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On the estimation of population variance using auxiliary attribute in absence and presence of non-response

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In this article we have proposed a new class of estimators for estimating the finite population variance using available auxiliary attribute in absence and presence of non-response problem. Properties such as bias and mean square error of the proposed class are derived up to the first order of approximation. It is shown that the proposed class is more efficient than the Singh et al. (1988), Shabbir and Gupta (2007), Singh and Solanki (2013a), usual sample variance and regression estimators.

keywords: mean square error, non-response, attribute, simple random sampling.

1 Introduction

In application, scientists studying with specific data in agriculture, industry, biology and medical studies have faced problems in assessing the finite population variance. For example a doctor needs a full comprehension of variation in the degree of human circulatory strain, body temperature and heartbeat rate for adequate remedy. Similarly, an agriculturist requires sufficient information of climatic variety to devise suitable arrangement for developing his product. A reasonable comprehension of variability is essential

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for better results in different fields of life. For these reasons various authors such as Isaki (1983), Singh et al. (1988), Upadhyaya and Singh (1999), Kadilar and Cingi (2006b), Kadilar and Cingi (2006a), Grover (2010), Singh and Solanki (2013a), Singh and Solanki (2013b), Yadav and Kadilar (2014) and Singh and Pal (2016) have paid their attention towards the enhanced estimation of population variance S_y^2 of the study variable Y in presence of auxiliary information X which is highly correlated with the study variable Y.
Another approach to gain the efficiency of the estimators is to utilize the information of available auxiliary character. Numerous analysts utilized diverse characteristics of the auxiliary character for acquiring better estimate of the variance of a study variable. For more detailed discussion on estimators and its modifications one may refer to Naik and Gupta (1996), Abd-Elfattah et al. (2010), Singh and Malik (2014).

2 Preliminaries and existing estimators

Let $A = \sum_{i=1}^{N} \phi_i$ and $a = \sum_{i=1}^{n} \phi_i$ denote the total number of units in the population $V = \{V_1, V_2, ..., V_N\}$ and in the sample respectively, possessing an auxiliary attribute ϕ. The corresponding population and sample proportions are $P = \frac{A}{N}$ and $p = \frac{a}{n}$, respectively. Let Y_i, $i = 1, ..., N$, denote the value of the i^{th} unit of study variable Y in the population.
To find mean square error of the proposed and existing estimators, let us define following terms

$$\delta_o = \frac{s_y^2 - S_y^2}{S_y^2}, \delta_1 = \frac{s_\phi^2 - S_\phi^2}{S_\phi^2}$$

where $s_y^2 = \frac{\sum_{i=1}^{n}(Y_i-\bar{y})^2}{n-1}$, $s_\phi^2 = \frac{\sum_{i=1}^{n}(\phi_i-p)^2}{n-1}$ are the sample and $S_y^2 = \frac{\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^2}{N-1}$,$S_\phi^2 = \frac{\sum_{i=1}^{N}(\phi_i-P)^2}{N-1}$ are the population variances of study variable and auxiliary attribute respectively.

Using these notations we have

$$E(\delta_1) = E(\delta_o) = 0,$$

$$E(\delta_o^2) = \lambda\beta_2(y)^\diamond = u_{20},\ E(\delta_1^2) = \lambda\beta_2(\phi)^\diamond = u_{02},\ E(\delta_o\delta_1) = \lambda\eta_{22}^\diamond = u_{11},$$

$$\beta_2(y)^\diamond = \beta_2(y) - 1 = \frac{\mu_{40}}{\mu_{20}^2} - 1,\ \beta_2(\phi)^\diamond = \beta_2(\phi) - 1 = \frac{\mu_{04}}{\mu_{02}^2} - 1,$$

$$\eta_{22}^\diamond = \eta_{22} - 1,\ \eta_{22} = \frac{\mu_{22}}{\mu_{20}\mu_{02}},\ \rho^\diamond = \frac{\eta_{22}^\diamond}{\sqrt{\beta_2(\phi)^\diamond\beta_2(y)^\diamond}}$$

where

$$\lambda = \left(\frac{1}{n} - \frac{1}{N}\right),\ \mu_{rk} = N^{-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^r (P_i - P)^k \ and\ u_{rk} = E\left\{\frac{(s_y^2 - S_y^2)^r(s_\phi^2 - S_\phi^2)^k}{(S_y^2)^r(S_\phi^2)^k}\right\}.$$

The usual variance estimator and its variance are given by

$$\hat{\mathfrak{T}}_o = s_y^2, \tag{1}$$

$$Var(\hat{\mathfrak{T}}_o) = S_y^4 u_{20}. \tag{2}$$

The regression estimator for the estimation of s_y^2 is

$$\hat{\mathfrak{T}}_{Reg} = s_y^2 + m^{Reg}\left(S_\phi^2 - s_\phi^2\right) \tag{3}$$

where $m^{Reg} = \dfrac{u_{11}}{u_{02}}\dfrac{S_y^2}{S_\phi^2}$.

The mean square error of $\hat{\mathfrak{T}}_{Reg}$ is given by

$$MSE_{min}(\hat{\mathfrak{T}}_{Reg}) = S_y^4 u_{20}(1-\rho^{\diamond 2}). \tag{4}$$

When S_ϕ^2 is known, Singh et al. (1988) estimator may be given as

$$\hat{\mathfrak{T}}_{ts1} = \mathfrak{m}_1^{ts1} s_y^2 + \mathfrak{m}_2^{ts1}\left(S_\phi^2 - s_\phi^2\right) \tag{5}$$

where $\mathfrak{m}_1^{ts1}$ and $\mathfrak{m}_2^{ts1}$ are derived constants.
The minimum MSE of $\hat{\mathfrak{T}}_{ts1}$ is given by

$$MSE_{min}(\hat{\mathfrak{T}}_{ts1}) = \frac{S_y^4 u_{20}(1-\rho^{\diamond 2})}{[1+u_{20}(1-\rho^{\diamond 2})]} \tag{6}$$

and optimum values are

$$\mathfrak{m}_1^{ts1(opt)} = \left[\frac{u_{02}}{u_{02}+u_{02}u_{20}-u_{11}^2}\right],\ \mathfrak{m}_2^{ts1(opt)} = \left[\frac{u_{11}}{u_{02}+u_{02}u_{20}-u_{11}^2}\right]\frac{S_y^2}{S_\phi^2}.$$

When the variance of the auxiliary attribute is known, Shabbir and Gupta (2007) estimator for S_y^2 will be like that

$$\hat{\mathfrak{T}}_{tg} = \left[\mathfrak{m}_1^{tg} s_y^2 + \mathfrak{m}_2^{tg}\left(S_\phi^2 - s_\phi^2\right)\right] exp\left(\frac{S_\phi^2 - s_\phi^2}{S_\phi^2 + s_\phi^2}\right) \tag{7}$$

where $\mathfrak{m}_1^{tg}$ and $\mathfrak{m}_2^{tg}$ are suitably derived constants.
Grover (2010) calculated the corrected mean square error of $\hat{\mathfrak{T}}_{tg}$ as given by

$$MSE(\hat{\mathfrak{T}}_{tg}) = S_y^4 + \mathfrak{m}_1^{tg^2} A^{\diamond\diamond} + \mathfrak{m}_2^{tg^2} B^{\diamond\diamond} + 2\mathfrak{m}_1^{tg}\mathfrak{m}_2^{tg} C^{\diamond\diamond} - 2\mathfrak{m}_1^{tg} D^{\diamond\diamond} - \mathfrak{m}_2^{tg} E^{\diamond\diamond} \tag{8}$$

where

$$A^{\diamond\diamond} = S_y^4\left[1+u_{20}+u_{02}-2u_{11}\right], \qquad B^{\diamond\diamond} = S_\phi^4 u_{02},$$
$$C^{\diamond\diamond} = S_y^2 S_\phi^2\left\{-u_{11}+u_{02}\right\}, \qquad E^{\diamond\diamond} = S_\phi^2 S_y^2 u_{02},$$
$$D^{\diamond\diamond} = S_y^4\left\{1+\frac{3u_{02}}{8}-\frac{u_{11}}{2}\right\}$$

which is minimum for

$$\mathfrak{m}_1^{tg(opt)} = \left[\frac{B^{\diamond\diamond}D^{\diamond\diamond} - \frac{C^{\diamond\diamond}E^{\diamond\diamond}}{2}}{A^{\diamond\diamond}B^{\diamond\diamond} - C^{\diamond\diamond 2}}\right] \; and \; \mathfrak{m}_2^{tg(opt)} = \left[\frac{-C^{\diamond\diamond}D^{\diamond\diamond} + \frac{A^{\diamond\diamond}E^{\diamond\diamond}}{2}}{A^{\diamond\diamond}B^{\diamond\diamond} - C^{\diamond\diamond 2}}\right].$$

The minimum mean square error of $\hat{\mathfrak{T}}_{tg}$ is given by

$$MSE_{min}(\hat{\mathfrak{T}}_{tg}) = \left[S_y^4 - \frac{B^{\diamond\diamond}D^{\diamond\diamond 2} + \frac{A^{\diamond\diamond}E^{\diamond\diamond 2}}{4} - C^{\diamond\diamond}D^{\diamond\diamond}E^{\diamond\diamond}}{A^{\diamond\diamond}B^{\diamond\diamond} - C^{\diamond\diamond 2}}\right]. \tag{9}$$

Using available information of auxiliary attribute, Singh and Solanki (2013a) class of estimators will be

$$\hat{\mathfrak{T}}_{ss} = \left[\mathfrak{m}_1^{ss}s_y^2 + \mathfrak{m}_2^{ss}(S_\phi^2 - s_\phi^2)\right]\eta_i^{'} \;\; for \;\; i = 1, 2, ..., 8, \tag{10}$$

where

$$\eta_1^{'} = \frac{S_\phi^2}{s_\phi^2}, \eta_2^{'} = \frac{NS_\phi^2 - S_\phi^2}{Ns_\phi^2 - S_\phi^2}, \eta_3^{'} = \frac{NS_\phi^2 - P^2}{Ns_\phi^2 - P^2}, \eta_4^{'} = \frac{nS_\phi^2 - S_\phi^2}{ns_\phi^2 - S_\phi^2}, \eta_5^{'} = \frac{\rho S_\phi^2 - P}{\rho s_\phi^2 - P},$$

$$\eta_6^{'} = \frac{nS_\phi^2 - P^2}{ns_\phi^2 - P^2}, \eta_7^{'} = \frac{n^2S_\phi^2 - P^2}{n^2s_\phi^2 - P^2}, \eta_8^{'} = \frac{n^2S_\phi^2 - S_\phi^2}{n^2s_\phi^2 - S_\phi^2}.$$

The mean square error of $\hat{\mathfrak{T}}_{ss}$ is given by

$$MSE(\hat{\mathfrak{T}}_{ss}) = S_y^4 + \mathfrak{m}_1^{ss^2}A^{''} + \mathfrak{m}_2^{ss^2}B^{''} - 2\mathfrak{m}_1^{ss}\mathfrak{m}_2^{ss}C^{''} - 2\mathfrak{m}_1^{ss}D^{''} - 2\mathfrak{m}_2^{ss}E^{''} \tag{11}$$

where

$$\begin{aligned} A^{''} &= S_y^4\left[1 + \left\{u_{20} + 3\theta^{'2}u_{02} - 4\theta^{'}u_{11}\right\}\right] & B^{''} &= S_\phi^4 u_{02}, \\ C^{''} &= S_y^2S_\phi^2\left\{u_{11} - 2\theta^{'}u_{02}\right\} & E^{''} &= \theta^{'}S_\phi^2S_y^2u_{02}, \\ D^{''} &= S_y^4\left\{1 + \theta^{'}\left(\theta^{'}u_{02} - u_{11}\right)\right\} & \theta^{'} &= \frac{aS_\phi^2}{aS_\phi^2 + b}. \end{aligned}$$

which is minimum for

$$\mathfrak{m}_1^{ss(opt)} = \frac{(B^{''}D^{''} + C^{''}E^{''})}{(A^{''}B^{''} - C^{''2})} \; and \; \mathfrak{m}_2^{ss(opt)} = \frac{(A^{''}E^{''} + C^{''}D^{''})}{(A^{''}B^{''} - C^{''2})}.$$

The minimum mean square error of $\hat{\mathfrak{T}}_{ss}$ is given by

$$MSE_{min}(\hat{\mathfrak{T}}_{ss}) = S_y^4 - \frac{\left(B^{''}D^{''2} + 2C^{''}D^{''}E^{''} + A^{''}E^{''2}\right)}{(A^{''}B^{''} - C^{''2})}. \tag{12}$$

Following Khan et al. (2014) and Ismail et al. (2015) we may write ratio and product estimators using square root transformation as given by

$$\hat{\mathfrak{T}}_{hr1} = s_y^2 \left(\frac{S_\phi^2}{s_\phi^2}\right)^{\frac{1}{2}} \; and \; \hat{\mathfrak{T}}_{hr2} = s_y^2 \left(\frac{s_\phi^2}{S_\phi^2}\right)^{\frac{1}{2}}.$$

The MSEs of $\hat{\mathfrak{T}}_{hr1}$ and $\hat{\mathfrak{T}}_{hr2}$ are given by

$$MSE(\hat{\mathfrak{T}}_{hr1}) = S_y^4 \left[u_{20} + \frac{1}{4}u_{02} - u_{11}\right] \; and \; MSE(\hat{\mathfrak{T}}_{hr2}) = S_y^4 \left[u_{20} + \frac{1}{4}u_{02} + u_{11}\right].$$

Note that Singh et al. (1988), Shabbir and Gupta (2007), Grover (2010), Singh and Solanki (2013a) proposed these estimators when the auxiliary variable is available. We are introducing these estimators when the auxiliary attribute is available. Further we have introduced these estimators in case of non-response in Section 4. Also taking inspiration from these studies, we are going to construct a new class of estimators for variance S_y^2 using auxiliary attribute in absence and presence of non-response under simple random sampling scheme.

3 Suggested class of estimator

From $\hat{\mathfrak{T}}_{hr2}$ and $\hat{\mathfrak{T}}_{tg}$, we suggest the following class of estimators as

$$\hat{\mathfrak{T}}_N = \left[\mathfrak{m}_1\hat{\mathfrak{T}}_a + \mathfrak{m}_2\left(\frac{s_\phi^2}{S_\phi^2}\right)^{\frac{1}{2}}\right] exp\left[\frac{\alpha(S_\phi^2 - s_\phi^2)}{2c_k S_\phi^2 + s_\phi^2 - S_\phi^2}\right]$$

where α and c_k are suitable constants or known population parameters of auxiliary attribute and

$$\hat{\mathfrak{T}}_a = \frac{s_y^2}{2}\left(\frac{S_\phi^2}{s_\phi^2} + \frac{s_\phi^2}{S_\phi^2}\right).$$

We can generate new estimators using suitable values of α and c_k as given in Table1.

We can rewrite $\hat{\mathfrak{T}}_N$ with δ terms as given by

$$\hat{\mathfrak{T}}_N = \mathfrak{m}_1 S_y^2\left\{1 + \delta_o - a\delta_1 + \left(b + \frac{1}{2}\right)\delta_1^2 - a\delta_o\delta_1\right\} + \mathfrak{m}_2\left\{1 + (\frac{1}{2} - a)\delta_1 + c\delta_1^2\right\} \quad (13)$$

where

$$a = \frac{\alpha}{2c_k}, \; b = \left(\frac{\alpha}{4c_k^2} + \frac{\alpha^2}{8c_k^2}\right) \; and \; c = \left(b - \frac{a}{2} - \frac{1}{8}\right).$$

The bias of $\hat{\mathfrak{T}}_N$ is

$$B(\hat{\mathfrak{T}}_N) = \mathfrak{m}_1 S_y^2\left\{1 + (b + \frac{1}{2})u_{02} - au_{11}\right\} + \mathfrak{m}_2\left(1 + cu_{02}\right) - S_y^2. \quad (14)$$

Table 1: Some members of suggested family in absence of non-response

$Est.$	α	c_k
$\hat{\mathfrak{T}}_{N1} = \left[\mathfrak{m}_1\hat{\mathfrak{T}}_a + \mathfrak{m}_2\left(\frac{s^2_\phi}{S^2_\phi}\right)^{\frac{1}{2}}\right] exp\left[\frac{\alpha(S^2_\phi - s^2_\phi)}{2\beta_2(\phi)S^2_\phi + s^2_\phi - S^2_\phi}\right]$	1	$\beta_2(\phi)$
$\hat{\mathfrak{T}}_{N2} = \left[\mathfrak{m}_1\hat{\mathfrak{T}}_a + \mathfrak{m}_2\left(\frac{s^2_\phi}{S^2_\phi}\right)^{\frac{1}{2}}\right] exp\left[\frac{\alpha(S^2_\phi - s^2_\phi)}{2\beta^\diamond_1(\phi)S^2_\phi + s^2_\phi - S^2_\phi}\right]$	1	$\beta^\diamond_1(\phi)$
$\hat{\mathfrak{T}}_{N3} = \left[\mathfrak{m}_1\hat{\mathfrak{T}}_a + \mathfrak{m}_2\left(\frac{s^2_\phi}{S^2_\phi}\right)^{\frac{1}{2}}\right] exp\left[\frac{\alpha(S^2_\phi - s^2_\phi)}{2C_\phi S^2_\phi + s^2_\phi - S^2_\phi}\right]$	1	C_ϕ
$\hat{\mathfrak{T}}_{N4} = \left[\mathfrak{m}_1\hat{\mathfrak{T}}_a + \mathfrak{m}_2\left(\frac{s^2_\phi}{S^2_\phi}\right)^{\frac{1}{2}}\right] exp\left[\frac{\alpha(S^2_\phi - s^2_\phi)}{2\rho_{pb} S^2_\phi + s^2_\phi - S^2_\phi}\right]$	1	ρ_{pb}

The MSE of $\hat{\mathfrak{T}}_N$ is

$$MSE(\hat{\bar{y}}_{N1}) = L_1 + \mathfrak{m}_1^2\Delta_{A1} + \mathfrak{m}_2^2\Delta_{B1} + 2\mathfrak{m}_1\mathfrak{m}_2\Delta_{C1} - 2\mathfrak{m}_1\Delta_{D1} - \mathfrak{m}_2\Delta_{E1}, \tag{15}$$

where

$$\Delta_{A1} = S_y^4\left\{1 + u_{20} + (a^2 + 2b + 1)u_{02} - 2au_{11}\right\} \quad \Delta_{E1} = 2S_y^2\left\{1 + cu_{11}\right\},$$

$$\Delta_{B1} = \left\{1 + \left(\left(\frac{1}{2} - a\right)^2 + 2c\right)u_{02}\right\} \quad \Delta_{D1} = S_y^4\left\{1 + \left(b + \frac{1}{2}\right)u_{02} - au_{11}\right\},$$

$$\Delta_{C1} = S_y^2\left\{1 + \left(c - a\left(\frac{1}{2} - a\right) + b + \frac{1}{2}\right)u_{02} + \left(\frac{1}{2} - 2a\right)u_{11}\right\}.$$

By minimizing MSE of $\hat{\mathfrak{T}}_N$, we get the optimum values of $\mathfrak{m}_1$, $\mathfrak{m}_2$ as given by

$$\mathfrak{m}_1^{opt} = \left[\frac{\Delta_{B1}\Delta_{D1} - \frac{\Delta_{C1}\Delta_{E1}}{2}}{\Delta_{A1}\Delta_{B1} - \Delta_{C1}^2}\right] \; and \; \mathfrak{m}_2^{opt} = \left[\frac{-\Delta_{C1}\Delta_{D1} + \frac{\Delta_{A1}\Delta_{E1}}{2}}{\Delta_{A1}\Delta_{B1} - \Delta_{C1}^2}\right].$$

Hence, minimum mean square error of $\hat{\mathfrak{T}}_N$ is given by

$$MSE_{min}(\hat{\mathfrak{T}}_N) = \left[S_y^4 - \frac{\Delta_{B1}\Delta_{D1}^2 + \frac{\Delta_{A1}\Delta_{E1}^2}{4} - \Delta_{C1}\Delta_{D1}\Delta_{E1}}{\Delta_{A1}\Delta_{B1} - \Delta_{C1}^2}\right]. \tag{16}$$

4 Non-response

Suppose a finite population $V_N = \{v_i, i = 1, 2, ..., N\}$ containing N units. Suppose y_i be the study variate and p_i be the supplementary attribute. Whenever non-response problem occurs in simple random sampling, we follow Hansen and Hurwitz (1946) sub-sampling scheme. Suppose n_1 be the responding units out of n and remaining $n_2 = n - n_1$ units are taken as non-respondents. Now a sub sample of size $n_g = \frac{n_2}{l}$ is selected by SRSWOR from n_2 non-respondent units with the inverse sampling rate l i.e ($l > 1$). Suppose that all n_g units fully respond on second call. Note that n_g must be an integer otherwise it is necessary to round it. The population is said to be distributed into two groups namely V_{N1} and V_{N2} of sizes N_1 and N_2. Further V_{N1} is a response group that would give response on the first call and V_{N2} is non-response group which could respond on the second call. Obviously V_{N1} and V_{N2} are non-overlapping and unknown quantities. Recently, Sinha and Kumar (2015) find the following unbiased estimator for handling the non-response issue in the estimation of population variance

$$\hat{\mathfrak{T}}'_o = s_y^{*2} = \frac{1}{n-1}\left(\sum_{U_{n1}} y_i^2 + l \sum_{U_{n2(ng)}} y_i^2 - n\hat{\bar{y}}'^2\right) \tag{17}$$

where $\hat{\bar{y}}'$ is Hansen and Hurwitz (1946) unbiased estimator for the estimation of $\bar{Y}$ in case of non-response.

$$V(\hat{\mathfrak{T}}'_o) = S_y^4 u_{20} + wS_{y(2)}^4(\beta_2(y_{(2)}) - 1) = S_y^4 u_{20} + wS_{y(2)}^4 \beta_2(y_{(2)})^{\diamond} \tag{18}$$

where

$$w = \frac{N_2(l-1)}{nN}.$$

The linear regression estimator in case of non-response is given by

$$\hat{\mathfrak{T}}'_{Reg} = s_y^{*2} + m^{Reg}\left(S_\phi^2 - s_\phi^2\right).$$

The mean square error of $\hat{\mathfrak{T}}'_{Reg}$ is given by

$$MSE_{min}(\hat{\mathfrak{T}}'_{Reg}) = S_y^4 u_{20}(1 - \rho^{\diamond 2}) + wS_{y(2)}^4 \beta_2(y_{(2)})^{\diamond}. \tag{19}$$

Singh et al. (1988) estimator under non-response will be

$$\hat{\mathfrak{T}}'_{ts1} = \mathfrak{m}_1^{ts1} s_y^{*2} + \mathfrak{m}_2^{ts1}\left(S_\phi^2 - s_\phi^2\right) \tag{20}$$

where $\mathfrak{m}_1^{ts1}$ and $\mathfrak{m}_2^{ts1}$ are suitably chosen constants. The minimum mean square error of $\hat{\mathfrak{T}}'_{ts}$ is

$$MSE_{min}(\hat{\mathfrak{T}}'_{ts1}) = \frac{S_y^4 \mathfrak{u}_{20}(1 - \rho^{\diamond 2})}{[1 + \mathfrak{u}_{20}(1 - \rho^{\diamond 2})]} \tag{21}$$

where

$$\mathfrak{u}_{20} = \frac{V(\hat{\mathfrak{T}}'_o)}{S_y^4} = u_{20} + \frac{wS_{y(2)}^4 \beta_2(y_{(2)})^{\diamond}}{S_y^4}.$$

In presence of non-response, Shabbir and Gupta (2007) estimator for S_y^2 will be like that

$$\hat{\mathfrak{T}}'_{tg} = \left[\mathfrak{m}_1^{tg} s_y^{*2} + \mathfrak{m}_2^{tg}\left(S_\phi^2 - s_\phi^2\right)\right] exp\left(\frac{S_\phi^2 - s_\phi^2}{S_\phi^2 + s_\phi^2}\right). \tag{22}$$

The minimum mean square error of $\hat{\mathfrak{T}}'_{tg}$ is

$$MSE_{min}(\hat{\mathfrak{T}}'_{tg}) = \left[S_y^4 - \frac{B^{\diamond\diamond}D^{\diamond\diamond 2} + \frac{A^{\diamond}E^{\diamond\diamond 2}}{4} - C^{\diamond\diamond}D^{\diamond\diamond}E^{\diamond\diamond}}{A^{\diamond}B^{\diamond\diamond} - C^{\diamond\diamond 2}}\right] \tag{23}$$

where

$$A^{\diamond} = S_y^4\left[1 + \mathfrak{u}_{20} + u_{02} - 2u_{11}\right].$$

Singh and Solanki (2013a) class of estimators under non-response will be

$$\hat{\mathfrak{T}}'_{ss} = \left[\mathfrak{m}_1^{ss} s_y^{*2} + \mathfrak{m}_2^{ss}(S_\phi^2 - s_\phi^2)\right] \eta'_i \quad for \quad i = 1, 2, ..., 8. \tag{24}$$

Hence the minimum mean square error of $\hat{\mathfrak{T}}'_{ss}$ is

$$MSE_{min}(\hat{\mathfrak{T}}'_{ss}) = S_y^4 - \frac{\left(B''D''^2 + 2C''D''E'' + A'E''^2\right)}{(A'B'' - C''^2)} \tag{25}$$

where

$$A' = S_y^4\left[1 + \left\{\mathfrak{u}_{20} + 3\theta'^2 u_{02} - 4\theta' u_{11}\right\}\right]. \tag{26}$$

Following Khan et al. (2014) and Ismail et al. (2015) one can write ratio and product estimators using square root transformation, in case of non-response as given by

$$\hat{\mathfrak{T}}'_{hr1} = s_y^{*2}\left(\frac{S_\phi^2}{s_\phi^2}\right)^{\frac{1}{2}} \; and \; \hat{\mathfrak{T}}'_{hr2} = s_y^{*2}\left(\frac{s_\phi^2}{S_\phi^2}\right)^{\frac{1}{2}}.$$

The MSEs of $\hat{\mathfrak{T}}'_{hr1}$ and $\hat{\mathfrak{T}}'_{hr2}$ are given by

$$MSE(\hat{\mathfrak{T}}'_{hr1}) = S_y^4\left[\mathfrak{u}_{20} + \frac{1}{4}u_{02} - u_{11}\right] \; and \; MSE(\hat{\mathfrak{T}}'_{hr2}) = S_y^4\left[\mathfrak{u}_{20} + \frac{1}{4}u_{02} + u_{11}\right].$$

4.1 Suggested class of estimators under non-response

Suggested class of estimators $\hat{\mathfrak{T}}_N$ under non-response is as follows

$$\hat{\mathfrak{T}}'_N = \left[\mathfrak{m}_1\hat{\mathfrak{T}}'_a + \mathfrak{m}_2\left(\frac{s^2_\phi}{S^2_\phi}\right)^{\frac{1}{2}}\right] exp\left[\frac{\alpha(S^2_\phi - s^2_\phi)}{2c_k S^2_\phi + s^2_\phi - S^2_\phi}\right]$$

where

$$\hat{\mathfrak{T}}'_a = \frac{s^{*2}_y}{2}\left(\frac{S^2_\phi}{s^2_\phi} + \frac{s^2_\phi}{S^2_\phi}\right).$$

Using suitable values of c_k and α in the $\hat{\mathfrak{T}}'_N$ estimator we can generate new estimators under non-response as given in Table2.

Table 2: Members of suggested family in presence of non-response

Est.	α	c_k
$\hat{\mathfrak{T}}'_{N1} = \left[\mathfrak{m}_1\hat{\mathfrak{T}}'_a + \mathfrak{m}_2\left(\frac{s^2_\phi}{S^2_\phi}\right)^{\frac{1}{2}}\right] exp\left[\frac{\alpha(S^2_\phi - s^2_\phi)}{2\beta_2(\phi)S^2_\phi + s^2_\phi - S^2_\phi}\right]$	1	$\beta_2(\phi)$
$\hat{\mathfrak{T}}'_{N2} = \left[\mathfrak{m}_1\hat{\mathfrak{T}}'_a + \mathfrak{m}_2\left(\frac{s^2_\phi}{S^2_\phi}\right)^{\frac{1}{2}}\right] exp\left[\frac{\alpha(S^2_\phi - s^2_\phi)}{2\beta^\diamond_1(\phi)S^2_\phi + s^2_\phi - S^2_\phi}\right]$	1	$\beta^\diamond_1(\phi)$
$\hat{\mathfrak{T}}'_{N3} = \left[\mathfrak{m}_1\hat{\mathfrak{T}}'_a + \mathfrak{m}_2\left(\frac{s^2_\phi}{S^2_\phi}\right)^{\frac{1}{2}}\right] exp\left[\frac{\alpha(S^2_\phi - s^2_\phi)}{2C_\phi S^2_\phi + s^2_\phi - S^2_\phi}\right]$	1	C_ϕ
$\hat{\mathfrak{T}}'_{N4} = \left[\mathfrak{m}_1\hat{\mathfrak{T}}'_a + \mathfrak{m}_2\left(\frac{s^2_\phi}{S^2_\phi}\right)^{\frac{1}{2}}\right] exp\left[\frac{\alpha(S^2_\phi - s^2_\phi)}{2\rho_{pb} S^2_\phi + s^2_\phi - S^2_\phi}\right]$	1	ρ_{pb}

By minimizing MSE of $\hat{\mathfrak{T}}'_N$, we get the optimum values of $\mathfrak{m}_1$, $\mathfrak{m}_2$ are given by

$$\mathfrak{m}^{opt}_1 = \left[\frac{\Delta_{B1}\Delta_{D1} - \frac{\Delta_{C1}\Delta_{E1}}{2}}{\Delta'_{A1}\Delta_{B1} - \Delta^2_{C1}}\right] \; and \; \mathfrak{m}^{opt}_2 = \left[\frac{-\Delta_{C1}\Delta_{D1} + \frac{\Delta'_{A1}\Delta_{E1}}{2}}{\Delta'_{A1}\Delta_{B1} - \Delta^2_{C1}}\right].$$

The minimum mean square error of $\hat{\mathfrak{T}}'_N$ is given by

$$MSE_{min}(\hat{\mathfrak{T}}'_N) = \left[S^4_y - \frac{\Delta_{B1}\Delta^2_{D1} + \frac{\Delta'_{A1}\Delta^2_{E1}}{4} - \Delta_{C1}\Delta_{D1}\Delta_{E1}}{\Delta'_{A1}\Delta_{B1} - \Delta^2_{C1}}\right] \quad (27)$$

where

$$\Delta'_{A1} = S_y^4 \left\{1 + u_{20} + (a^2 + 2b + 1)u_{02} - 2au_{11}\right\}.$$

5 Efficiency Comparison

In current section, we find the efficiency conditions for the proposed class by looking at the minimum mean square error of the existing estimators in absence of non-response as Observation (A):

$$MSE(\hat{\mathfrak{T}}_N) < MSE(\hat{\mathfrak{T}}_{ti}),$$

if

$$\left[\frac{\Delta_{B_1}\Delta_{D_1}^2 + \frac{\Delta_{A_1}\Delta_{E_1}^2}{4} - \Delta_{C_1}\Delta_{D_1}\Delta_{E_1}}{\Delta_{A_1}\Delta_{B_1} - \Delta_{C_1}^2}\right] - S_y^4\left[1 - u_{20}(1 - \rho^{\diamond 2})\right] > 0$$

Observation (B):

$$MSE(\hat{\mathfrak{T}}_N) < MSE(\hat{\mathfrak{T}}_{ts1}),$$

if

$$\left[\frac{\Delta_{B_1}\Delta_{D_1}^2 + \frac{\Delta_{A_1}\Delta_{E_1}^2}{4} - \Delta_{C_1}\Delta_{D_1}\Delta_{E_1}}{\Delta_{A_1}\Delta_{B_1} - \Delta_{C_1}^2}\right] - S_y^4\left[1 - \frac{1}{1 + \frac{1}{u_{20}(1-\rho^{\diamond 2})}}\right] > 0$$

Observation (C):

$$MSE(\hat{\mathfrak{T}}_N) < MSE(\hat{\mathfrak{T}}_{tg}),$$

if

$$\left[\frac{\Delta_{B_1}\Delta_{D_1}^2 + \frac{\Delta_{A_1}\Delta_{E_1}^2}{4} - \Delta_{C_1}\Delta_{D_1}\Delta_{E_1}}{\Delta_{A_1}\Delta_{B_1} - \Delta_{C_1}^2}\right] - \left[\frac{B^{\diamond\diamond}D^{\diamond\diamond 2} + \frac{A^{\diamond\diamond}E^{\diamond\diamond 2}}{4} - C^{\diamond\diamond}D^{\diamond\diamond}E^{\diamond\diamond}}{A^{\diamond\diamond}B^{\diamond\diamond} - B^{\diamond\diamond 2}}\right] > 0,$$

Observation (D):

$$MSE(\hat{\mathfrak{T}}_N) < MSE(\hat{\mathfrak{T}}_{ss}),$$

if

$$\left[\frac{\Delta_{B_1}\Delta_{D_1}^2 + \frac{\Delta_{A_1}\Delta_{E_1}^2}{4} - \Delta_{C_1}\Delta_{D_1}\Delta_{E_1}}{\Delta_{A_1}\Delta_{B_1} - \Delta_{C_1}^2}\right] - \left[\frac{B''D'' + \frac{A''E''}{4} - C''D''E''}{A''B'' - B''}\right] > 0,$$

Similarly, one can make the efficiency conditions for $MSE(\hat{\mathfrak{T}}'_N)$.
If the observations in A-D are found to be true then we can argue that the new estimators perform better than all of the reviewed estimators.

6 Numerical Illustration

We use following data sets as follows

Population 1 We use the data set presented in Sukhatme (1957) concerning the number of villages containing in a circle considered as (Y) and only those circles containing more than 5 villages considered as (ϕ). Descriptives of the population are $N = 89$, $\bar{Y} = 3.3595$, $P = 0.123$, $S_y = 2.0183$, $S_\phi = 0.3309$, $\rho_{pb} = 0.7662$, $\beta_2(y) = 3.8980$, $\beta_2(\phi) = 6.2319$, $\beta_1(\phi) = 5.2319$, $\eta_{22} = 4.0412$ and $n = 23$. We consider 10 %, 20 % and 30 % weights for non-response (missing values). So numerical results are provided only for 10 %, 20 % and 30 % percent weight of missing values by considering last 9, 18, and 27 values as non-respondents respectively. Some important results from the population of non-respondents are as follows:

- For 10 %
  $l = 2$, $S^2_{Y2} = 3.194444$, $\beta_2(y_{(2)}) = 2.5153$, $N_2 = 9$.
- For 20 %
  $l = 2$, $S^2_{Y2} = 4.852941$, $\beta_2(y_{(2)}) = 2.359339$, $N_2 = 18$.
- For 30 %
  $l = 2$, $S^2_{Y2} = 4.25641$, $\beta_2(y_{(2)}) = 2.627123$, $N_2 = 27$.

Table 3: PRE of reviewed and proposed estimators in absence of non-response

Estimator	PopI	PopII	Estimator	PopI	PopII	Estimator	PopI	PopII
$\hat{\mathfrak{T}}_o$	100	100	$\hat{\mathfrak{T}}_{ss4}$	174.87	158.33	$\hat{\mathfrak{T}}_{hr1}$	248.81	177.68
$\hat{\mathfrak{T}}_{Reg}$	256.42	179.92	$\hat{\mathfrak{T}}_{ss5}$	116.46	161.61	$\hat{\mathfrak{T}}_{N1}$	322.12	422.01
$\hat{\mathfrak{T}}_{ts1}$	265.77	192.33	$\hat{\mathfrak{T}}_{ss6}$	183.48	162.43	$\hat{\mathfrak{T}}_{N2}$	390.16	511.12
$\hat{\mathfrak{T}}_{ss1}$	184.88	163.08	$\hat{\mathfrak{T}}_{ss7}$	106.859	163.05	$\hat{\mathfrak{T}}_{N3}$	586.89	768.64
$\hat{\mathfrak{T}}_{ss2}$	182.29	161.87	$\hat{\mathfrak{T}}_{ss8}$	184.45	162.88	$\hat{\mathfrak{T}}_{N4}$	3639.83	1555.57
$\hat{\mathfrak{T}}_{ss3}$	184.52	162.91	$\hat{\mathfrak{T}}_{tg}$	268.51	196.07			

Population 2 We use the data set presented in Sukhatme (1957) concerning the Area (in acres) under the wheat crop within the circles considered as (Y) and only those circles containing more than 5 villages considered as (ϕ). Descriptives of the population are $N = 89$, $\bar{Y} = 1102$, $P = 0.123$, $S_y = 716.65$, $S_\phi = 0.3309$, $\rho_{pb} = 0.6235$, $\beta_2(y) = 4.8494$, $\beta_2(\phi) = 6.2319$, $\beta_1(\phi) = 5.2319$, $\eta_{22} = 3.9910$ and $n = 23$. Some important results from the population of non-respondents are as follows:

- For 10 %
  $l = 2$, $S^2_{Y2} = 160686.6$, $\beta_2(y_{(2)}) = 2.589502$, $N_2 = 9$.
- For 20 %
  $l = 2$, $S^2_{Y2} = 317734.4$, $\beta_2(y_{(2)}) = 3.868826$, $N_2 = 18$.
- For 30 %
  $l = 2$, $S^2_{Y2} = 160686.6$, $\beta_2(y_{(2)}) = 2.589502$, $N_2 = 27$.

Table 4: PRE of reviewed and proposed estimators in presence of non-response

Estimator	PopI 10%	PopI 20%	PopI 30%	PopII 10%	PopII 20%	PopII 30%
$\hat{\mathfrak{T}}'_o$	100	100	100	100	100	100
$\hat{\mathfrak{T}}'_{Reg}$	230.56	208.02	195.21	169.93	161.10	165.14
$\hat{\mathfrak{T}}'_{ts1}$	266.49	267.40	268.11	193.31	194.45	193.89
$\hat{\mathfrak{T}}'_{ss1}$	179.16	173.65	170.32	160.70	158.66	159.58
$\hat{\mathfrak{T}}'_{ss2}$	177.01	171.92	168.84	159.70	157.85	158.68
$\hat{\mathfrak{T}}'_{ss3}$	178.86	173.41	170.11	160.56	158.55	159.46
$\hat{\mathfrak{T}}'_{ss4}$	170.81	166.89	164.51	156.75	155.45	156.03
$\hat{\mathfrak{T}}'_{ss5}$	119.03	130.99	136.94	165.47	169.71	167.62
$\hat{\mathfrak{T}}'_{ss6}$	178.00	172.72	169.52	160.16	158.23	159.10
$\hat{\mathfrak{T}}'_{ss7}$	179.11	173.61	170.29	160.68	158.64	159.56
$\hat{\mathfrak{T}}'_{ss8}$	178.80	173.36	170.01	160.53	158.53	159.43
$\hat{\mathfrak{T}}'_{tg}$	243.78	222.38	210.37	187.07	179.35	182.85
$\hat{\mathfrak{T}}'_{hr1}$	224.82	203.71	191.64	168.08	159.57	163.47
$\hat{\mathfrak{T}}'_{N1}$	345.73	375.80	399.35	454.93	493.10	474.34
$\hat{\mathfrak{T}}'_{N2}$	418.85	455.28	483.78	550.90	597.03	574.35
$\hat{\mathfrak{T}}'_{N3}$	630.16	684.98	727.81	828.30	897.41	863.43
$\hat{\mathfrak{T}}'_{N4}$	3909.31	4248.72	4512.70	1674.17	1811.13	1743.85

The percentage relative efficiencies of all the ratio type proposed and existing estimators available in Table 3 and 4.
Note that
In absence of non-response $PRE(.) = \dfrac{MSE(\hat{\mathfrak{T}}_o)}{MSE(.)} \times 100,$
In presence of non-response $PRE(.) = \dfrac{MSE(\hat{\mathfrak{T}}'_o)}{MSE(.)} \times 100.$
For non-response problem we take $(10\%, 20\% \; and \; 30\%)$ values as non-response. By assuming all these three possible choices we notice that the PRE of the proposed and existing estimators available in Table 4 are not affected by different weights of missing values. Surely, numerical results are not similar/same on all these weights but the behavior of the numerical results is almost similar in all situations.

7 Conclusion

In this study, the issue of estimating the population variance is investigated alongside the non-response problem. Mean square error of suggested class has been compared with some of the existing estimators. The result of these comparisons, possibilities under which the proposed class has smaller mean square error than the others have been found. Theoretical results are also verified with the help of natural populations. From the numerical discussion, we found that new class of estimators is more efficient than the existing ones. Hence it is advisable to utilize the proposed class of estimator $\hat{\mathfrak{T}}_N$.

Acknowledgement

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References

Abd-Elfattah, A. M., El-Sherpieny, E. A., Mohamed, S. M., and Abdou, O. F. (2010). Improvement in estimating the population mean in simple random sampling using information on auxiliary attribute. *Applied Mathematics and Computation*, 215(12):4198–4202.

Grover, L. K. (2010). A correction note on improvement in variance estimation using auxiliary information. *Communications in Statistics-Theory and Methods*, 39(5):753–764.

Hansen, M. H. and Hurwitz, W. N. (1946). The problem of non-response in sample surveys. *Journal of the American Statistical Association*, 41(236):517–529.

Isaki, C. T. (1983). Variance estimation using auxiliary information. *Journal of the American Statistical Association*, 78(381):117–123.

Ismail, M., Hanif, M., and Shahbaz, M. Q. (2015). Generalized estimators for population mean in the presence of non-response for two-phase sampling. *Pak. J. Statist*, 31(3):295–306.

Kadilar, C. and Cingi, H. (2006a). Improvement in variance estimation using auxiliary information. *Hacettepe Journal of Mathematics and Statistics*, 35(1):11–115.

Kadilar, C. and Cingi, H. (2006b). Ratio estimators for the population variance in simple and stratified random sampling. *Applied Mathematics and Computation*, 173(2):1047–1059.

Khan, H., Sanaullah, A., Khan, M. A., and Siddiqi, A. F. (2014). Improved exponential ratio type estimators for estimating population mean regarding full information in survey sampling. *World Applied Science journal*, 26(5):1897–1902.

Naik, V. and Gupta, P. (1996). A note on estimation of mean with known population proportion of an auxiliary character. *Jour. Ind. Soc. Agr. Stat*, 48(2):151–158.

Shabbir, J. and Gupta, S. (2007). On improvement in variance estimation using auxiliary information. *Communications in Statistics-Theory and Methods*, 36(12):2177–2185.

Singh, H., Upadhyaya, L., and Namjoshi, U. (1988). Estimation of finite population variance. *Current Science*, pages 1331–1334.

Singh, H. P. and Pal, S. K. (2016). An efficient class of estimators of finite population variance using quartiles. *Journal of Applied Statistics*, 43(10):1945–1958.

Singh, H. P. and Solanki, R. S. (2013a). Improved estimation of finite population variance using auxiliary information. *Communications in Statistics-Theory and Methods*, 42(15):2718–2730.

Singh, H. P. and Solanki, R. S. (2013b). A new procedure for variance estimation in simple random sampling using auxiliary information. *Statistical Papers*, 54(2):479–497.

Singh, R. and Malik, S. (2014). Improved estimation of population variance using information on auxiliary attribute in simple random sampling. *Applied Mathematics and Computation*, 235:43–49.

Sinha, R. and Kumar, V. (2015). Families of estimators for finite population variance using auxiliary character under double sampling the non-respondents. *National Academy Science Letters*, 38(6):501–505.

Sukhatme, P. V. (1957). *Sampling theory of surveys with applications.* The Indian Society Of Agricultural Statistics; New Delhi.

Upadhyaya, L. N. and Singh, H. P. (1999). Use of transformed auxiliary variable in estimating the finite population mean. *Biometrical Journal: Journal of Mathematical Methods in Biosciences*, 41(5):627–636.

Yadav, S. K. and Kadilar, C. (2014). A two parameter variance estimator using auxiliary information. *Applied Mathematics and Computation*, 226:117–122.