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Acceptance sampling plans based on truncated life tests in the inverse-Gamma model

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In this article, acceptance sampling plans are suggested for the life test truncated at a preassigned time. The lifetime of the test units are assumed to follow the Inverse-Gamma distribution. The minimum sample size necessary to ensure the specified mean life is obtained. The operating characteristic function values of the proposed sampling plans and producers risk are provided. Our procedure is explained by numerical examples.

keywords: Acceptance sampling plans; consumers risk; Inverse-Gamma distribution; operating characteristic; Producers risk.

1 Introduction

Acceptance sampling (AS) is a survey procedure applied in statistical quality control. There are several research fields considering AS such as ecology, agriculture, industry, management and quality maintenance. Applying AS in industrial is helping the managers as well as the decision-making process for the purpose of selecting a high standard quality of a given item. There are several types of sampling plans, Single, Double, Sequential and so on. In the single AS plan, which will be considered in this article; the main problem is establishing the minimum sample size that should be used for testing a lot of items based on a randomly selected sample. It is implicitly assumed in the usual single sampling plan that only a single item is put in a tester. On the basis of quality standards information that pre-specified, we accept or reject the lot. If a good lot is rejected on the basis of the obtained information, its probability is called the type-I error (producers risk, α). On the other hand, the probability of accepting a bad lot is known as type-II error (consumers risk, β). In general, we put a random sample on test and accept the entire lot if no more than c(AS number) failures occur during the experiment time (Al-Nasser and Gogah, 2017 ; Gogah and Al-Nasser, 2018).

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The problem we are considering is that of finding the minimum sample size necessary to ensure a certain average life when the life test is terminated at a preassigned time (t) and when number of failures does not exceed a given acceptance number (c). The lot is accepted if the specified life can be established with preassigned probability (P^*) specified by the consumer. The single AS of life tests is discussed by Srinivasa et al. (2012) for the inverse Rayleigh model. Aslam and Jun (2013) applied the two-point approach to the designing of acceptance sampling plans based on a truncated life test for various life distributions and log-logistic distributions. Baklizi (2003) considered the Pareto model of the second kind. Baklizi and El Masri (2004) considered the Birnbaum Saunders model. Baklizi et al. (2005) considered the Rayleigh model. Balakrishnan et al. (2007) generalized BirnbaumSaunders distribution. Al-Omari (2014) developed the acceptance sampling plans based on truncated lifetime when the life of an item follows a generalized inverted exponential distribution. Al-Nasser and Al-Omari (2013) considered the exponentiated Frchet distribution. Al-Omari (2014) applied the three parameters Kappa distribution. Aslam et al. (2010) proposed variables sampling plan for life testing in a continuous process under Weibull distribution. Al-Omari et al. (2016) proposed a two points acceptance sampling method were used to draw a decision on accepting or rejecting a tested product. It is also, assumed that the life time product following a new distribution that formulated based on Weibull and Pareto life time distributions that it is known as new Weibull-Pareto (NWP) distribution. Ramaswamy and Jayasri (2013) and Ramaswamy and Jayasri (2014) suggested time truncated chain sampling plan for Marshall-Olkin extended exponential and generalized Rayleigh distribution.
In this article, a new single sampling plan based on inverse gamma distribution will be developed. The Inverse Gamma distribution is given in section 2. The proposed sampling plan and the operating characteristic (OC) are given in section 3. The results and a descriptive examples are made in section 4.

2 Inverse Gamma distribution

The Inverse Gamma (IG) distribution with parameters α and σ , is mentioned infrequently in statistical literature, and usually for a specific purpose. It is useful in many problems of diffraction theory and corrosion problems in new machines. Also, it is most often used as a conjugate prior distribution in Bayesian statistics. Some other applications and motivations for this model can be found in Mead (13).
Glen (2011) gave number of properties that are useful when considering the IG as a lifetime of a product. An inverse gamma random variable T can be derived by transforming a random variable Y $\sim$ gamma(α, σ) with the multiplicative inverse, i.e., $T = 1/Y$. The gamma PDF is

$$f(y;\alpha,\sigma) = \frac{\sigma^{\alpha}}{\Gamma(\alpha)} y^{\alpha-1} exp(-\sigma y), \quad y > 0,\ \alpha,\ \sigma > 0, \tag{1}$$

Therefore, $T \sim$ IG(α, σ), has the following PDF, CDF, survivor function, and hazard function

$$f\left(t;\alpha,\sigma\right) = \frac{\sigma^{\alpha}}{\Gamma\left(\alpha\right)} t^{-\alpha-1} exp\left(-\frac{\sigma}{t}\right),\ t > 0 \tag{2}$$

$$F\left(t;\alpha,\sigma\right) = \frac{\Gamma\left(\alpha,\frac{\sigma}{t}\right)}{\Gamma\left(\alpha\right)},\ t > 0 \tag{3}$$

$$S\left(t;\alpha,\sigma\right) = 1 - \frac{\Gamma\left(\alpha,\frac{\sigma}{t}\right)}{\Gamma\left(\alpha\right)},\ t > 0 \tag{4}$$

$$h\left(t;\alpha,\sigma\right)=\frac{f\left(t\right)}{1-F\left(t\right)}=\frac{\sigma^{\alpha}}{\Gamma\left(\alpha,\frac{\sigma}{t}\right)}t^{-\alpha-1}exp\left(-\frac{\sigma}{t}\right),\ t>0 \tag{5}$$

where, the shape parameter is α, and the scale parameter is $\sigma > 0$. Recall that $\Gamma(\cdot)$ is the Euler gamma function, and $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function.
Furthermore, the rth moment about the origin is

$$E\left(T^{r}\right)=\frac{\sigma^{r}\Gamma\left(\alpha-r\right)}{\Gamma\left(\alpha\right)},\ \alpha>r,\ r=1,2,.... \tag{6}$$

Therefore, the mean and variance are

$$E(T)=\mu=\frac{\sigma}{\alpha-1},\alpha>1\ and\ Var(T)=\frac{\sigma^{2}}{(\alpha-1)(\alpha-2)^{2}},\alpha>2 \tag{7}$$

3 The sampling plans

A common practice in life testing is to terminate the life test by a pre-determined time t and note the number of failures. One of the objectives of these experiments is to set a lower confidence limit on the average life. It is then desired to establish a specified average life with a given probability of at least P^* (probability of rejecting a bad lot). The decision to accept the specified average life occurs if and only if the number of observed failures at the end of the fixed time t does not exceed a given number c- called the acceptance number (maximum number of allowable bad items to accept the lot). The test may get terminated before the time t is reached when the number of failures exceeds c in which case, the decision is to reject the lot. For such a truncated life test and the associated decision rule, we are interested in obtaining the smallest sample size necessary to achieve the objective. The acceptance sampling plan under a truncated life test is to set up the minimum sample size n for this given acceptance number c such that the consumers risk, the probability of accepting a bad lot, does not exceed $1-P^*$.
A sampling plan consists of

n: The number of units on test.

c: The acceptance number.

t/σ_0, where σ_0 is the specified average life.

The consumer's risk, i.e., the probability of accepting a bad lot (the one for which the true average life is below the specified life σ_0) not to exceed $1-P^*$, so that P^* is a minimum confidence level with which a lot of true average life below σ_0 is rejected, by the sampling plan. For a fixed P^* our sampling plan is characterized by $(n, c, t/\sigma_0)$. Here we consider sufficiently large lots so that the binomial distribution can be applied. The problem is, for given values of P^* $(0 < P^* < 1)$, σ_0 and c the smallest positive integer n is to be determined such that

$$\sum_{k=0}^{c}\binom{n}{k}p^{k}\left(1-p\right)^{n-k}\leq 1-P^{*} \tag{8}$$

Where $p = F\left(t;\sigma_0\right)$ indicates the failure probabilities before time t, which depends only on the ratio t/σ_0, it is sufficient to specify this ratio for designing the experiment and is a monotonically increasing function of t/σ_0 and known as the probability of a failure observed during the time t.
If the number of observed failures within the time t is at most c, then from Inequality (8) we can

confirm with probability P^* that $F(t;\sigma) \leq F(t;\sigma_0)$, which implies $\sigma_0 \leq \sigma$.
The minimum sample size that satisfy above inequality for $t/\sigma_0 = 0.942$, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712, with $P^* = 0.75$, 0.9, 0.95, 0.99 and $c = 0$, 1, 2,..., 10 are presented in Table 1 for $\alpha = 1$. The values of t/σ_0 and P^* presented in this work are the same with the corresponding values of many authors (Baklizi and El Masri, 2004; Kantam et al., 2001 and Gupta and Groll, 1961).
For the sampling plan $(n, c, t/\sigma_o)$, the probability of accepting the lot (operating characteristic function OC) is given by

$$L(p) = \sum_{k=0}^{c} \binom{n}{k} p^k (1-p)^{n-k} \tag{9}$$

Where $p = F(t;\sigma)$ $(\alpha = 1)$ considered as a function of σ (the lot quality parameter), and values of the $L(p)$ is a function of σ/σ_o for some selected sampling plan are given in Table 2.
The producers risk is defined as the probability of rejecting the lot when $\sigma_0 \leq \sigma$. For a given value of the producers risk , say 0.05, we are interested in knowing the value of σ/σ_0 to ensure the producers risk is less than or equal to 0.05. This value of σ/σ_o is the smallest number for which $F\left(\frac{t}{\sigma_o}\frac{\sigma_o}{\sigma}\right)$ satisfies the inequality

$$\sum_{k=0}^{c} \binom{n}{k} p^k (1-p)^{n-k} \geq 0.95 \tag{10}$$

For a given sampling plan $(n, c, t/\sigma_o)$ at specified confidence level P^*, the minimum values of σ/σ_0 satisfying Equation (10) are computed and presented in Table 3.

4 Illustration of the tables and example for test plan

Assume that the life distribution is an inverse Gamma distribution and the experimenter is interested in showing that the true unknown average life is at least 1000 hours. Let the consumers risk be set to $1 - P^* = 0.05$. It is desired to stop the experiment at $t = 942$ hours. Then for an acceptance number $c = 2$, the required n is the entry in Table 1 is 16. If during 942 hours no more than 2 failures out of 16 are observed then the experimenter can assert with a confidence level of 0.95 that the average life is at least 1000 hours. For the sampling plan $(n = 16, c = 2, t/\sigma_o = 0.942)$, the operating characteristic values from Table 2 are follows.

σ/σ_0	2	4	6	8	10	12
t/σ_0	0.70171	0.99857	1.00000	1.00000	1.00000	1.00000

This means that if the true mean life is twice the specified mean life, then the producers risk is about 0.30 while it is about zero when the true mean life is 6 times the specified mean life.
Table 3 used to get the value of σ/σ_0 for various values of $c, t/\sigma_0$ when the producers risk may not exceed 0.05. For example, for $c = 2, t/\sigma_0 = 0.942$, $P^* = 0.95$, the value of σ/σ_0 is 2.77. This means that the product should have an average life of 2.77 times the specified average life if 1000 hours in order that the product be accepted with probability 0.95.

Table 1: Minimum sample size n necessary to assert the average life to exceed a given value σ_o with probability P^* and the corresponding acceptance number c

		t/σ_0							
P*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.712
0.75	0	7	4	3	2	2	2	1	1
	1	13	7	6	5	4	3	3	3
	2	19	11	8	7	5	5	4	4
	3	25	14	11	9	7	6	6	5
	4	30	17	13	11	9	8	7	7
	5	36	21	16	13	10	9	9	8
	6	41	24	18	15	12	11	10	9
	7	47	27	20	17	14	12	11	11
	8	52	30	23	19	15	14	13	12
	9	58	33	25	21	17	15	14	13
	10	63	37	28	23	19	17	15	15
0.9	0	11	6	4	4	3	2	2	2
	1	18	10	8	6	5	4	4	3
	2	25	14	10	9	7	6	5	5
	3	31	18	13	11	9	7	7	6
	4	38	21	16	13	10	9	8	8
	5	44	25	19	16	12	11	10	9
	6	50	28	21	18	14	12	11	11
	7	56	32	24	20	16	14	13	12
	8	62	35	27	22	17	15	14	13
	9	68	39	29	24	19	17	15	15
	10	73	42	32	27	21	18	17	16
0.95	0	14	8	5	4	3	3	2	2
	1	22	12	9	7	6	5	4	4
	2	29	16	12	10	8	7	6	6
	3	36	20	15	12	10	8	7	7
	4	43	24	18	15	11	10	9	8
	5	49	28	21	17	13	12	11	10
	6	56	32	23	20	15	13	12	11
	7	62	35	26	22	17	15	13	13
	8	68	39	29	24	19	16	15	14
	9	74	42	32	26	20	18	16	16
	10	80	46	34	29	22	19	18	17
0.99	0	21	11	8	7	5	4	4	3
	1	30	17	12	10	7	6	6	5
	2	38	21	16	13	10	8	7	7
	3	46	26	19	15	12	10	9	8
	4	54	30	22	18	14	12	11	10
	5	61	34	25	21	16	13	12	11
	6	68	38	28	23	18	15	14	13
	7	74	42	31	26	20	17	15	14
	8	81	46	34	28	21	19	17	16
	9	88	49	37	30	23	20	18	17
	10	94	53	39	33	25	22	20	19

Table 2: Operating characteristic values for the sampling plan $(n, c, t/\sigma_o)$ for a given P^* when $c = 2$

			σ/σ_0					
P*	n	t/σ_0	2	4	6	8	10	12
0.75	19	0.628	0.95816	1	1	1	1	1
	11	0.942	0.86424	0.99956	1	1	1	1
	8	1.257	0.78871	0.99658	0.99997	1	1	1
	7	1.571	0.69199	0.98675	0.99965	0.99999	1	1
	5	2.356	0.63335	0.95425	0.99574	0.99964	0.99997	1
	5	3.141	0.44572	0.86253	0.97433	0.99574	0.99933	0.9999
	4	3.972	0.51719	0.85844	0.96408	0.99145	0.99803	0.99955
	4	4.712	0.42969	0.7872	0.9307	0.97882	0.99375	0.99819
0.9	25	0.628	0.91715	0.99999	1	1	1	1
	14	0.942	0.76981	0.99905	1	1	1	1
	10	1.257	0.66659	0.99312	0.99993	1	1	1
	9	1.571	0.51718	0.97176	0.9992	0.99998	1	1
	7	2.356	0.36075	0.87979	0.98677	0.99881	0.9999	0.99999
	6	3.141	0.29104	0.78067	0.95426	0.99198	0.99871	0.9998
	5	3.972	0.30987	0.74062	0.92487	0.98074	0.99537	0.99892
	5	4.712	0.22879	0.63335	0.86248	0.95425	0.98576	0.99574
0.95	29	0.628	0.88317	0.99998	1	1	1	1
	16	0.942	0.70171	0.99857	1	1	1	1
	12	1.257	0.54526	0.98814	0.99987	1	1	1
	10	1.571	0.43792	0.96197	0.99887	0.99997	1	1
	8	2.356	0.25992	0.8329	0.98006	0.99814	0.99984	0.99999
	7	3.141	0.18177	0.69224	0.92862	0.98678	0.99781	0.99966
	6	3.972	0.17318	0.61598	0.87395	0.96528	0.99129	0.99793
	6	4.712	0.11299	0.48665	0.7806	0.92079	0.97404	0.99198
0.99	38	0.628	0.79275	0.99996	1	1	1	1
	21	0.942	0.53223	0.99678	0.99999	1	1	1
	16	1.257	0.33832	0.97333	0.99969	1	1	1
	13	1.571	0.24954	0.92394	0.99744	0.99994	1	1
	10	2.356	0.12642	0.72819	0.96203	0.99621	0.99967	0.99997
	8	3.141	0.10971	0.6031	0.89804	0.98007	0.9966	0.99946
	7	3.972	0.09206	0.49732	0.81444	0.94518	0.98567	0.99651
	7	4.712	0.0529	0.36075	0.69216	0.87979	0.95858	0.98677

Table 3: Minimum ratio of true mean life to specified mean life for the acceptability of a lot with producers risk of 0.05

		t/σ_0							
P*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.972	4.712
0.75	0	3.09	4.11	5.13	5.78	8.67	11.55	11.9	14.12
	1	2.25	2.77	3.48	4.04	5.49	6.29	7.95	9.43
	2	1.96	2.4	2.77	3.23	3.93	5.23	5.53	6.56
	3	1.81	2.14	2.52	2.8	3.52	4.1	5.19	5.05
	4	1.69	1.97	2.26	2.54	3.26	3.9	4.28	5.07
	5	1.63	1.91	2.18	2.36	2.81	3.35	4.23	4.32
	6	1.57	1.82	2.03	2.22	2.72	3.3	3.71	3.76
	7	1.53	1.75	1.93	2.12	2.65	2.96	3.31	3.92
	8	1.49	1.69	1.9	2.04	2.41	2.96	3.38	3.54
	9	1.47	1.65	1.82	1.97	2.39	2.71	3.09	3.22
	10	1.44	1.64	1.81	1.92	2.36	2.73	2.84	3.37
0.9	0	3.38	4.49	5.49	6.86	9.61	11.55	14.61	17.33
	1	2.46	3.12	3.86	4.35	6.06	7.31	9.25	9.43
	2	2.14	2.64	3.07	3.66	4.83	5.9	6.62	7.85
	3	1.95	2.39	2.75	3.15	4.2	4.69	5.92	6.15
	4	1.85	2.18	2.55	2.83	3.55	4.34	4.93	5.85
	5	1.76	2.08	2.41	2.72	3.32	4.1	4.74	5.02
	6	1.7	1.97	2.25	2.54	3.15	3.63	4.18	4.95
	7	1.65	1.92	2.17	2.4	3.02	3.53	4.12	4.43
	8	1.61	1.85	2.12	2.3	2.76	3.22	3.74	4.01
	9	1.58	1.82	2.02	2.21	2.69	3.18	3.43	4.06
	10	1.54	1.77	1.99	2.2	2.63	2.95	3.45	3.75
0.95	0	3.53	4.76	5.77	6.86	9.61	12.81	14.61	17.33
	1	2.59	3.29	4.02	4.61	6.52	8.08	9.25	10.97
	2	2.23	2.77	3.31	3.84	5.18	6.44	7.46	8.85
	3	2.04	2.49	2.94	3.3	4.47	5.17	5.92	7.03
	4	1.93	2.32	2.71	3.08	3.8	4.73	5.49	5.85
	5	1.83	2.2	2.55	2.82	3.53	4.42	5.19	5.62
	6	1.77	2.11	2.37	2.72	3.33	3.93	4.59	4.95
	7	1.71	2.01	2.28	2.57	3.18	3.79	4.12	4.89
	8	1.67	1.96	2.21	2.45	3.06	3.46	4.07	4.44
	9	1.63	1.89	2.16	2.34	2.83	3.39	3.73	4.43
	10	1.6	1.86	2.07	2.32	2.75	3.14	3.72	4.1
0.99	0	3.78	5.06	6.36	7.73	10.81	13.71	17.33	19.22
	1	2.78	3.63	4.39	5.19	6.91	8.7	11	12.12
	2	2.41	3.03	3.69	4.27	5.75	6.91	8.15	9.66
	3	2.2	2.75	3.26	3.68	4.95	5.96	7.07	7.76
	4	2.07	2.54	2.98	3.38	4.43	5.37	6.41	7.09
	5	1.97	2.39	2.78	3.18	4.07	4.7	5.59	6.15
	6	1.89	2.28	2.63	2.96	3.81	4.44	5.3	5.89
	7	1.83	2.19	2.52	2.85	3.6	4.23	4.79	5.3
	8	1.78	2.12	2.43	2.71	3.32	4.07	4.65	5.18
	9	1.74	2.04	2.35	2.59	3.19	3.77	4.28	4.77
	10	1.7	1.99	2.26	2.54	3.09	3.67	4.21	4.71

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