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A technical note on periodic inventory model with controllable lead time under service level constraint

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In this article, a periodic review inventory model having variable lead time, along with the service level constraint has been discussed. The demand during the protection interval is assumed to follow the Normal distribution. A wide range of expected shortages during the protection interval have been considered with their corresponding values of safety factors which directly affect the total expected annual cost of the supplier. Keeping this in mind, we have returned to both Ouyang and Chuang (2000) and Liang et al. (2008) model. Further, it has been observed that both the models do not succeed in providing the solution for service level other than 98% and 99%. In the light of this fact, we revisit the solution procedure of both the papers and are able to obtain the result for any level of service. Detail comparative analysis has also been presented.

keywords: Periodic inventory, Lead-Time, Shortages, Service level.

1 Introduction

In competitive business arena, each firm is aware of the importance of time, price, quality and service in order to be in place in the market. Therefore, each firm always tries to reduce the leadtime, an effort to reach to his customer at the earliest, which is, of course, not at all compromising with other aspects of the product. Infact, the application of Just-In-Time (JIT) philosophy includes the crashing of lead-time to raise the productivity.

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In several practical situations, lead-time is controllable i.e. can be reduced at an extra cost. However, several continuous review inventory models have been developed to consider lead-time as a decision variable (Liao and Shyu, 1991; Ben-Daya and Raouf, 1994; Ouyang and Wu, 1997; Moon Choi, 1998; Moon et al., 2014; Ouyang and Chuang, 1998; Wu and Tsai, 2001; Pan and Hsiao, 2001; Pan and Hsiao, 2005; Sarkar and Moon, 2014; Sarkar et al., 2015). Lee et al. (2006) developed two computational algorithms to find optimal order quantity and optimal lead time under service level constraint. Lee et al. (2007) discussed a computational algorithmic procedure to optimize lead time, order quantity, backorder discount and ordering cost. Tempelmeier (2007) included the minimization of the setup cost and holding costs, assuming that the inventory at the end of any period cannot be negative. In all these papers, authors have mainly emphasized the benefits of reducing the leadtime. However, the lead time reduction in periodic review inventory model with a service level constraint has received very little attention of the researchers. Ouyang and Chuang (2000) provided the periodic review model with variable lead time with service level constraint. Ouyang et al. (2002) further investigated the stochastic inventory model with the reduction of the setup cost and the lead time for unknown distribution of demand with probabilistic backorder rate. In 2008, Liang et al. have presented an alternative approach to find the better solution of Ouyang and Chuang (2000).

In this paper, the periodic review inventory model with lead time reduction under the service level constraint has been re-evaluated. In the past, Ouyang and Chuang (2000) had assumed that protection interval demand X follows normal distribution with p.d.f . f_x having finite mean $D(T+L)$ and standard deviation $\sigma\sqrt{(T+L)}$ with the target level $R = D(T+L) + k\sigma\sqrt{T+L}$ where D represent average demand per year and L stands for lead-time. The length of periodic review is denoted by T and k is defined safety factor. $E(X-R)^+$ represents expected demand shortage at the end of cycle given by $E(X-R)^+ = \int_R^\infty (x-R) f_x dx$.

They tried to minimize the total expected annual cost (EAC) which is the sum of the ordering cost (A), holding cost (h) and lead time crashing cost ($C(L)$). Symbolically, the problem is to minimize $EAC(T, L)$ where

$$EAC(T,L) = \frac{A}{T} + h\left[\frac{DT}{2} + k\sigma\sqrt{T+L} + (1-\beta)G(k)\right] + \frac{C(L_i)}{T}.$$

$$\text{subject to } \frac{E(X-R)^+}{D\sqrt{(T_i+L_i)}} \leq \alpha \text{ (Ouyang and Chuang, 2000, equation (3))}$$

where α = Proportion of demands that are not met from stock with $(1-\alpha)$ as a service level. They have suggested the solution of the model by taking the condition of optimality i.e. by equating the derivative of above equation with respect to decision variable and equating to zero. Whereas, the definition of second order derivative has been used for checking the convexity and concavity of the function, but one can check that no feasible solution exists for $\alpha = 0.01$ and moreover if the value of α increases 0.02 onwards, then the optimal solution for the expected annual cost does not change. It is a well established fact that if one reduces the service level i.e. $(1-\alpha)$ then the total expected cost is likely to decrease, which is not apparent in this paper. Moreover, their findings do not show any change with respect to change in service level, which implies that the model is insensitive with respect to change in service level.

Further, Liang et al. (2008) have examined Ouyang and Chuang (2000) approach and commented that $EAC(T, L)$ is not necessarily a convex function of review period, T and have provided the alternative approach by defining the function as

$$f_i(T) = \omega_1 T^2 + \omega_2 \frac{T^2}{\sqrt{T + L_i}} - \theta_i \quad \text{for } i = 0, 1, \ldots, n$$

where $\theta_i = A + C(L_i)$, $\omega_1 = \left(\frac{Dh}{2}\right)$, $\omega_2 = (h\sigma/2)(k + (1-\beta)G(k))$ (refer equation (12)).

This ensures the existence of unique solution for $T_i^{\wedge}$ when $f_i(T) = 0$. However, this solution has been obtained without considering the constraints. However, there are two constraints i.e.

(a) $T_i^{\wedge}$ should satisfy the assumption as $L_{\leq} T_i^{\wedge}$ and

(b) $T_i^{\wedge}$ should satisfy the service level constraint i.e. $\frac{\sigma G(k)}{D\sqrt{(T_i^{\wedge} + L_i)}} \leq \alpha$.

He has successfully showed that both the constraints are satisfied by taking the optimal review period as $T_i^* = \max\left\{T_i^{\wedge}, L_i, \left(\frac{\sigma G(k)}{D\alpha}\right)^2 - L_i\right\}$ to minimize the expected annual cost.

By showing this, Liang et al. (2008) could add only one more level of service i.e. 99% to that of Ouyang and Chuang (2000) approach. Unfortunately, this model is also insensitive to the change of level of service except 98% and 99%.

2 Notations and Assumptions

The following notations and assumptions have been used in this paper.

A	=	Fixed Ordering cost per order
D	=	Average Demand per year
h	=	Inventory holding cost per order per year
L	=	Length of lead-time, a decision variable
T	=	Length of Periodic review, a decision variable
X	=	Demand of protection interval, $T + L$, which has probability density function f_x, finite mean $D(T + L)$ and standard deviation $\sigma\sqrt{T + L}$.
α	=	Proportion of demands that are not met from stock so $1 - \alpha$ is the service level
β	=	Fraction of demand backordered during the stock out period.
$C(L)$	=	Lead time crashing cost
$EAC(T, L)$	=	Total expected annual cost.

Assumptions

1. The Inventory level is reviewed every T units of time. A sufficient ordering quantity is ordered up to the target level R, and the ordering quantity is arrived at after L units of time.

2. The length of the lead-time L is not greater than the review period length T so that there is never more than a single order taking place in any cycle.

3. The target level $R =$ expected demand during protection interval + safety stock (SS), and $SS = k\sigma\sqrt{T+L}$, that is $R = D(T+L) + k\sigma\sqrt{T+L}$ where k is the safety factor.

4. If X has a normal distribution function $F(x)$, then $E(X-R)^+ = \sigma\sqrt{T+L}G(k)$ where $G(k) = \int_k^\infty (z-k)\, f_Z(z)dz$ and $f_Z(z)$is the probability density function of the standard normal random variable Z.

5. The lead-time L includes n mutually independent components. The ith component has a minimum duration a_i and normal duration b_i and a crashing cost per unit time c_i. Further, we assume that $c_1 \le c_2 \le \cdots \le c_n$. The lead-time components are crashed one at a time starting with the component of least c_i, and so on.

6. If we let $L_0 = \sum\limits_{j=1}^{n} b_j$ and L_i be the length of lead-time with components $1, 2, \ldots, i$ crashed to their minimum duration, then $L_i = \sum\limits_{j=i+1}^{n} b_j + \sum\limits_{j=1}^{i} a_j$. The lead-time crashing cost $C(L)$ per cycle for a given $L \in [L_i, L_{i-1}]$, is given by $C(L) = c_i(L_{i-1}, L) + \sum\limits_{j=1}^{i-1} c_j(b_j - a_j)$.

7. When X has a normal distribution function, the service level constraint becomes $\dfrac{E(X-R)^+}{D(T+L)} = \dfrac{\sigma G(k)}{D\sqrt{T+L}} \le \alpha$.

3 Improved Solution Procedure

In general, the term service level refers to the availability of stock in a probabilistic or expected sense without considering other dimensions of service. Arnold (1998) suggested that the value of safety factor changes with a service level, which depends on the probability of stock out or fill rate. After the careful review of these two papers, it has been observed in both the papers that the expected total cost of the system behaves independent of the level of service. The reason for this, according to our observation was that they were not considering the proper values of the safety factor, which actually depends on the probability of stock out in the normal distribution or the fill rate (Refer Silver and Peterson, 1985, pp. 699-708). Owing to this fact, we could establish that both the authors have considered the value of safety factor $k = 0.845$ with the probability of stock out of 20% i.e. $q = 0.2$ which implies the level of service of 80%. But, they have used this data for providing the level of service 98% which is a contradiction. Since, it is not possible for a firm to provide the service level of 98% when the probability of stock out (q) is considered to be 20%. As they suggested, $(1-\alpha)$ represents the service level where $0 < \alpha < 1$. Therefore, the value of safety factor k and expected shortages $E(X-R)^+$ based on service level $G(k)$ should change with respect to different values of α. Therefore, there is no need to specify the probability of stock out i.e. q separately for the specification of safety factor k, once the proportion of demands that are not met from stock has given (refer Zeng and Hayya, 1999). For different value of service level, $(1-\alpha) = G(k)$ the respective values of safety factor, k is given as

$G(k)$	0.900	0.925	0.950	0.975	0.990	
k	1.28	1.43	1.64	1.96	2.33	(refer Thomopoulos, 2006)

Therefore, if one changes the value of safety factor, k depending upon the value of service level $(1-\alpha)$, then, the optimal solution can be obtained for different level of services by covering wide range of expected shortages.

4 Numerical Examples

First of all, we have considered the data utilized by Ouyang and Chuang (2000) and Liang et al. (2008) respectively and obtained the results using their approaches. Later, we have obtained the results by rectifying the error, which was overlooked by these authors.

Example 1 (Ouyang and Chuang, 2000 data)**.** $D = 625$ Units per year, $A = \$350$ per order, $\sigma = 7$ units per week, $\beta = 1$, $h = \$35$ per unit per year, and employed both the approaches. The lead-time has three components, which have given in Table 1.

Table 1: Lead time data

Lead time component i	Normal duration (days) b_i	Minimum duration (days) a_i	Unit crashing cost c_i
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Using this data we have obtained the solution by Ouyang and Chuang (2000) and Liang et al. (2008) approach for different values of $\alpha = (0.01, 0.02, 0.04, 0.20, 0.30)$ with the respective service levels of 99%, 98%, 96%, 80% and 70% with $\beta = 1$ (which represents the case of complete backlogged demand) and results have been illustrated in Table 2

Table 2: Results of Ouyang and Chuang (2000) and Liang et al. (2008) for Example 1

Service level of 99% with $\alpha = 0.01$ and $k = 0.845$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	No Feasible Solution			33.76	240	8975.77
1	6				35.76	216	9374.44
2	4				37.76	194	9789.01
3	3				38.76	187	10031.16
Service level of 98% with $\alpha = 0.02$ and $k = 0.845$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	8.80	226	4764.73	8.78	226	4764.69
1	6	8.84	201	4745.68	8.82	201	4745.65
2	4	8.97	177	4771.89	8.96	177	4771.89
3	3	9.37	169	4941.21	9.37	169	4941.16
Service level of 96% with $\alpha = 0.04$ and $k = 0.845$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	8.80	226	4764.73	8.78	226	4764.69
1	6	8.84	201	4745.68	8.82	201	4745.65
2	4	8.97	177	4771.89	8.96	177	4771.89
3	3	9.37	169	4941.21	9.37	169	4941.16
Service level of 80% with $\alpha = 0.20$ and $k = 0.845$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	8.80	226	4764.73	8.78	226	4764.69
1	6	8.84	201	4745.68	8.82	201	4745.65
2	4	8.97	177	4771.89	8.96	177	4771.89
3	3	9.37	169	4941.21	9.37	169	4941.16
Service level of 70% with $\alpha = 0.30$ and $k = 0.845$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	8.80	226	4764.73	8.78	226	4764.69
1	6	8.84	201	4745.68	8.82	201	4745.65
2	4	8.97	177	4771.89	8.96	177	4771.89
3	3	9.37	169	4941.21	9.37	169	4941.16

* k - Safety factor

In Table 2, it is clearly observed that both approaches fail to obtain the correct results for service level of 98% and below. The expected annual cost seems to be insensitive and coming out to be 4745.68 approximately for different levels of service and does not affected by crashing of lead time which sounds to be illogical. Moreover, the optimal solution involves the crashing of only one component of lead time for various levels of service.

Example 2 (Liang et al., 2008 data)**.** $D = 500$ units per year, $A = \$400$ per order, $\sigma = 7$ units per week, $\beta = 1$, $h = \$40$ per unit per year, and obtained the results by using both approaches with the same lead-time data shown in Table 1. Using this data, we have obtained the solution by Ouyang and Chuang (2000) and Liang et al. (2008) approach for different values of $\alpha = (0.01, 0.02, 0.04, 0.20, 0.30)$ with the respective service levels of 99%, 98%, 96%, 80% and 70% with $\beta = 1$ (which represents the case of complete backlogged demand) and results have been provided in Table 3.

Table 3: Results of Ouyang and Chuang (2000) and Liang et al. (2008) for Example 2

Service level of 99% with $\alpha = 0.01$ and $k = 0.845$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	No Feasible Solution			57.26	170	13285.42
1	6				59.26	151	13662.69
2	4				61.26	133	14047.57
3	3				62.26	127	14263.39
Service level of 98% with $\alpha = 0.02$ and $k = 0.845$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	9.71	170	5005.12	9.71	170	5005.12
1	6	9.74	201	4977.18	10.31	151	4984.02
2	4	9.86	177	4989.89	12.31	133	5095.64
3	3	10.25	169	5138.65	13.31	127	5291.56
Service level of 96% with $\alpha = 0.04$ and $k = 0.845$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	9.71	170	5005.12	9.71	170	5005.11
1	6	9.74	201	4977.18	9.74	151	4977.16
2	4	9.86	177	4989.89	9.87	133	4989.89
3	3	10.25	169	5138.65	10.25	127	5138.62
Service level of 80% with $\alpha = 0.20$ and $k = 0.845$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	9.71	170	5005.12	9.71	170	5005.12
1	6	9.74	201	4977.18	9.74	151	4977.16
2	4	9.86	177	4989.89	9.87	133	4989.89
3	3	10.25	169	5138.65	10.25	127	5138.65
Service level of 70% with $\alpha = 0.30$ and $k = 0.845$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	9.71	170	5005.12	9.71	170	5005.11
1	6	9.74	201	4977.18	9.74	151	4977.16
2	4	9.86	177	4989.89	9.87	133	4989.89
3	3	10.25	169	5138.65	10.25	127	5138.62

* k - Safety factor

It is evidently seen from Table 3; the expected annual cost remains to be static and does not change with level of service of 96% and below. Further, the optimal solution involves the crashing of only one component of lead time for different levels of service which ultimately shows that expected annual cost is insensitive with respect to level of service.

Now, let us revisit the above examples with the rectified approach i.e. by using the right values of k (safety factor) which is different for different level of services. Table 4 and Table 5 illustrate the results of Ouyang and Chuang (2000) and Liang et al. (2008) data respectively.

Table 4: Outcomes of Example-1 for Different Service Levels with rectified approach

Service level of 99% with $\alpha = 0.01$ and $k = 2.33$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	9.27	275	6285.63	8.03	258.09	6241.03
1	6	9.35	248	6180.50	8.03	229.76	6129.98
2	4	9.53	223	6120.61	8.10	202.27	6062.16
3	3	9.97	215	6262.79	8.44	192.8	6198.82
Service level of 98% with $\alpha = 0.02$ and $k = 2.05$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	9.28	267	6000.73	8.16	252.02	5965.75
1	6	9.35	241	5912.01	8.16	224.32	5872.37
2	4	9.53	215	5868.60	8.25	197.52	5822.75
3	3	9.97	208	6016.08	8.60	188.38	5965.91
Service level of 96% with $\alpha = 0.04$ and $k = 1.75$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	9.28	259	5695.42	8.31	245.54	5669.56
1	6	9.35	233	5624.27	8.32	218.50	5594.97
2	4	9.53	208	5598.49	8.42	192.43	5564.60
3	3	9.97	200	5751.65	8.78	183.66	5714.57
Service level of 80% with $\alpha = 0.20$ and $k = 0.84$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	9.29	232	4764.73	8.79	226.03	4764.69
1	6	9.36	208	4745.68	8.83	201.01	4745.65
2	4	9.55	184	4771.89	8.97	177.14	4771.90
3	3	9.99	177	4941.21	9.37	169.45	4941.16
Service level of 70% with $\alpha = 0.30$ and $k = 0.52$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	9.30	223	4442.95	8.98	219	4440.52
1	6	9.37	199	4443.75	9.03	195	4441.00
2	4	9.55	176	4490.18	9.18	172	4487.00
3	3	9.99	169	4666.51	9.60	164	4663.04

$*$ k - Safety factor

Table 5: Outcomes of Example-2 for Different Service Levels with rectified approach

Service level of 99% with $\alpha = 0.01$ and $k = 2.33$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	10.36	246	6795.52	8.74	228	6729.90
1	6	10.43	224	6672.42	8.72	204	6598.68
2	4	10.61	203	6590.24	8.77	181	6505.83
3	3	11.04	196	6708.78	9.08	173	6617.09
Service level of 98% with $\alpha = 0.02$ and $k = 2.05$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	10.37	238	6459.89	8.9	222	6408.31
1	6	10.44	216	6354.98	8.89	199	6297.00
2	4	10.61	195	6291.01	8.96	176	6224.65
3	3	11.04	189	6415.45	9.28	168	6343.40
Service level of 96% with $\alpha = 0.04$ and $k = 1.75$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	10.37	229	6100.19	9.09	215	6061.97
1	6	10.44	208	6014.75	9.08	193	5971.80
2	4	10.62	187	5970.28	9.16	171	5921.11
3	3	11.05	181	6101.04	9.5	164	6047.67
Service level of 80% with $\alpha = 0.20$ and $k = 0.84$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	10.39	202	5014.51	9.71	195	5005.12
1	6	10.46	182	4987.74	9.74	175	4977.17
2	4	10.64	163	5001.97	9.86	155	4989.90
3	3	11.07	157	5151.75	10.25	149	5138.65
Service level of 70% with $\alpha = 0.30$ and $k = 0.52$							
i	L_i	Ouyang and Chuang (2000)			Liang et al. (2008)		
		T_i	R_i	$EAC(\cdot)$	T_i	R_i	$EAC(\cdot)$
0	8	10.39	192	4624.41	9.96	188	4620.77
1	6	10.46	173	4618.67	10.00	169	4614.60
2	4	10.64	155	4653.96	10.15	150	4649.30
3	3	11.08	149	4810.54	10.55	144	4805.49

$*$ k - Safety factor

5 Observations

It is clearly evident from Tables 4 and 5 that, we are able to obtained the results for different level of service with its right value of safety factor but total expected cost also changes with the variation in the level of service, which otherwise, is not reflecting in the results obtained by the authors (see Table 2 and Table 3) using their respective approaches. Further, one can check that total expected annual cost decreases with reduction of lead time and crashing of two components of lead time is desirable in order to provide higher level of service i.e.96% and above.

6 Conclusion

It is concluded that reduction in lead time plays an important role to run the system profitably as it helps the supplier to reduce the overall cost of the system by reducing the loss caused by shortages and improving the level of service to the customers. This paper revisits the solution procedure of Ouyang and Chuang (2000) and Liang et al. (2008) for the wide range of the levels of service when demand during protection interval $(T + L)$ is normally distributed. Owing to the fact that the value of safety factor varies with the level of service i.e. $(1 - \alpha)$ is being offered. The comparative study of two approaches has been provided. The findings clearly suggest that the Liang et al. (2008) approach has got an edge over the Ouyang and Chuang (2000) approach.

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