size $1, 2, ..., r$ elements respectively from the study population and rank the elements in each sample from lowest to largest by a personal judgment without actually quntifying them.

Step 2. Implement MMRSS on every single set of the choosen samples to pick an MMRSS of size r elements (assuming that r is even), yielding a MiniMax ranked sets of size r.

$X_{(1:1)k}$				
$X_{(2:1)k}$	$X_{(2:2)k}$			
$X_{(r:1)k}$	$X_{(r:2)k}$	.....	$X_{(r:(r-1))k}$	$X_{(r:r)k}$

					MMRSS
$X_{(1:1)k}$					$\boldsymbol{Z_1 = X_{(1:1)k}}$
$X_{(2:1)k}$	$X_{(2:2)k}$				$\boldsymbol{Z_2 = X_{(2:2)k}}$
$X_{(r:1)k}$	$X_{(r:2)k}$	.....	$X_{(r:(r-1))k}$	$X_{(r:r)k}$	$\boldsymbol{Z_r = X_{(r:r)k}}$

Step 3. Conduct MRSS on the choosen elements produced from Step 2 to obtain MRSS(MMRSS) of size r (assuming that r is even).

					MRSS(MMRSS)
$Z_{(1:1)k}$	$Z_{(1:2)k}$	.....	$Z_{(1:(r-1))k}$	$Z_{(1:r)k}$	$\boldsymbol{Y_1 = \frac{Z_{(1:\frac{r}{2})k}+Z_{(1:\frac{r}{2}+1)k}}{2}}$
$Z_{(2:1)k}$	$Z_{(2:2)k}$	.....	$Z_{(2:(r-1))k}$	$Z_{(2:r)k}$	$\boldsymbol{Y_2 = \frac{Z_{(2:\frac{r}{2})k}+Z_{(2:\frac{r}{2}+1)k}}{2}}$
$Z_{(r:1)k}$	$Z_{(r:2)k}$	.....	$Z_{(r:(r-1))k}$	$Z_{(r:r)k}$	$\boldsymbol{Y_r = \frac{Z_{(r:\frac{r}{2})k}+Z_{(r:\frac{r}{2}+1)k}}{2}}$

Step 4. The entire procedure (Steps 1 through 3) can be reprocessed m times to obtain a sample of size rm.

3.3 Population mean Estimation

Assuming that we have one cycle ($m = 1$), we can define the mean estimators under RSS(MMRSS) and MRSS(MMRSS) using the following formulas respectively:

$$\overline{W}_{(RSS(MMRSS))} = \frac{1}{r}\left\{\sum_{i=1}^{r} W_i\right\} \text{ and } \overline{Y}_{(MRSS(MMRSS))} = \frac{1}{r}\left\{\sum_{i=1}^{r} Y_i\right\}$$

It follows that the expected value of the sample mean from RSS(MMRSS) and MRSS(MMRSS) can respectively be defined as:

$$E\left(\overline{W}_{(RSS(MMRSS)\ )}\right) = \frac{1}{r}\left\{\sum_{i=1}^{r} E\left(Z_{(i:i)}\right)\right\}$$

and

$$E\left(\overline{Y}_{(MRSS(MMRSS))}\right)=\begin{cases}\frac{1}{r}\left\{\sum_{i=1}^{r}E\left(Z_{\left(i:\frac{i+1}{2}\right)}\right)\right\} \; if\ r\ is\ odd\\ \frac{1}{r}\left\{\sum_{i=1}^{\frac{r}{2}}E\left(Z_{\left(i:\frac{i}{2}\right)}\right)+\sum_{i=1}^{\left(\frac{r}{2}\right)-1}E\left(Z_{\left(i:\frac{i}{2}+1\right)}\right)\right\} \; if\ r\ is\ even\end{cases}$$

The variance expressions of $\overline{W}_{(RSS(MMRSS)\ )}$ and $\overline{Y}_{(MRSS(MMRSS))}$ are, respectively, given by:

$$Var\left(\overline{W}_{(RSS(MMRSS))}\right)=\frac{1}{r^2}\left\{\sum_{i=1}^{r}Var\left(Z_{(i:i)}\right)\right\}$$

and

$$Var\left(\overline{Y}_{(MRSS(MMRSS))}\right)=\begin{cases}\frac{1}{r^2}\left\{\sum_{i=1}^{r}Var\left(Z_{\left(i:\frac{i+1}{2}\right)}\right)\right\} \; if\ r\ is\ odd\\ \frac{1}{r^2}\left\{\sum_{i=1}^{\frac{r}{2}}Var\left(Z_{\left(i:\frac{i}{2}\right)}\right)+\sum_{i=1}^{\frac{r}{2}}Var\left(Z_{\left(i:\frac{i}{2}+1\right)}\right)\right\} \; if\ r\ is\ even\end{cases}$$

Moreover, the relative efficiency (RE) of the estimator of the population mean based on RSS(MMRSS) and MRSS(MMRSS) techniques with respect to the traditional SRS can be defined by:

$RE=\frac{MSE\left(\overline{Z}_{SRS}\right)}{MSE\left(\overline{W}_{(RSS(MMRSS)\ )}\right)}$ and $RE=\frac{MSE\left(\overline{Z}_{SRS}\right)}{MSE\left(\overline{Y}_{(MRSS(MMRSS))}\right)}$

4 Simulation Study

To investigate the efficiency of the suggested estimators compared with RSS, DRSS, ERSS, DERSS and MMRSS estimators, a simulation study is conducted for a variety of symmetrical and non-symmetrical probability distributions for sample of sizes 2,3,4,5 and 6, and based on 1 million replications using R-Language (R Core Team (2021)). The highest efficiency value is highlighted to make comparison easier.

Based on Tables 2-11, the concluding remarks are:

1. It is observed that there is an improvement in the RE when using MRSS(MMRSS) instead of SRS, DRSS, ERSS and DERSS in many cases.

2. Simulation results are showing that RE of the proposed estimators are better than MMRSS in almost all cases.

3. Although proposed mean estimators under asymmetrical distributions are biased, they are more efficient as compared to many other existing mean estimators.

4. It is also noteworthy from simulation findings that MRSS(MMRSS) can be applied as an effective substitute to DRSS when the ranking cost is not negligible and/or when there is shortage of sampling elements for asymmetrical distributions.

Table 2: RE comparisons between RSS, DRSS, ERSS, DERSS, MMRSS, RSS(MMRSS) and MRSS(MMRSS) for m= 2 relative to SRS

Distribution		**RSS**	**DRSS**	**ERSS**	**DERSS**	**MMRSS**	**RSS (MMRSS)**	**MRSS (MMRSS)**
Symmetric	*U(0,1)*	1.4966	1.9222	1.4966	1.9227	1.0000	1.3324	1.7133
	N(0,1)	1.4707	1.7918	1.4721	1.7888	1.0034	1.3161	1.7268
	Logistic(5,2)	1.4353	1.7031	1.4337	1.7069	1.0016	1.2988	1.7371
	Studentt(4)	1.3822	1.582	1.3792	1.5644	0.9976	1.2597	1.7667
	Beta(3,3)	1.4864	1.8511	1.4859	1.8529	0.9980	1.3252	1.7147
	ArcSin(0,1)	1.4891	1.9416	1.4883	1.941	1.0003	1.3288	1.7178
Asymmetric	*Beta(5,2)*	1.4613	1.7948	1.4659	1.7971	1.1046	1.4206	1.8802
	Rayleigh(1)	1.4591	1.7754	1.4557	1.7736	0.9110	1.2113	1.5817
	Half Normal(2)	1.4295	1.7273	1.4312	1.7179	0.8701	1.1585	1.5390
	Exponential(1)	1.3347	1.5135	1.3265	1.5175	0.7979	1.0275	1.4524
	Gamma (2, 3)	1.3951	1.6272	1.3884	1.63	0.8400	1.1044	1.5084
	ChiSquare(3)	1.3701	1.5873	1.3753	1.5863	0.8251	1.0746	1.4866
	LogNormal(0,1)	1.1847	1.2683	1.2017	1.2545	0.7407	0.8591	1.3960
	Pareto(1,3)	1.2396	1.3349	1.2726	1.2642	0.7610	0.9218	1.4917
	Weibull(0.5,1)	1.1364	1.1851	1.1348	1.1908	0.7208	0.8083	1.3646
	Gamma(0.5,1)	1.2562	1.3779	1.2564	1.3751	0.7589	0.9477	1.4114

5 Conclusion

In this article, we have developed two new double-stage RSS schemes which is an extension of MMRSS to improve the efficiency of the mean estimator. A simulation study is conducted to evaluate the performances of the estimators based on the introduced techniques using both symmetric and asymmetric distributions. It is noticed that MRSS(MMRSS) and RSS(MMRSS) provide more flexibility to the experimenter in selecting more representative samples from the population of interest as compared with the other techniques. Moreover, according to the results of simulation studies, the mean estimate based on RSS(MMRSS) and MRSS(MMRSS) techniques are more precise than their counterparts based on MMRSS and SRS techniques in almost all cases. In addition, when estimating the mean of asymmetric population, RSS(MMRSS) and MRSS(MMRSS) techniques provide more precise mean estimates than the mean estimates with RSS technique in many cases. In summary, we recommend using MRSS(MMRSS) technique for estimating the population mean since it is more efficient than SRS, RSS, DRSS, ERSS, DERSS, MMRSS to estimate the population mean for many cases and distributions considered in this article especially for asymmetric cases. A future direction is to find a new Mixed RSS that is more efficient and use less measurements in the sampling procedure.

Table 3: RE comparisons between RSS, DRSS, ERSS, DERSS, MMRSS, RSS(MMRSS) and MRSS(MMRSS) for m= 3 relative to SRS

Distribution		**RSS**	**DRSS**	**ERSS**	**DERSS**	**MMRSS**	**RSS (MMRSS)**	**MRSS (MMRSS)**
Symmetric	*U(0,1)*	2.0064	3.038	2.0075	3.0358	1.3667	2.2579	1.6877
	N(0,1)	1.9149	2.6288	1.916	2.6392	1.2915	2.0286	2.2875
	Logistic(5,2)	1.8446	2.4229	1.8472	2.4268	1.2581	1.8999	2.6402
	Studentt (4)	1.6958	2.0698	1.6913	2.0821	1.1859	1.6635	3.2967
	Beta(3,3)	1.9636	2.819	1.9646	2.8237	1.3270	2.1424	2.0075
	ArcSin(0,1)	1.9715	3.0497	1.9772	3.046	1.3849	2.2835	1.4310
Asymmetric	*Beta(5,2)*	1.9123	2.6692	1.9074	2.67	1.2650	2.0067	2.1438
	Rayleigh(1)	1.9038	2.6221	1.8999	2.6279	1.3277	2.0899	2.0614
	Half Normal(2)	1.8357	2.4976	1.835	2.4996	1.3219	2.0290	1.8837
	Exponential(1)	1.6305	2.0209	1.6373	2.019	1.2503	1.7468	2.0393
	Gamma(2,3)	1.7527	2.2655	1.749	2.2757	1.2899	1.8945	2.0707
	ChiSquare (3)	1.7102	2.182	1.712	2.1776	1.2791	1.8462	2.0560
	LogNormal(0,1)	1.3524	1.481	1.3426	1.4926	1.1543	1.3696	3.0776
	Pareto(1,3)	1.3166	1.3288	1.3176	1.1974	1.1586	1.2607	3.9675
	Weibull(0.5,1)	1.2324	1.3239	1.2383	1.3352	1.0949	1.2595	3.3657
	Gamma(0.5,1)	1.4779	1.7294	1.485	1.7334	1.2007	1.5621	2.1239

Table 4: RE comparisons between RSS, DRSS, ERSS, DERSS, MMRSS, RSS(MMRSS) and MRSS(MMRSS) for m= 4 relative to SRS

Distribution		**RSS**	**DRSS**	**ERSS**	**DERSS**	**MMRSS**	**RSS (MMRSS)**	**MRSS (MMRSS)**
Symmetric	*U(0,1)*	2.4955	4.279	3.1243	8.5793	1.3309	2.2139	1.7020
	N(0,1)	2.3477	3.5303	2.0337	2.6889	1.2170	1.9537	2.2361
	Logistic(5,2)	2.2116	3.1251	1.7088	1.8573	1.1592	1.8121	2.5458
	Studentt(4)	1.9603	2.5052	1.3145	1.0978	1.0786	1.5502	3.1689
	Beta(3,3)	2.4443	3.9108	2.4305	4.1418	1.2682	2.0830	1.9974
	ArcSin(0,1)	2.4519	4.2951	3.8698	18.7027	1.3848	2.2649	1.5080
Asymmetric	*Beta(5,2)*	2.3516	3.6248	2.0916	2.3451	1.3789	2.2171	1.7291
	Rayleigh(1)	2.3189	3.5118	1.9727	2.1198	1.0915	1.7378	2.5023
	Half Normal(2)	2.2375	3.3235	1.7681	1.4304	1.0295	1.6295	2.7976
	Exponential(1)	1.9221	2.5295	1.1727	0.6776	0.8653	1.2937	3.7838
	Gamma(2,3)	2.0929	2.9068	1.4525	1.0178	0.9510	1.4672	3.2858
	ChiSquare(3)	2.0204	2.7455	1.3329	0.857	0.9123	1.3872	3.4711
	LogNormal(0,1)	1.4725	1.6767	0.7538	0.3815	0.7219	0.9301	6.3205
	Pareto(1,3)	1.3785	1.5198	0.6257	0.3414	0.6809	0.8984	8.1993
	Weibull(0.5,1)	1.3297	1.4703	0.6477	0.3056	0.6642	0.8308	6.0323
	Gamma(0.5,1)	1.6978	2.0898	0.908	0.4564	0.7764	1.1096	4.2574

Table 5: RE comparisons between RSS, DRSS, ERSS, DERSS, MMRSS, RSS(MMRSS) and MRSS(MMRSS) for m= 5 relative to SRS

Distribution		**RSS**	**DRSS**	**ERSS**	**DERSS**	**MMRSS**	**RSS (MMRSS)**	**MRSS (MMRSS)**
Symmetric	*U(0,1)*	2.9997	5.6526	3.61	8.1351	1.7609	3.5419	2.0010
	N(0,1)	2.7754	4.447	2.4063	3.2581	1.4913	2.8454	2.9731
	Logistic(5,2)	2.5768	3.815	1.9914	2.2228	1.3742	2.4766	3.5504
	Studentt(4)	2.2187	2.9216	1.4776	1.2534	1.1961	1.9068	4.6130
	Beta(3,3)	2.9092	5.0598	2.8829	4.888	1.6007	3.1826	2.5315
	ArcSin(0,1)	2.923	5.684	4.1515	9.7285	1.8780	3.6578	1.6026
Asymmetric	*Beta(5,2)*	2.7921	4.6471	2.4695	2.7443	1.4303	2.6445	2.9648
	Rayleigh(1)	2.7452	4.4415	2.3123	2.475	1.5776	3.1117	2.2704
	Half Normal(2)	2.6225	4.1563	2.0498	1.6402	1.55263	2.9865	1.7892
	Exponential(1)	2.1785	3.0079	1.3159	0.76488	1.3018	2.1993	1.6300
	Gamma(2,3)	2.4353	3.5592	1.6689	1.1684	1.4314	2.5961	1.9007
	ChiSquare(3)	2.3326	3.3315	1.5213	0.9788	1.3799	2.4409	1.7755
	LogNormal(0,1)	1.5906	1.8014	0.8117	0.4059	1.0056	1.3461	2.1875
	Pareto(1,3)	1.3121	1.4107	0.7087	0.3238	0.9094	1.0643	2.5822
	Weibull(0.5,1)	1.4391	1.5861	0.6879	0.3271	0.8984	1.1596	2.1527
	Gamma(0.5,1)	1.8949	2.4351	1.0084	0.5125	1.1354	1.7571	1.4830

Table 6: RE comparisons between RSS, DRSS, ERSS, DERSS, MMRSS, RSS(MMRSS) and MRSS(MMRSS) for m= 6 relative to SRS

Distribution		**RSS**	**DRSS**	**ERSS**	**DERSS**	**MMRSS**	**RSS (MMRSS)**	**MRSS (MMRSS)**
Symmetric	*U(0,1)*	3.4945	7.1776	5.438	33.5223	1.6895	3.1191	1.5910
	N(0,1)	3.1872	5.4177	2.4024	3.5036	1.3916	2.5404	2.2773
	Logistic(5, 2)	2.9251	4.531	1.7993	1.9321	1.2660	2.2385	2.6826
	Studentt (4)	2.4514	3.3011	1.1919	0.8215	1.0863	1.7226	3.4597
	Beta (3, 3)	3.3872	6.2918	3.2894	7.6771	1.5124	2.8210	1.9618
	ArcSin(0,1)	3.3948	7.2135	8.9368	208.8118	1.8429	3.2523	1.3306
Asymmetric	*Beta(5,2)*	3.2276	5.7262	2.176	1.3144	1.7054	3.3911	1.4473
	Rayleigh(1)	3.1487	5.4148	2.0096	1.2072	1.1635	1.9586	2.9922
	Half Normal(2)	2.9934	5.0233	1.4525	0.5593	1.0270	1.6538	3.7001
	Exponential(1)	2.4435	3.508	0.7506	0.2267	0.7721	1.1645	5.4009
	Gamma(2,3)	2.7332	4.1778	1.0892	0.386	0.9089	1.4286	4.7266
	ChiSquare(3)	2.6362	3.9372	0.9409	0.3081	0.8543	1.3222	5.0963
	LogNormal(0,1)	1.7016	1.9705	0.4528	0.1246	0.5850	0.7857	6.9398
	Pareto(1,3)	1.4395	1.5799	0.4020	0.1092	0.5308	0.6560	7.7401
	Weibull(0.5,1)	1.5155	1.6976	0.3726	0.0976	0.5255	0.6971	4.7264
	Gamma(0.5,1)	2.0909	2.7627	0.5242	0.1453	0.6485	0.9387	4.8699

Table 7: Bias of estimators based on SRS, RSS, DRSS, ERSS, DERSS, MMRSS, RSS(MMRSS) and MRSS(MMRSS) for m= 2 relative to SRS

Distribution		**SRS**	**RSS**	**DRSS**	**ERSS**	**DERSS**	**MMRSS**	**RSS (MMRSS)**	**MRSS (MMRSS)**
Symmetric	*U(0,1)*	-0.00008	-0.00001	-0.00012	0.00004	-0.00018	0.08324	0.08329	0.08315
	N(0,1)	-0.00092	-0.00089	0.00018	-0.00036	-0.00016	0.28188	0.28267	0.28210
	Logistic(5,2)	-0.00112	-0.00047	-0.00155	0.00142	-0.00214	0.99811	0.99860	0.99828
	Studentt(4)	-0.00013	0.00103	0.00083	0.00005	-0.00108	0.36857	0.36850	0.36833
	Beta(3,3)	0.00001	-0.00002	0.00004	-0.00007	0.00005	0.05412	0.05411	0.05417
	ArcSin(0,1)	-0.00001	-0.00004	-0.00024	-0.00018	0.00008	0.10137	0.10140	0.10152
Asymmetric	*Beta(5,2)*	-0.00013	-0.00015	0.00003	0.00000	0.00006	0.04496	0.04491	0.04497
	Rayleigh(1)	0.00021	-0.00079	0.00010	-0.00017	0.00004	0.18319	0.18330	0.18406
	Half Normal(2)	-0.00021	-0.00048	0.00007	-0.00019	0.00002	0.10375	0.10340	0.10365
	Exponential(1)	-0.00063	0.00008	0.00051	0.00077	-0.00121	0.25029	0.24988	0.24967
	Gamma(2,3)	0.00605	0.00008	-0.00460	-0.00162	0.00128	1.12255	1.12489	1.12557
	ChiSquare(3)	0.00059	-0.00150	-0.00160	-0.00034	-0.00277	0.63844	0.63624	0.63564
	LogNormal(0,1)	-0.00093	-0.00073	-0.00111	-0.00155	0.00065	0.42844	0.42985	0.42898
	Pareto(1,3)	0.00028	-0.00041	-0.00010	0.00059	-0.00029	0.14970	0.14873	0.14977
	Weibull (0.5,1)	0.00224	-0.00220	-0.00132	0.00229	-0.00186	0.74442	0.74987	0.75360
	Gamma(0.5,1)	-0.00005	0.00055	-0.00074	0.00050	-0.00032	0.15943	0.15901	0.15939

Table 8: Bias of estimators based on SRS, RSS, DRSS, ERSS, DERSS, MMRSS, RSS(MMRSS) and MRSS(MMRSS) for m= 3 relative to SRS

Distribution		**SRS**	**RSS**	**DRSS**	**ERSS**	**DERSS**	**MMRSS**	**RSS (MMRSS)**	**MRSS (MMRSS)**
Symmetric	*U(0,1)*	0.00004	0.00024	-0.00013	-0.00008	-0.00017	-0.02749	-0.02775	-0.03079
	N(0,1)	0.00026	0.00049	-0.00003	0.00093	0.00071	-0.09381	-0.09430	-0.08946
	Logistic(5,2)	0.00048	0.00408	-0.00135	-0.00129	-0.00284	-0.33054	-0.33309	-0.29816
	Studentt(4)	-0.00040	0.00008	0.00089	-0.00086	-0.00023	-0.12154	-0.12285	-0.10323
	Beta(3,3)	-0.00002	0.00007	0.00007	0.00009	0.00001	-0.01787	-0.01813	-0.01816
	ArcSin(0,1)	0.00033	-0.00002	0.00013	-0.00007	-0.00002	-0.03389	-0.03395	-0.04141
Asymmetric	*Beta* (5, 2)	0.00006	0.00008	-0.00002	0.00011	-0.00006	-0.01677	-0.01687	-0.00346
	Rayleigh(1)	0.00005	-0.00039	0.00035	0.00026	-0.00010	-0.05460	-0.05434	-0.10393
	Half Normal(2)	0.00053	0.00000	-0.00001	-0.00008	-0.00022	-0.02769	-0.02789	-0.07743
	Exponential(1)	-0.00011	-0.00042	0.00004	-0.00054	0.00003	-0.05519	-0.05467	-0.25000
	Gamma (2, 3)	0.00110	0.00049	0.00242	0.00082	-0.00175	-0.28551	-0.28641	-0.90486
	ChiSquare(3)	-0.00029	0.00072	-0.00060	0.00072	-0.00023	-0.15335	-0.15357	-0.55932
	LogNormal(0,1)	0.00029	0.00121	-0.00063	0.00187	0.00047	-0.07645	-0.07561	-0.51981
	Pareto(1,3)	0.00069	0.00093	0.00067	-0.00039	-0.00037	-0.02488	-0.02419	-0.19155
	Weibull(0.5,1)	0.00241	-0.00026	0.00037	0.00166	-0.00305	-0.09291	-0.09235	-1.13682
	Gamma(0.5,1)	-0.00023	-0.00001	0.00010	0.00017	0.00034	-0.02856	-0.02833	-0.19914

Table 9: Bias of estimators based on SRS, RSS, DRSS, ERSS, DERSS, MMRSS, RSS(MMRSS) and MRSS(MMRSS) for m= 4 relative to SRS

Distribution		**SRS**	**RSS**	**DRSS**	**ERSS**	**DERSS**	**MMRSS**	**RSS (MMRSS)**	**MRSS (MMRSS)**
Symmetric	*U(0,1)*	-0.00002	0.00007	0.00004	-0.00004	0.00007	0.05412	0.05406	0.07585
	N(0,1)	0.00016	0.00003	-0.00032	0.00021	0.00007	0.18667	0.18704	0.22742
	Logistic (5, 2)	-0.00129	0.00108	0.00025	-0.00012	0.00231	0.66592	0.66483	0.76858
	Studentt(4)	-0.00014	-0.00058	0.00036	0.00012	0.00076	0.24679	0.24764	0.26650
	Beta(3,3)	0.00015	-0.00001	0.00003	-0.00013	-0.00002	0.03557	0.03552	0.04567
	ArcSin(0,1)	0.00018	-0.00007	-0.00001	0.00001	-0.00005	0.06523	0.06476	0.09871
Asymmetric	*Beta(5,2)*	0.00009	0.00000	-0.00002	-0.01093	-0.02732	0.02544	0.02551	0.04706
	Rayleigh(1)	0.00011	0.00005	-0.00005	0.04193	0.10617	0.13667	-0.02699	0.11360
	Half Normal(2)	-0.00006	0.00001	-0.00027	0.04104	0.10319	0.08360	0.08362	0.04964
	Exponential(1)	-0.00001	-0.00022	-0.00028	0.16617	0.42143	0.22928	0.22887	0.04499
	Gamma(2,3)	0.00133	-0.00033	-0.00016	0.52699	1.33848	0.94273	0.94652	0.42146
	ChiSquare(3)	-0.00122	-0.00025	-0.00058	0.34528	0.87741	0.55286	0.55436	0.19263
	LogNormal(0,1)	0.00100	0.00083	-0.00152	0.39724	1.03745	0.43919	0.43977	-0.07585
	Pareto(1,3)	-0.00061	0.00027	-0.00016	0.14990	0.39530	0.15847	0.15735	-0.04079
	Weibull(0.5,1)	0.00134	0.00283	-0.00085	0.94605	2.47466	0.86917	0.87482	-0.42886
	Gamma(0.5,1)	-0.00005	0.00009	0.00020	0.14753	0.37368	0.16219	0.16239	-0.01599

Table 10: Bias of estimators based on SRS, RSS, DRSS, ERSS, DERSS, MMRSS, RSS(MMRSS) and MRSS(MMRSS) for m= 5 relative to SRS

Distribution		**SRS**	**RSS**	**DRSS**	**ERSS**	**DERSS**	**MMRSS**	**RSS (MMRSS)**	**MRSS (MMRSS)**
Symmetric	*U(0,1)*	-0.00001	-0.00003	-0.00008	-0.00018	0.00010	-0.02345	-0.02335	-0.03274
	N(0,1)	0.00019	-0.00006	-0.00019	-0.00013	-0.00009	-0.08309	-0.08299	-0.09191
	Logistic(5,2)	-0.00110	0.00033	-0.00103	-0.00230	0.00328	-0.30150	-0.29963	-0.30384
	Studentt(4)	-0.00063	-0.00003	0.00009	-0.00040	0.00054	-0.11268	-0.11226	-0.10324
	Beta(3,3)	-0.00001	0.00010	-0.00001	0.00005	0.00003	-0.01574	-0.01569	-0.01892
	ArcSin(0,1)	-0.00002	0.00005	-0.00007	0.00007	-0.00002	-0.02738	-0.02740	-0.04472
Asymmetric	*Beta(5,2)*	-0.00004	0.00005	-0.00001	-0.00978	-0.02347	-0.01950	-0.01948	-0.00122
	Rayleigh(1)	0.00012	-0.00012	0.00001	0.03783	0.09201	-0.02928	-0.02886	-0.11477
	Half Normal(2)	0.00036	0.00021	0.00010	0.03680	0.08852	-0.00609	-0.00598	-0.08753
	Exponential(1)	-0.00073	0.00059	-0.00022	0.15013	0.36428	0.02291	0.02351	-0.28787
	Gamma(2,3)	0.00022	0.00156	-0.00134	0.47841	1.15874	-0.02379	-0.02232	-1.02586
	ChiSquare(3)	-0.00088	0.00038	0.00006	0.31287	0.75886	0.01402	0.01528	-0.63905
	LogNormal(0,1)	0.00108	0.00055	0.00116	0.36418	0.92557	0.09906	0.10043	-0.59040
	Pareto(1,3)	-0.00051	0.00070	-0.00019	0.13753	0.35606	0.04073	0.04110	-0.21705
	Weibull(0.5,1)	0.00135	-0.00215	-0.00005	0.86815	2.20853	0.31139	0.31430	-1.29644
	Gamma(0.5,1)	-0.00003	0.00003	-0.00045	0.13282	0.32341	0.03802	0.03846	-0.23126

Table 11: Bias of estimators based on SRS, RSS, DRSS, ERSS, DERSS, MMRSS, RSS(MMRSS) and MRSS(MMRSS) for m= 6 relative to SRS

Distribution		**SRS**	**RSS**	**DRSS**	**ERSS**	**DERSS**	**MMRSS**	**RSS (MMRSS)**	**MRSS (MMRSS)**
Symmetric	*U(0,1)*	0.00008	0.00002	-0.00007	-0.00006	-0.00003	0.04012	0.04004	0.07100
	N(0,1)	-0.00002	-0.00017	0.00008	-0.00028	-0.00033	0.14119	0.14214	0.20463
	Logistic(5,2)	0.00091	-0.00014	-0.00040	-0.00102	-0.00202	0.51202	0.51003	0.68195
	Studentt(4)	-0.00027	0.00032	0.00003	-0.00020	0.00015	0.19114	0.19178	0.23342
	Beta(3,3)	0.00013	-0.00002	-0.00002	0.00002	-0.00005	0.02682	0.02677	0.04169
	ArcSin(0,1)	-0.00024	0.00008	0.00000	-0.00007	0.00000	0.04767	0.04731	0.09494
Asymmetric	*Beta(5,2)*	0.00010	0.00007	0.00002	-0.02015	-0.04882	0.01356	0.01356	0.04630
	Rayleigh(1)	-0.00019	-0.00023	0.00005	0.07742	0.19645	0.12514	0.12527	0.09010
	Half Normal(2)	-0.00003	-0.00008	-0.00006	0.07562	0.18639	0.08408	0.08432	0.03222
	Exponential(1)	-0.00042	-0.00024	-0.00059	0.30851	0.77541	0.26089	0.26114	-0.01411
	Gamma(2,3)	-0.00011	-0.00018	-0.00006	0.27205	0.76433	0.21220	0.21176	-0.06322
	ChiSquare(3)	0.00031	-0.00072	-0.00070	0.63994	1.61526	0.60020	0.60003	0.06135
	LogNormal(0,1)	0.00044	-0.00094	0.00059	0.74430	2.05009	0.54727	0.54931	-0.19507
	Pareto(1,3)	-0.00011	-0.00018	-0.00006	0.28205	0.79433	0.20220	0.20176	-0.08322
	Weibull(0.5,1)	-0.00063	-0.00137	0.00057	1.77534	4.90301	1.17670	1.17514	-0.69498
	Gamma(0.5,1)	0.00032	0.00022	0.00010	0.27268	0.68809	0.20116	0.20149	-0.06459

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