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Using Markov-Switching models in Italian, British, U.S. and Mexican equity portfolios: a performance test

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In this paper we test the use of Markov Switching models in equity trading strategies, following Brooks and Persand (2001), Kritzman et al. (2012) and Hauptmann et al. (2014), who suggest their use as warning systems of bad performing periods. We extend their reviews by testing again (with the impact of trading fees) the U.S. and U.K. markets and by extending our tests to the Italian and Mexican case. The rationale behind our Markov-Switching strategy is to invest in equity index tracking ETFs in low volatility or "good performing" periods and in the local risk-free asset in high-volatility or "bad performing" ones. Our results show that in a weekly simulation from January 4, 2001 to July 30, 2017 with a 0.35% trading fee plus taxes, our system is useful to create alpha in all the simulated markets even if the Italian case showed several deep distress moments due to a financial or political crisis.

keywords: Markov Switching models; active portfolio management; automated trading strategy; warning systems

1 Introduction

Markov-Switching models are popular in time series analysis by the fact that they model the random behavior of the breaks in different regimes[1] or states in the data sample.

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[1] In statistical terms we mean multiple regimes or states to the number of subsets of data that, although they are part of a bigger stochastic process or time series modeled with a given probability function,

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The rationale behind these time series models is the one of a stochastic process with multimodal elliptic probability density functions, that is, the fact that instead of having data with a single location, scale and shape parameter set, the random behavior of the variable is modeled with a number of parameter sets that are related with a number of regimes and, therefore, with a multishape probability density function (pdf). One particular case is the multimodal Gaussian pdf that could be used in normal-mixture models. The normal-mixture model (for the sake of simplicity and following our purposes in this paper, with two regimes or $k = 2$.) allows a stochastic process to be modeled with two means and two standard deviations in a parameter vector $\theta = [\mu_{k=1}, \mu_{k=2}, \sigma_{k=1}, \sigma_{k=2}]$. Therefore, the normal-mixture probability density function for a vector of observed returns $\mathbf{r} = [r_t]$ is given in the following expression:

$$f(\mathbf{r}, \theta) = \left[\pi_{k=1} \frac{1}{\sqrt{2\pi}\sigma_{k=1}} e^{-\frac{1}{2}\left(\frac{r_t - \mu_{k=1}}{\sigma_{k=1}}\right)} + \pi_{k=2} \frac{1}{\sqrt{2\pi}\sigma_{k=2}} e^{-\frac{1}{2}\left(\frac{r_t - \mu_{k=2}}{\sigma_{k=2}}\right)}\right] \quad (1)$$

The shape of this pdf is the one of a double continuous bell. And the term $\pi_{k=i}$ in (1) is the mixing proportion that the Gaussian pdf of the k-th regime has in the data. This concept leads to a mixing law given by $\mathbf{p} = [\pi_{k=1}, \pi_{k=2}]'$ where the sum of the mixing proportions adds to 1 or 100% of probability, and to the next parameter vector:

$$\theta = [\mu_{k=1}, \mu_{k=2}, \sigma_{k=1}, \sigma_{k=2}, \pi_{k=1}, \pi_{k=2}] \quad (2)$$

The normal-mixture pdf has been studied and used in finance by authors such as Haas et al. (2013), who propose the use of normal mixture GARCH(1,1) models to model the returns of the NASDAQ100 index and to also give an introductory proposal to determine the proper number of regimes in a given time series. Following them, Alexander and Lazar (2006) test a two-regime normal mixture model and a normal skewed one, against the single regime normal, t-Student. These three models were used, as in Haas et al. (2004) with symmetric and asymmetric GARCH(1,1) variances. Their results are in line with the previous work and show that, if they are used in FX rates such as GBPUSD, EURUSD and USDJPY, the normal mixture models have a better performance to model these exchange rates and they also have a better fitting for Value at Risk (VaR) calculations. Following them, Chung (2009) tests a bivariate normal mixture model with a BEKK covariance matrix in spot and future prices of commodities such as wheat and corn in the Chicago Board of Trade[2].The author found that in the short-term (with daily time periods), the standard BEKK-GARCH outperforms the mixture case and the constant correlation GARCH model of Bollerslev (1986) to model the covariance of a hedged portfolio of each commodity. The author also found that, in periods far from $t + 10$, the normal mixture BEKK-GARCH outperforms the other competing models. Later, Haas et al. (2013) study again the usefulness of the normal mixture GARCH models and relax the assumption of constant mixing proportions in (1). This change allows these parameters to change over time with two weighing methods: one relating the weight of the current observation against past ones and

they behave as if they were coming from different probability sub-functions.

[2]Nowadays part of the Chicago Mercantile Exchange (CME).

another one relating the weight of the current log likelihood function with previous realized ones. Their results show that their new model outperforms the non-time varying normal mixture one and find a relationship between the leverage effect of the asymmetric GARCH component and the time varying change of the mixture components. Finally, we can mention the work of Rosales Contreras (2016) who applies the normal-mixture model in several Mexican financial assets and finds evidence in favor of its use. Even though normal mixture models (and its GARCH extension) are an important advance to calculate the risk (standard deviation) and expected returns of different states[3] or regimes[4], these models are not good to tell if the actual period is a good performing or a bad performing (please refere to footnotes in this page) one and are also less useful to tell, in future periods $(t + q)$, the probability that the current regime will be the same or not. In order to solve this, Hamilton (1989) proposed a model known in the time series analysis literature as Markov-Switching models (henceforth MS models or simply MS). This model implies that the return r_t at time t could be generated from one of the k stochastic processes or regimes in the time series, a behavior that is modeled with k states (or regimes in terms of this paper) of a Markov chain with a time varying mixing law or probabilities ($\mathbf{P} = [\pi_{k=1,t}, \pi_{k=1,t}]$) and a time fixed transition probability of changing from regime i to regime j $(\pi_{i,j})$:

$$\mathbf{\Pi} = \begin{bmatrix} \pi_{1,1} & \pi_{2,1} \\ \pi_{1,2} & \pi_{2,2} \end{bmatrix} \tag{3}$$

With this in mind, this model (also known as "Hamilton's filter") can tell us if, at the moment t, a financial asset or portfolio is in a "bad performing" ("good performing") period, where higher (lower) volatility levels are usually observed along with possibly uncomfortable ("comfortable") expected return levels. Departing from this, Hamilton's filter can give us a parameter vector similar to (2) and also the transition probability matrix $\mathbf{\Pi}$ of (3) as outputs. The only difference from (2) is the fact that the mixing probabilities in $\mathbf{P}$ are not fixed for all the time series but filtered through the data and smoothed over time as Hamilton (1989, 2005) and Kim (1994) propose. This leads to use instead of $\mathbf{P}$ in (2), a $T \times K$ matrix $\mathbf{SP}$ with the smoothed probability $\pi_{k=i,t}$ of being in regime k at time t in each column vector:

$$\mathbf{SP} = [[\pi_{K=1,t}], [\pi_{K=2,t}]] = [\xi_{k=1}, \xi_{k=2}] \tag{4}$$

Based on this feature, the difference with the mixture probabilities given in (2) is the fact that the probability of being in regime k is changing over time and can be estimated for $t + q$ periods ahead with the following expression:

$$\mathbf{P_{t+q}} = [\pi_{k=1,t+q}, \pi_{k=2,t+q}] = [\xi_{k=1,t}, \xi_{k=2,t}]\,\mathbf{\Pi}^q \tag{5}$$

[3] One related to a low standard deviation (known henceforth as normal period or good performing) and one related to a high standard deviation (henceforth crisis periods or bad performing).

[4] The proper term is state instead of regime due to the fact that time series analysis is also widely used in Physics and Biostatistics, but in Economics and Finance the term regime is a synonym of state. We will use this term in order to be consistent with the Economic cycle terminology.

Departing from the previous definitions, the normal-mixture models are a special case of the Markov-switching model due to the fact that in the former, the mixing proportions are changing at t in the MS model and fixed in the normal-mixture one. This leads to the outputs given in the MS model parameters for two regimes:

$$\theta = [\mu_{k=1}, \mu_{k=2}, \sigma_{k=1}, \sigma_{k=2}, \xi_{k=1,t}, \xi_{k=2,t}, \mathbf{\Pi}] \tag{6}$$

Based on this model, it is useful, for active investment management, to model the historical performance of the return on financial assets (r_t), in order to know if the asset (or market) is in a high volatility (standard deviation) or bad performing regime ($k = 1$) or it is in a low volatility (good performing) one ($k = 2$) at t. More specifically, with the use of Hamiltons filter, it would be interesting for the institutional investor to know the current and future volatility regime in order to allocate its capital in a risky asset in good performing periods (that is, the low-volatility regime) or in a risk-free one in bad performing periods (that is, the high-volatility regime). This very basic rationale has been explored in papers such as the one by Perez-Quiros and Timmermann (2000), who use MS models to test the asymmetry of small firms in Economic recession stages. In the same line with our work, we mention the one by Brooks and Persand (2001), who use the bond-equity yield ratio in an MS model and test a very basic rule of investing either in Government bonds or equities with the simulated investment proportions (w_i) in each asset given by the current regime probability vector ($w_i = \pi_{k=i,t}$). Their results applied in the U.K., U.S. and German markets showed an improvement in the performance of the portfolio against a buy and hold strategy. Although they found over performance in their simulated portfolio, the return is not enough to cover the trading costs involved. Following Brooks and testing their cointegration strategy for enhanced index tracking with MS models, Alexander and Dimitriu (2005) found an improvement in the performance of a portfolio that invests in the Dow Jones Industrial Average (DJIA) from January, 1990 to December, 2001. They test Alexanders (1999) enhanced index tracking method and its MS version in two different strategies. The first one invests in the DJIA if the probability of being in the crisis period at t ($\pi_{k=2,t}$) is lower than or equal to 50% (0.50) or sold otherwise. The second shorts the DJIA if that probability is higher than 50%. Their results show a significant improvement in performance but the financial costs are high enough to support their model. We also found the work of Ang and Bekaert (2002, 2004) of interest and closely related to this paper. They use MS models to develop asset allocation activities by incorporating the impact of changing regimes in the asset allocation process of a world equity portfolio (or index). Even though their results ignore the impact of financial costs, they support the benefits of MS models in the asset allocation process with the presence of a risk-free asset against a buy and hold strategy. In addition, we must mention the work of Kritzman et al. (2012), who propose the use of Baum and Welsh algorithm (1970) for MS models to perform forecasts of turbulent times and make some changes or tilts (given the observed regime in market turbulence, recession or inflation historical data) in a starting portfolio of assets such as global stocks and bonds, U.S. stocks and bonds and currencies. Their results are in line[5] with this paper by the fact that the MS strategy reduces downward movements and has no significant financial cost impact. Finally, we mention the work of Hauptmann et al. (2014) who develop a warning system based on MS models and probit regressions with three regimes to determine if a financial market at t is in a bullish (normal), crisis with negative expected return (or crisis bearish) or crisis with positive return periods (or crisis bullish). Their results show an improvement in the asset allocation process with their warning system and also find advances in the research of the driven factors of the regime change process. With this literature review we found no research evidence about the practical usefulness of MS models as warning systems for the Italian and Mexican markets. We also found that only Kritzman et al. (2012) describe the computer algorithm of the MS model that they use. With this in mind, several questions on the use of MS models for active equity portfolio management arise. The first one is: Would the use of these models in emerging markets give the same results as the observed ones in developed economies such as the U.S., the British or the German ones studied in the aforementioned papers? One of the most recent situations that Italy has suffered since 2011 is a financial and political crisis that has affected its Sovereign debt markets and its financial institutions stability. Based on this and despite the fact that Italy is a developed country, would the two-regime MS model be useful for the Italian case as a warning system to sort crisis or poor performing periods by investing in a risk-free asset? In a similar way, would the warning system that we present in this paper be useful for the Mexican equity market as an example of an emerging economy with the most liquid currency (among emerging economies) in the futures markets? We respond to these questions for these four economies from a U.S., a U.K., an Italian and a Mexican investor perspective (in their base currencies) and we test the results in two scenarios with 0.00% and 0.35% trading fees plus added value tax. On the basis of the objectives in the paper, we will describe briefly the main general steps or pseudo-code of our active trading strategy in the next section. In the third section, we describe the data used to simulate, along with the results and findings. Finally, in the fourth section, we present our conclusions and main research guidelines to follow.

2 The use of MS models as part of an active equity management algorithm

So far, we have made a general introduction and review of the rationale of MS models and also their Financial and Econometric applications. It is not our purpose to talk in detail about the different investment policies and market microestructure in the United States, the United Kingdom, Italy and Mexico. That purpose is outside the scope of this paper. We only want to point out that by following all the supporting evidence in favor of MS models against other time series analysis, this model could be the core of an active management that will filter the input data in order to determine if the investor is in a regime of crisis or bad performing or in a normal or good performing one at t. That is, in terms of MS models, we will infer if the current regime at t is $K = 1$y or a $k = 2$

[5] And contrary to the ones of Alexander and Dimitriu (2005) and Brooks and Persand (2001).

respectively. Following this, we propose and simulate a scenario where the fund manager of an institutional investor must decide if she should invest the portfolio proceedings in risky equity assets or in a risk-free one, given the actual volatility regime. The practical application of this rationale and the cost-benefit of its use is the main target in our paper, along with the related pseudo-code and M-files presented as additional material. Therefore, we will proceed to describe the general steps of the computer code used for investment decision-making.

2.1 The computer algorithm for the automated trading strategy with MS models

In this section we will omit the description of the quasi-maximum likelihood algorithm (QSML algorithm) suggested by Hamilton (1990, 1994) for the MS model and we will leave it for a detailed review of the interested reader. We will describe the pseudo-code that we used, which is similar to the one of Brooks and Persand (2001), Kritzman et al. (2012) and Hauptmann et al. (2014). As a starting point, we use a quasi-maximum likelihood (QSML) algorithm[6]. We will do this in our simulations by using Perlins (2013) M-file in our simulation code. For the sake of simplicity, we will assume that the investor lives in a two-regime ($k = 2$) Gaussian world where she could invest her portfolio proceedings in two assets: an index tracking portfolio or RA (a theoretical ETF) in $k = 1$ or good performing times and in a risk-free asset (rf) in $k = 2$ or bad performing ones. Other likelihood functions and number of regimes are left as guidelines for further research. Therefore the pseudo-code that we programmed in our automated trading strategy will follow these steps at each trading date (t):

1. To estimate the parameter vector $\hat{\theta}$ of the Gaussian MS model by using the QSML algorithm described in Hamilton (1990, 1994). The parameter vector will be estimated for the entire time series or information set, or $r = [r_t] = [r_0, r_1, \ldots, r_t]'$, of data available up to t.

2. From the parameter vector $\hat{\theta}$ we will use the last observation of the smoothed probabilities ($\xi_{k=1,t}, \xi_{k=2,t}$) in **SP** for each regime, that is, the ones that determine if an investor is in a good performing or bad performing regime at t respectively.

3. With the smoothed probabilities at t, the next trading rule must de determined:

If $\xi_{k=2,t} > 0.5$
Invest all the portfolio proceedings in rf.
Else
Invest all the portfolio proceedings in RA
End

[6]For more detail of the QSML algorithm, please refer to Hamilton (1989, 1994) to estimate the MS model.

4. Once the trading rule is given, the portfolio management activities such as valuation and rebalancing (if necessary) are executed with the next steps:

   a) To determine the actual portfolio balance PB with the next sum:

$$\begin{aligned} PB = {} & (\#\text{ stocks hold in of } RA \cdot \text{actual price of } RA)+ \\ & (\#\text{ stocks hold in of } rf \cdot \text{actual price of } rf)+ \\ & \text{actual cash balance} \end{aligned} \tag{7}$$

   b) To determine if the trading rule suggests buying the same asset in position by following the next conditional:
   If the actual asset in portfolio is equal to the suggested one (that is, if the trading rule says to buy RA and the actual asset in the portfolio is RA):

      i. To update the value of the portfolio balance (PB) by updating the last market price of the actual asset (either RA or rf) and by using (7).

   Else

      i. To update the value of the portfolio balance (PB) by updating the last market price of the actual asset (either RA or rf) and by using (7).

      ii. To determine the net sale amount (NSA) of the actual position to be sold (either RA or rf), given the financial cost (trading commission) fc and the related tax rate as follows (#stocksell is the number of stocks \$sell is the actual price of the asset to be sold):

$$NSA = \#\text{stocksell} \cdot \left(\frac{\$\text{sell}}{(1+(\text{fc} \cdot (1+\text{tax}\%)))} \right) \tag{8}$$

      iii. Once the net sale amount is calculated, to determine if the portfolio has enough cash to buy the stocks of the new asset by following the next steps:

         A. To determine the net cash in the portfolio ($NCIP$), given the net sell amount determined with (8):

$$NCIP = NSA + \text{actual cash balance} \tag{9}$$

         B. To determine the net buy amount (NBA) of the asset to be bought (#stockbuy and \$buy are the number of stocks and actual price of the asset to be bought respectively):

$$NBA = \#\text{stockbuy} \cdot [\$\text{buy} \cdot (1+(\text{fc} \cdot (1+\text{tax}\%)))] \tag{10}$$

         Where:

$$\#\text{stockbuy} = \frac{NCIP}{[\$\text{buy} \cdot (1+(\text{fc} \cdot (1+\text{tax}\%)))]} \tag{11}$$

C. To run the next conditional:

If $NCIP \geq NBA$

- To buy the number of necessary stocks, given #stockbuy in (11)

Else

- To run the next while operator:
  While $NBA \leq NCIP$
  - To update the number of stocks to be bought
    $\#stockbuy = \#stockbuy - 1$
  - To update NBA with the new value of *#stockbuy* in (10).

  End
- To buy the number of necessary stocks, given the number of stocks *#stockbuy* from the previous step.

End

End

5. **End of algorithm**

It is important to mention that these are the general steps or pseudo-code and we present, in the additional on-line material of this paper, a full Matlab code for used in the back test for practical implementation.

3 Empirical test of the MS automated trading strategy in the United States, the United Kingdom, Italian and Mexican markets

3.1 Simulation data

Once we have presented our active management system with MS models, we will test its practical usefulness in four different equity markets: The United States, The United Kingdom, the Italian and the Mexican ones. We did this because up to the moment of writing the present paper, we did not know about the practical usefulness of the use of MS models in emerging economies such as Mexico and developed ones with observable periods of distress like Italy.
We chose Mexico from a personal interest and due to the fact that it has the second most liquid currency in FX Emerging markets and future markets, according to the Bank for International Settlements (2016). We also chose Italy by the fact that this country has suffered the impact of several financial and political crises that makes it an interesting distress scenario to test our algorithm [7]. Finally, we test the algorithm in the United States and United Kingdom markets as reference to the two

[7] Our aim on this matter is also to contribute with a computer solution for active equity management that could also be of use in their country.

previous ones, and also in order to follow and to review again the results of previous literature on the subject. We also test these two markets because these two countries have the most liquid equity markets and because they have suffered observable financial distress periods such as the ones of the years 2001-2001 and 2007-2008. In order to test our algorithm we used the weekly historical close prices of the main market cap indexes of these four equity markets and theoretical risky (ETFs) and the risk-free assets funds of each country described in Table 1. We used these equity benchmarks or market indices to perform the MS analysis and to determine the probability $\xi_{k=i,t}$ of being in a normal (good performing) or crisis (bad performing) period at t. Given this, the previously described computer algorithm determined to invest either in a theoretical exchange traded fund (ETF) with perfect index tracking[8] (as theoretical assumption) and a starting price of 100 units (USD, GBP, EUR or MXN) at January 1st, 2001. We simulated each market of interest by performing the automated trading systems algorithm described in the previous section each week until July 30, 2017. In order to make the MS analysis to determine $\pi_{k=i,t}$, the computer used the weekly time series of the returns (or percentage variation in the level) of each index in each market with data from t_0 (as specified in table 1 for each market) to the simulated date t.

This simulated process was executed each end of week date (Friday or previous labor day) for a time interval of $t \in [t_0 = \text{January 4th, 2001}, T = \text{July 30, 2001}]$ to perform investment decisions either in the risky or the risk-free asset. The starting value of the simulated portfolio is USD/GBP/EUR/MXN 100,000.00. In order to contemplate the impact of financial costs two scenarios were assumed: one with no trading fees and another one with a trading commission of 0.35% in the risky asset or equity index ETF[9]. To simulate the investment in a risk-free asset, we assumed that the investor buys an overnight banking funding rate mutual fund[10]. with no management costs. Finally we assumed, for the trading fee in the risky asset, a Value Added Tax rate of 10% for the United States, United Kingdom and Italian cases and 16% in the Mexican one. As a final parameter in our simulation we assumed that we could buy the risky asset and the risk-free one at the opening price in the simulated week, that is the close value of the previous week. In order to have a reference or benchmark to test the practical usefulness of our algorithm, we followed Brooks and Persand (2001), Kritzman et al. (2012) and Hauptmann et al. (2014) by testing our simulated portfolios with a buy and hold strategy in the risky asset, that is, against the performance of the market index during the simulation period.

[8] For the sake of simplicity, we will use the 100 base value at January 4, 2001 of each simulated markets benchmark, as the theoretical value of the corresponding ETF.

[9] No other financial cost, such as slippage, are taken into account because we assume to invest in liquid ETFs.

[10] For the United States, the United Kingdom and the Italian cases we will proxy the performance of this fund with a base 100 index that invest in the overnight LIBOR rate in US Dollars, British pounds and Euros correspondingly. For the Mexican case, we will use the S&P-VALMER overnight banking funding index as mentioned in table 1.

Table 1: The benchmarks, the risky and risk-free assets to be used in the three simulated markets

Market	Index	Risky ("good performing" volatility regime) asset	Risk-Free ("bad performing" volatility regime)	Index start date for MS analysis	Start date for portfolio simulation
Italy	FTSE-MIB	A theoretical ETF determined with the base EUR 100 value of the index at Jan. 4th, 2001	A base 100 index benchmark of the overnight USD based LIBOR rate as theoretical banking funding mutual fund	Dec. 31st, 1997	Jan. 4th, 2001
U. S.	S&P500	A theoretical ETF determined with the base USD 100 value of the index at Jan. 4th, 2001	A base 100 index benchmark of the overnight Euro based LIBOR rate as theoretical banking funding mutual fund	Dec. 31st, 1963	Jan. 4th, 2001
U.K.	FTSE 100	A theoretical ETF determined with the base GBP 100 value of the index at Jan. 4th, 2001	A base 100 index benchmark of the overnight GBP based LIBOR rate as theoretical banking funding mutual fund	Feb. 1st, 1987	Jan. 4th, 2001
Mexico	IPC	A theoretical ETF determined with the base MXN 100 value of the index at Jan. 4th, 2001	S&P-VALMER overnight banking funding index as a theoretical banking funding mutual fund	Jan. 2nd, 1987	Jan. 4th, 2001

3.2 Simulation results review

As a starting point we present, in Table **??**, a statistical and performance resume of the four simulated portfolios in each of the two trading fee scenarios. For the U.S. market our simulated portfolio had a cumulative return (please refer to panel a of Table **??**) of 377.16% in the non-trading fee scenario and an accumulated return of 278.92%, net of trading fees and taxes, in the 0.35% trading commission one. This is an important difference against the buy and hold strategy that only paid 85.27% in the same period. With a detailed review in our result[11] and with the support of Figure 1 we note that the difference comes thanks to the sensitivity and the "warning signals" that the algorithm, with the use of the MS model, suggest properly to sell the risky asset and buy the risk-free one with a proper timing. In Figure 1, we placed rectangles in the periods when the probability of being in the second regime (high volatility or crisis period) is higher than 50%. With those rectangles we marked the trading suggestion made by the algorithm (that is, buy the USD overnight LIBOR fund as risk-free asset) and we also marked the performance of the simulated portfolio in the two trading fee scenarios against the S&P500 or buy and hold strategy (panel c of Table **??**).

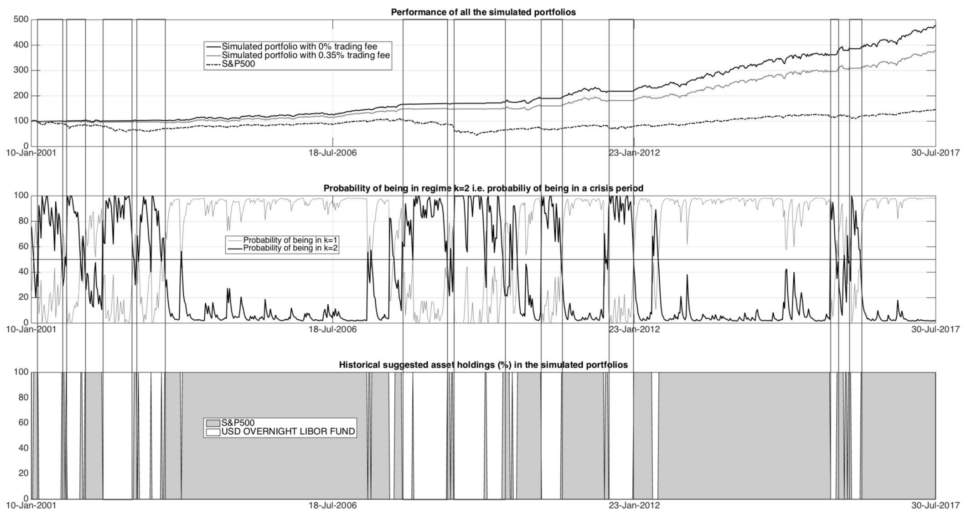

Figure 1: Accumulated return of the simulated portfolios in the U.S. market v.s. the buy and hold strategy.

As noted, the algorithm sent a warning signal (that is, that the investor was in the bad performing period at t) and suggested properly to sell the S&P500 ETF. In the specific case of this simulated portfolio we had financial cost impact on the 0.35% fee scenario

[11] We present, in the supplementary material of this paper and in order to allow the replications of our results, the M-file and the financial data used in our simulations. The supplementary material can also be downloaded from [Mendeley DOI web link for the used data set and codes lies here if the paper is accepted]

Table 2: Accumulated return and main statistics of the three simulated portfolios

Panel a. Turnover, CVaR and mean return in the simulated portfolios given the simulated scenario (expressed in %)					
Country	Commission rate	Cumulative return (%)	mean % CVaR (Prob.=95%)	mean % CVaR (Prob.=98%)	Mean ret.(%)
Italy	0	-12.9533	4.4143	5.1804	-0.8189
	0.35	-16.7150	4.4143	5.1804	-1.0797
U.S.A.	0	377.1672	2.1430	2.5133	9.0538
	0.35	278.9276	2.1430	2.5133	7.7182
U.K.	0	249.9590	3.2420	3.8066	7.5369
	0.35	151.9573	3.2420	3.8066	5.5601
Mexico	0	1,193.3580	6.0630	7.1125	15.4020
	0.35	1,060.9182	6.0630	7.1125	14.7520
Panel b. standard deviation max drawdown and the trading costs paid during the simulations					
Country	Commission rate	Return Std. Dev. (%)	Max Drawdown (%)	Total trading fees paid (EUR, USD, GBP or MXN)	Total taxes Paid (EUR, USD, GBP or MXN)
Italy	0	16.7808	-8.5446	0	0
	0.35	16.5016	-8.0040	10,064.2626	1,006.4262
U.S.A.	0	8.3970	-3.9747	0	0
	0.35	8.4009	-3.9733	32,070.6733	3,207.0673
U.K.	0	11.3662	-6.0650	0	0
	0.35	11.7559	-7.9980	27,477.0691	2,747.7069
Mexico	0	16.2330	-7.9724	0	0
	0.35	29.7134	-42.4301	18,892.2008	1,889.2200

Panel c. Cumulative return (%) of the "buy and hold" strategy in each of the four simulated markets			
FTSE-MIB turnover	-49.6508	S&P500 turnover	85.27158
FTSE100 turnover	19.1800	IPC index turnover	743.4028
EUR LIBOR fund	31.2993	USD LIBOR fund	29.9296
GBP LIBOR fund	34.4946	MXN bank funding rate fund	177.8836

Source: Own elaboration based on data of the Italian stock exchange, the Financial Times, Standard and Poors and the Mexican stock exchange through Reuters datalink and Valmer.

but, contrary to Brooks and Persand (2001), our simulated algorithm and the trading strategy leads to a proper turnover that covers the trading costs and taxes incurred (USD 32,070.67 and USD 3,207.06 respectively as shown in table **??**). Following the United States case, we present the British one with the active management of the FTSE100 ETF and the GBP LIBOR fund as risk-free asset. In panel a of Table **??**, we present an accumulative return of 249.95% in the 0.00% trading fee scenario and one of 151.95% in the 0.35% one. As noted, the financial cost incurred and also contrary to Brooks and Persand (2001) had a notable impact (14.70% and 8.93% of accumulated return in yearly basis respectively) but still show benefits with the presence of trading fees. This active trading accumulative return is higher than the 19.18% of accumulated return of the FTSE100 buy and hold strategy and the 34.94% (2.02% yearly) of the GBP LIBOR fund. The reasons that lead to these accumulated returns are shown in figure 2 for the United Kingdom case.

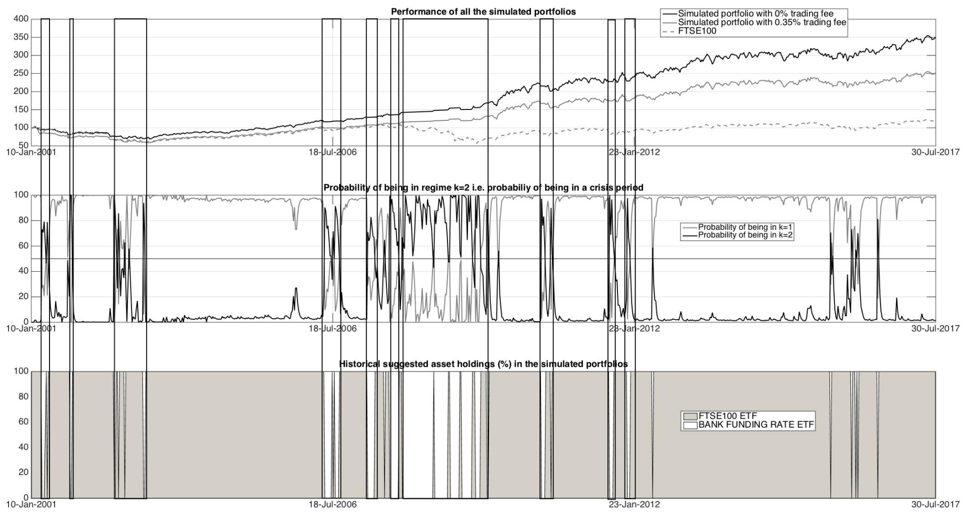

Figure 2: Accumulated return of the simulated portfolios in the U.K. market v.s. the buy and hold strategy.

The results of our simulation for the Italian market were a case of special interest by the fact that this country has suffered several financial, sovereign debt and political situations in 2008, 2011 and almost at the end of the simulated period. As noted in panel a of Table **??** and also in figure 3, this country showed a loss in the buy and hold strategy and another (lesser) loss with the use of the automated trading system given with the algorithm that we present.

Despite this, our automated trading strategy showed better results by paying a -12.95% return before trading fees and taxes and a -16.71% net of these, against a -49.65% turnover of the passive investment strategy (buy and hold) in the FTSE-MIB index. One result of particular interest is the fact that the simulated portfolio in the 0.35%

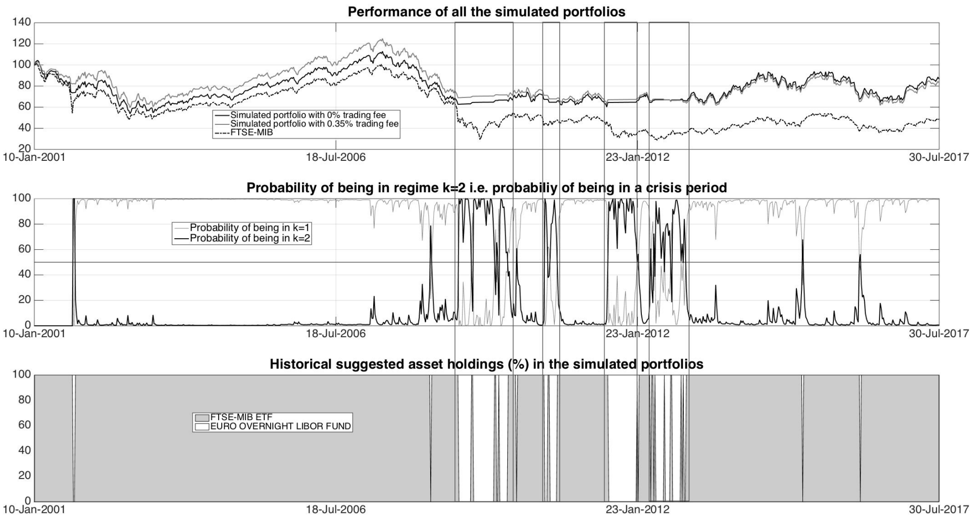

Figure 3: Accumulated return of the simulated portfolios in the Italian market v.s. the buy and hold strategy.

trading fee scenario had moments of over performance against the 0% fees case. After a review of our results we noted that, in this specific market, the trading costs incurred allowed us to buy fewer stocks than in the 0% fees scenario. With a lower exposure to the risky asset in our portfolio, the drawdowns in it were lower in periods of the high-volatility or crisis regime. Another result that arises from our simulations, and as noted in Figure 3, is the fact that, thanks to the trading signals given, we reduced the impact of downward movements in the periods of crisis or high volatility by the fact that the simulated portfolio had invested its proceedings in the EURO denominated overnight LIBOR rate fund (or risk-free asset). Finally, we also found results of interest in the Mexican stock market. This market is notably different to the previous ones and had a notably different performance by the fact that it is an emerging market.

The first result arises from panel a) of Table **??**. The two simulated portfolios (without trading fees and taxes and the one that pays them) paid a 1,193.35% and 1,057.93% respectively. This is a notable difference against the buy and hold strategy in the IPC that paid 743.40%. Even though Mexico was also exposed to internal and external political and financial issues during the simulation period, the number of regime changes is relatively low (the 9-11 influence, the 2006 presidential elections in July and the 2007-2008 U.S. financial crisis). Based on this, Figure 4 shows the performance of the simulated portfolios and the improvements in the performance during the second regime or crisis against the passive strategy are noted. Among the main findings from our simulations and our results review is the fact that the use of two-regime MS models to generate automated trading signs is useful for active investment management. We also found that the impact of financial costs is not high during weekly periods rebalancing

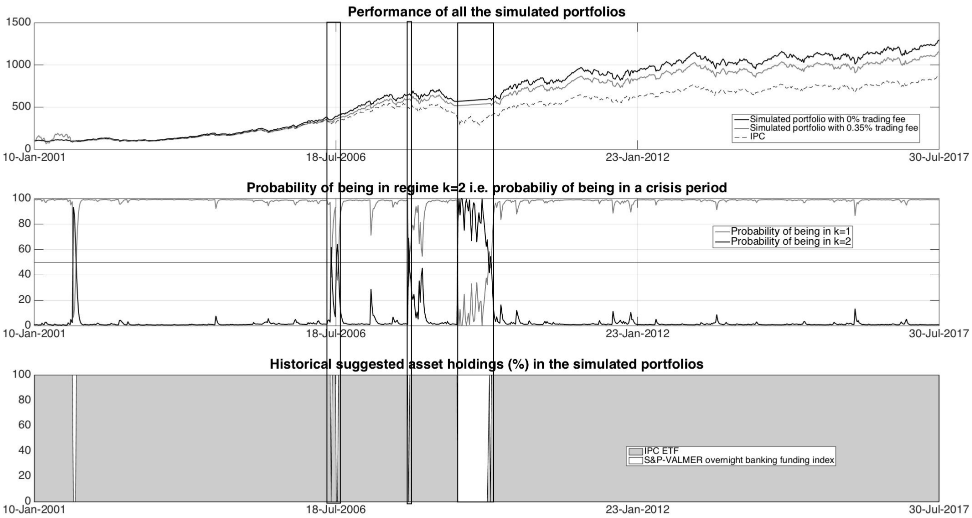

Figure 4: Accumulated return of the simulated portfolios in the Mexican market v.s. the buy and hold strategy.

and analysis. Due to space restrictions, we couldnt run and present the monthly nor daily (or smaller) periodicity simulations. Therefore, we will also leave that issue as a suggested task for further research.

4 Conclusions

In the present paper we describe the pseudo-code for an automated trading strategy that uses univariate Markov-Switching models (Hamilton, 1989, 1990) as the core of the decision-making process, based on the earlier warning systems of Brooks and Persand (2001), Kritzman et al. (2012) and Hauptmann et al. (2014). Following them we propose to use a two regime MS model with one regime for low-volatility or normal periods in the behavior of a financial market and another one for the high-volatility or crisis one. Given our literature review, we noted that the use of MS models has been tested only in U.S., U.K. and German equity and derivative markets and there is no evidence related to the benefits of automated trading strategies that use MS models in Italy and Mexico. Based on the fact that the literature on the subject is focused on the theoretical and practical usefulness of active investment strategies with MS models and that only Kritzman et al. (2012) give a description of the pseudo-algorithm used to estimate the MS model used in their automated trading strategy, we propose our own pseudo-algorithm proposal and test it in the U.S., U.K., Italian and Mexican equity markets. With weekly data from January 4, 2001 to July 31st, 2017 we simulated the use of our algorithm by investing in theoretical ETFs that track the main equity index in each simulated market (as risky asset) and in a theoretical mutual fund that pays the local overnight interbank funding rate as risk-free asset. Our results in the U.S. and Mexican markets show a notable over performance with the use of our automated trading system against a buy and hold passive strategy in their corresponding equity benchmarks. We also found that, with a 0.35% trading fee in the ETF plus taxes, our conclusions are similar, given the fact that the financial costs incurred do not affect or reduce the benefits of using our strategy. For the Italian case we found that even though the Italian market lost value in the simulated period due to financial and political situations, the use of our automated trading strategy would have helped to reduce significantly the lost incurred in the 0% and in the 0.35% trading fee scenarios. With these results we found evidence in favor of using MS models for active management activities in equity markets and we left, as a guideline for further research that will extend our work, the use of the trading strategy in shorter time periods such as day-by-day or even intraday. In addition, the use of this automated trading system in other and more distressed markets would be of interest to probe its practical usefulness, along with the test in more than three regimes as suggested research task.

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