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Modelling Recession in Two European Countries: the Generalized Binomial Heterogeneous Autoregressive Model

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The time series of German and Italian recessions are analyzed using a novel approach for binary time series introduced in Startz (2008). For the German recessions a purely autoregressive model for binary data has provided satisfactory results. For the Italian recessions, on the other hand, to reach a significant fit we have applied a model including some lags of the German driving economy. An interpretation in terms of short-run and long-run effects is proposed.

keywords: binary variable, economic recession, time series.

1 Introduction

The constitution of the European Monetary Union and the recent globalization of economies have raised several interesting issues. Among them, one of paramount relevance concerns the existence of a common cycle among countries. For this reason more and more often several economists and policy makers (Lumsdaine and Prasad, 2003; Gregory et al., 1997; Canova et al., 2003) assume the existence of this cycle and estimate it to calculate its importance in explaining country specific movements. At the same time, many others (Krolzigy and Toro, 2001; Mansour, 2003; Del Negro and Ottrok, 2003; Artis et al., 2004) assume the existence of a European-specific business cycle,

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that is a business cycle determined by Euro-area specific factors. Some recent recession events, like the failure of important credit institutes or the diffusion of the crisis among European countries, appear to be consistent with this thesis.

In this paper we want to shed light on the possible presence of a relationship between recessions in European countries, distinguishing countries with a solid economy which can drive the economic cycle (leader countries) and countries partially depending on the business cycle of the driving economy (follower countries). In particular, we want to analyze whether there is a causality relationship between the recession pattern of the most important European economy, the German one, and that of another country (Italy). Actually, German market is the main end market of Italian exports, so a recession (expansion) in Germany could involve with some delay a recession (expansion) in Italy.

The approach that we follow makes use of models for binary time series of recession data. From a methodological point of view, we exploit a modern parametric approach, introduced by Startz (2008), based on the comparison between the empirical pattern of the estimated autopersistence functions from a binary time series and the theoretical pattern of the autopersistence functions associated to the specific model used for the time-series. Startz (2008) has proposed the Binomial Autoregressive Moving Average model for a very long time series of the U.S. recessions. The same approach has been followed by De Luca and Carfora (2010) where the novel Binomial Heteregeneous Autoregressive model for binary time series has been introduced taking into account the long memory nature of the autopersistence graphs of the same time series. Moreover, De Luca and Carfora (2014) have studied the forecasting performance of the model suggesting a combination of forecasts, so to take advantage of the dynamics of both the Binomial Heterogenous Autoregressive model and the probit model by Kauppi and Saikkonen (2008) which introduces in the equation of the recession data some macroeconomic variables. In this work the Binomial Heterogenous Autoregressive model is generalized to take into account a possible number of exogenous components, such that it looses its purely autoregressive nature. In fact, the generalized model considers a mix of endogenous and exogenous components and is defined as Generalized Binomial Heterogenous Autoregressive model.

The paper is structured as follows. Section 2 characterizes the concept of business cycles and illustrates the business cycles chronology of the two European countries of the study (Germany and Italy). Section 3 and Section 4 describe, respectively, the Binomial Heterogenous Autoregressive model and its generalization. In Section 5, the recession data of Germany and Italy are analyzed. Section 6 concludes.

2 Business Cycle Dates of European countries

Many different methods have been used to identify business cycle turning points. In the United States, the official U.S. business cycle dates are diffused by the National Bureau of Economic Research (NBER). The NBER Business Cycle Dating Committee meets on a regular basis to analyze available information for the United States and reaches a consensus about the timing of turning points based, to a large extent, on the definition of business cycle given in Burns and Mitchell (1946). The NBER does not define a recession only in terms of two consecutive quarters of decline in real GDP. Rather, a recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales. Applying the same methodology used by the NBER, the Economic Cycle Research Institute (ECRI) diffuses a business cycle peak and trough date chronologies of 22 Countries from 1948. In Table 1 the peaks and the troughs of two European countries (Germany and Italy) are reported. Following the approach of Startz (2008), we can obtain from the monthly data of the ECRI a quarterly binary time series. The series is created coding each quarter with 1 for recession, if any month in a quarter is identified as being in a recession, with 0 otherwise.

3 The Binomial Heterogeneous AR model

For a binary time series, conditional probabilities are more relevant than autocorrelations or serial dependences (Cai and Ding, 2009; Harding and Pagan, 2012). Startz (2008) has proposed two novel tools for the analysis of binary time series, the autopersistence functions (APF) and the autopersistence graphs (APG). For an ergodic binary time series $\{Y_t\}$, where each realization y_t takes the value 0 and 1, the autopersistence functions are defined as

$$\mathrm{APF}^0(k) = P(Y_{t+k} = 1 | Y_t = 0)$$

and

$$\mathrm{APF}^1(k) = P(Y_{t+k} = 1 | Y_t = 1).$$

The autopersistence graphs are based on their empirical counterparts, defined, respectively, as

$$\mathrm{APG}^0(k) = \frac{n}{(n-k)} \frac{\sum_{t=1}^{n-k} I\{Y_{t+k} = 1, Y_t = 0\}}{\sum_{t=1}^{n} I\{Y_t = 0\}}$$

and

$$\mathrm{APG}^1(k) = \frac{n}{(n-k)} \frac{\sum_{t=1}^{n-k} I\{Y_{t+k} = 1, Y_t = 1\}}{\sum_{t=1}^{n} I\{Y_t = 1\}}.$$

The APF and the APG can be considered as the analogues of the autocorrelation function and the estimated autocorrelation function, respectively. For this reason they are very intuitive measures of dependence for binary time series. As a result, the APG provide empirical measures of dependence for an observed binary time series. The statistical properties of APG as estimators of APF have been investigated in Wang and Li (2011). A good model for binary time series has to provide an APF similar to the APG of the data (Wang and Li, 2011).

Startz (2008) has analyzed the autopersistence measures of the binary time series of U.S. recessions, applying a Binomial Autoregressive Moving Average (BARMA) model. The same time series has been studied by De Luca and Carfora (2010) who have proposed

Table 1: Business cycle peak and trough dates. NA represent periods for which data are not available. Source: Economic Cycle Research Institute (ECRI) www.businesscycle.com

Period	Peak or Trough	Germany	Italy
1948-1950	P		NA
	T		NA
1951-1952	P		NA
	T		NA
1953-1955	P		NA
	T		NA
1956-1959	P		
	T		
1960-1961	P		
	T		
1962-1966	P	3/66	1/64
	T		3/65
1967-1968	P		
	T	5/67	
1969-1973	P		10/70
	T		8/71
1973-1975	P	8/73	4/74
	T	7/75	4/75
1976-1978	P		
	T		
1979-1980	P	1/80	5/80
	T		
1981-1983	P		
	T	10/82	5/83
1984-1986	P		
	T		
1987-1988	P		
	T		
1989-1991	P	1/91	
	T		
1992-1994	P		2/92
	T	4/94	10/93
1994-1996	P		
	T		
1997-1999	P		
	T		
2000-2001	P	1/01	
	T		
2002-2003	P		8/02
	T	8/03	5/03
2004-2011	P	4/08	2/08
	T	1/09	2/09

the Binomial Heterogeneous Autoregressive (BHAR) model to take into account a long-memory feature of the autopersistence graphs.

Define $Y^t = [Y_t, Y_{t-1}, \ldots]$ the history of the time series Y_t up to time t. The binary variable Y_t conditionally on Y^{t-1} is Bernoulli distributed with probability μ_t. Formally, the BHAR model for quarterly data is defined as

$$P(Y_t = 1|Y^{t-1}) = \mu_t,$$

$$\mu_t = \frac{\exp(\eta_t)}{1 + \exp(\eta_t)},$$

$$\eta_t = \boldsymbol{Y}'_{t-1}\boldsymbol{\theta}$$

where

$$\boldsymbol{Y}_{t-1} = \begin{bmatrix} 1 \\ Y_{t-1} \\ Y_{t-1}^{(1)} \\ \vdots \\ Y_{t-1}^{(2^k)} \end{bmatrix}$$

with $k \in \{0, 1, 2, \ldots\}$ and $Y_t^{(i)}$ defined as a cumulative sum as follows,

$$Y_t^{(i)} = \sum_{j=0}^{4i-1} Y_{t-j},$$

and

$$\boldsymbol{\theta} = \begin{bmatrix} \phi_0 \\ \phi_1 \\ \gamma_0 \\ \vdots \\ \gamma_k \end{bmatrix}.$$

The conditional (on the $4 \cdot 2^k$ initial values) likelihood function of the BHAR model is

$$\ell(\theta) = \sum_{t=4\cdot 2^k+1}^{T} (y_t \log \mu_t + (1 - y_t) \log(1 - \mu_t))$$

where T is the length of the time-series.

The performance of the model can be evaluated in many ways. Firstly, we propose a statistics based on the Mean Absolute Error between the APF of the estimated model and the APG of the time-series at lags $1, \ldots, m$,

$$\text{MAE} = \frac{\text{MAE}^0 + \text{MAE}^1}{2} \tag{1}$$

where

$$\mathrm{MAE}^0 = \frac{1}{m}\sum_{i=1}^{m}\left|\mathrm{APF}^0(i) - \mathrm{APG}^0(i)\right|$$

and

$$\mathrm{MAE}^1 = \frac{1}{m}\sum_{i=1}^{m}\left|\mathrm{APF}^1(i) - \mathrm{APG}^1(i)\right|.$$

Considering that APF^j(i) and APG^j(i), $j = 0, 1$, are probabilities, the MAE ranges between 0 and 1.

Moreover, we can use

$$R_M^2 = 1 - \frac{\ell_1}{\ell_0}$$

where ℓ_1 is the maximized log-likelihood of the estimated model and ℓ_0 is the maximized log-likelihood of the basic model $\eta_t = \phi_0$ and

$$R_p^2 = 1 - \frac{\sum(y_t - \mu_t)^2}{\sum(y_t - \bar{\mu})^2}.$$

The references of these two goodness-of-fit measures are, respectively, McFadden (1974) and Efron (1978).

4 The Generalized BHAR model

In the analysis of recession data of some economies, an important role is due to the influence of cycles of countries that act as drivers, that is countries that possess the driving forces of economic growth. We call such countries as leader countries. Our idea is to model the probability of recession in a follower country also conditionally on the cyclical trends in a single leader country. As an intermediate step, we first estimate a model with the recessions of the follower country depending only on the recessions of a leader country. In this case we define Y_t the binary time-series of recessions in the follower country and X_t the binary time-series of recessions in the driving country. Then, we introduce the notion of Exogenous Persistence Function (XPF),

$$\mathrm{XPF}^0(k) = P(Y_{t+k} = 1 | X_t = 0)$$

and

$$\mathrm{XPF}^1(k) = P(Y_{t+k} = 1 | X_t = 1).$$

Their empirical counterparts are now defined using the exogenous persistence graph (XPG),

$$\mathrm{XPG}^0(k) = \frac{n}{(n-k)}\frac{\sum_{t=1}^{n-k} I\{Y_{t+k} = 1, X_t = 0\}}{\sum_{t=1}^{n} I\{X_t = 0\}}$$

and

$$\mathrm{XPG}^1(k) = \frac{n}{(n-k)}\frac{\sum_{t=1}^{n-k} I\{Y_{t+k} = 1, X_t = 1\}}{\sum_{t=1}^{n} I\{X_t = 1\}}.$$

In this case, a good model for the binary time series of recessions in a follower country has to provide an XPF similar to the XPG of the data.

In order to quantify the effective influence of a leading nation in the cyclic movement of a country we define the simple Binomial Heterogeneous Regression (BHR) model. We assume that the variable to be predicted (probability of economic recession in a follower country at time t) is affected by the lags of an exogenous variable (the economic recessions of the leader country). The BHR model defines

$$P(Y_t = 1|X^{t-1}) = \mu_t^F$$

where superscript F stands for *follower*, and

$$\mu_t^F = \frac{\exp(\eta_t^F)}{1 + \exp(\eta_t^F)},$$

$$\eta_t^F = \boldsymbol{X}'_{t-1}\boldsymbol{\theta}^F$$

where

$$\boldsymbol{X}_{t-1} = \begin{bmatrix} 1 \\ X_{t-1} \\ X_{t-1}^{(1)} \\ \vdots \\ X_{t-1}^{(2^k)} \end{bmatrix}$$

and

$$\boldsymbol{\theta}^F = \begin{bmatrix} \phi_0^F \\ \phi_1^F \\ \gamma_0^F \\ \vdots \\ \gamma_k^F \end{bmatrix}.$$

To measure the goodness of fit we can compute the MAE as in formula (1), after defining

$$\mathrm{MAE}^0 = \frac{1}{n}\sum_{i=1}^{n}\left|\mathrm{XPF}^0(i) - \mathrm{XPG}^0(i)\right|$$

and

$$\mathrm{MAE}^1 = \frac{1}{n}\sum_{i=1}^{n}\left|\mathrm{XPF}^1(i) - \mathrm{XPG}^1(i)\right|,$$

as well as the usual R_M^2 and R_p^2.

The Generalized BHAR (GBHAR) model is then obtained assuming that η_t^F depends on Y_{t-1} and some cumulative lags that can be given by their own lags, or alternatively by the lags of the driving country. In this case, the binary variable Y_t conditionally on Y^{t-1} and X^{t-1} is still Bernoulli distributed with probability μ_t^F. Formally, we have

$$P(Y_t = 1|Y^{t-1}, X^{t-1}) = \mu_t^F$$

with

$$\mu_t^F = \frac{\exp(\eta_t^F)}{1+\exp(\eta_t^F)},$$
$$\eta_t^F = \boldsymbol{Z}'_{t-1}\boldsymbol{\theta}^F$$

where

$$\boldsymbol{Z}_{t-1} = \begin{bmatrix} 1 \\ Y_{t-1} \\ W_{t-1}^{(1)} \\ \vdots \\ W_{t-1}^{(2^k)} \end{bmatrix}$$

and $W_{t-1}^{(j)} \in \{Y_{t-1}^{(j)}, X_{t-1}^{(j)}\}$, that is $W_{t-1}^{(j)} = Y_{t-1}^{(j)}$, or $W_{t-1}^{(j)} = X_{t-1}^{(j)}$ according to the specific time series.

The goodness-of-fit measures, in this case, cannot include the MAE, but still include R_M^2 and R_p^2.

5 Analysis of European recessions

5.1 German recessions

The quarterly German binary time series of recessions from 1948, I quarter to 2011, IV quarter is represented in Figure 1.

It has been recently analyzed (Nyberg, 2010) using financial variables as predictors following the approach that some authors (Estrella and Mishkin, 1998; Kauppi and Saikkonen, 2008) have pursued in the analysis of U.S. time series of recessions. In this work we use the autopersistence measures to identify a model.

The APG of quarterly German binary time series of recession are in Figure 2 with a number of lags equal to 20 quarters (5 years). The two curves are constantly far from the level of the marginal probability (0.2187) except for lag 8, where the two curves cross. So, there is no fast convergence to the marginal probability and a BHAR model can be a good choice. In order to identify the order of the BHAR model, we used the Akaike Information Criterion (AIC) to compare the models from $k = 0$ to $k = 3$. The estimates and p-values, obtained in R making use of the package MaxLik, are reported in Table 2. The model with the lowest AIC is the model with $k = 2$. Formally, the model is given by

$$P(Y_t|Y_{t-1}, \ldots, Y_{t-16}) = \mu_t,$$
$$\mu_t = \frac{\exp(\eta_t)}{1+\exp(\eta_t)}$$

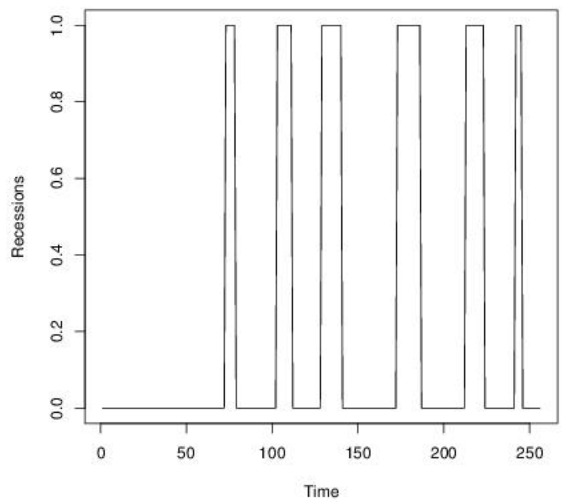


Figure 1: German quarterly recession data, 1948.1 to 2011.4

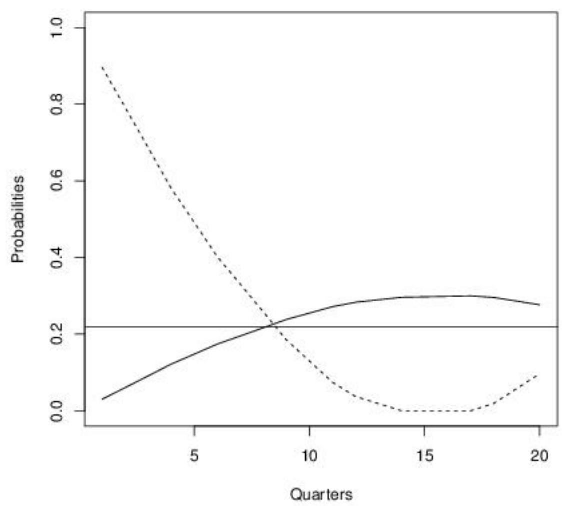


Figure 2: $\mathrm{APG}^0(\mathrm{k})$ (solid line) and $\mathrm{APG}^1(\mathrm{k})$ (dotted line) for German quarterly recession data, 1948.1 to 2011.4

and

$$\eta_t = \phi_0 + \phi_1 Y_{t-1} + \gamma_0 Y_{t-1}^{(1)} + \gamma_1 Y_{t-1}^{(2)} + \gamma_2 Y_{t-1}^{(4)}.$$

In the logit link function, μ_t depends on η_t which, in turn, depends on Y_{t-1}, then on the cumulative one-year, two-years and four-years lagged components. The parameters are significant, the unique p-value above the traditional significance levels of 0.10 concerns γ_1 (0.131).

The fitted values are represented in Figure 3. The comparison between actual and estimated autopersistences is graphically displayed in Figure 4. In Table 3 some values of the MAE^0, MAE^1 and MAE are reported for the model with $k = 2$ considering different values of n ($n = 4, 8, 12, 16$). In the same Table, the goodness-of-fit measures R^2_M and R^2_p shows satisfactory values, respectively, 0.7112 and 0.7673.

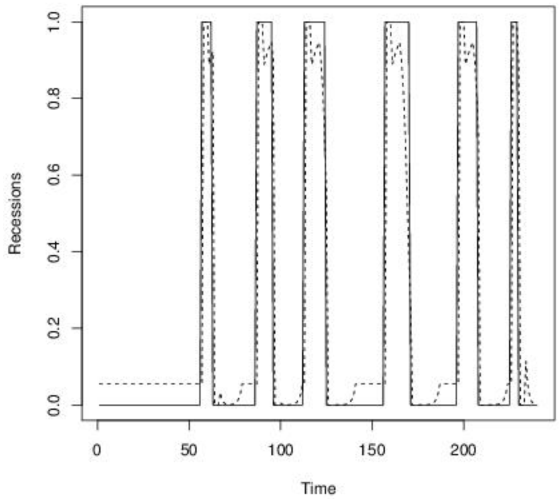


Figure 3: German quarterly recession data (solid line) and fitted data (dotted line), BHAR model, $k = 2$, 1952.1 to 2011.4.

5.2 Italian recessions

The second time-series is the Italian quarterly recessions in the period from 1956, I quarter to 2011, IV quarter, represented in Figure 5.

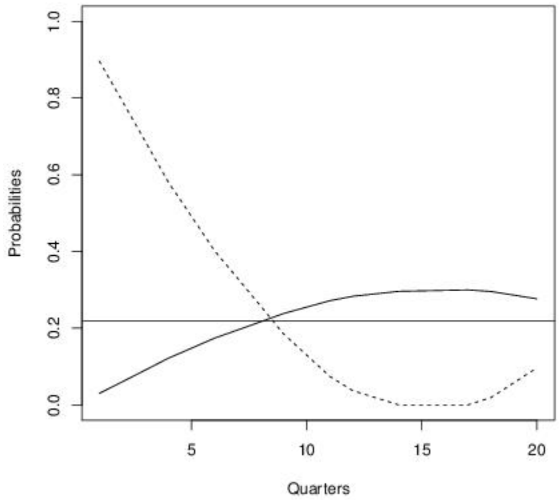
Figure 6 contains the autopersistence function for 20 lags. It presents similar features to the German case, that is the APG^0 and the APG^1 are still far from the marginal probability (0.2054) at lag 20. For this reason again a BHAR model seems to be appropriate. We have estimated different models with k from 0 to 3 (see Table 4). The AIC suggests to choose the parsimonious model with $k = 0$, that is the model where the one-lagged recession and the four-quarters cumulative sums are effective in explaining recession at time t. On the other hand, we have to remark the lack of significance of the parameters γ_1, γ_2 and γ_3 in the remaining models, implying that the associated cumulative terms

Table 2: Parameter estimates (p-values) of the BHAR models, $k = 0, 1, 2, 3$. German quarterly recession data, 1948.1 - 2011.4

Parameter	$k = 0$	$k = 1$	$k = 2$	$k = 3$
ϕ_0	-3.356 (0.000)	-3.257 (0.000)	-2.850 (0.000)	-2.739 (0.000)
ϕ_1	36.091 (0.447)	36.209 (0.031)	48.114 (0.013)	50.593 (0.033)
γ_0	-7.765 (0.513)	-7.286 (0.087)	-10.996 (0.024)	-11.645 (0.052)
γ_1	–	-0.312 (0.298)	0.858 (0.131)	0.862 (0.122)
γ_2	–	–	-0.659 (0.023)	-0.677 (0.021)
γ_3	–	–	–	0.013 (0.871)
AIC	0.3636	0.3712	0.3554	0.3809

Table 3: MAE^0, MAE^1 and MAE, BHAR model, $k = 2$. German quarterly recession data, 1948.1 - 2011.4

n	MAE^0	MAE^1	MAE
4	0.004	0.005	0.0045
8	0.005	0.004	0.0045
12	0.006	0.004	0.0050
16	0.005	0.005	0.0050
R^2_M		0.7112	
R^2_p		0.7673	



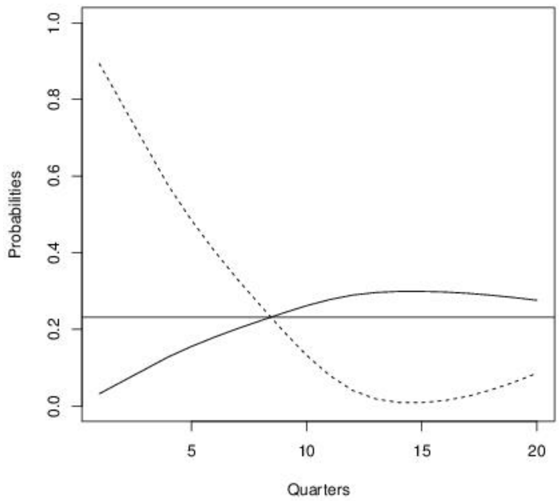


Figure 4: APG^0(k) (solid line) and APG^1(k) (dotted line) for quarterly German recession data, 1948.1 to 2011.4 (left), APF^0(k) (solid line) and APF^1(k) (dotted line), BHAR model, $k = 2$, German quarterly recession data (right).

have no role. For the sake of completeness we report the graphics of the theoretical recessions (Figure 7) and of the theoretical APF (Figure 8) for the BHAR model with $k = 0$. In Table 5, the values of MAE^0, MAE^1 and MAE are presented, together with R^2_M (0.6485) and R^2_p (0.7119).

The lack of significance of long-run terms in the estimated models is taken into account

Table 4: Parameter estimates (p-values) of the BHAR models, $k = 0, 1, 2, 3$. Italian quarterly recession data, 1956.1 - 2011.4.

Parameter	$k = 0$	$k = 1$	$k = 2$	$k = 3$
ϕ_0	-3.219 (0.000)	-3.142 (0.000)	-2.816 (0.000)	-2.146 (0.000)
ϕ_1	50.629 (0.000)	50.683 (0.000)	50.620 (0.005)	36.045 (0.016)
γ_0	-11.528 (0.000)	-11.314 (0.000)	-11.563 (0.010)	-7.950 (0.035)
γ_1	–	-0.154 (0.543)	0.321 (0.454)	0.333 (0.431)
γ_2	–	–	-0.311 (0.181)	-0.211 (0.397)
γ_3	–	–	–	-0.148 (0.207)
AIC	0.3411	0.3518	0.3593	0.3767

Table 5: MAE^0, MAE^1 and MAE, BHAR model, $k = 0$. Italian quarterly recession data, 1956.1 - 2011.4.

n	MAE^0	MAE^1	MAE
4	0.0007	0.0084	0.0046
8	0.0044	0.0128	0.0086
12	0.0159	0.0470	0.0315
16	0.0236	0.0668	0.0452
R^2_M		0.6485	
R^2_p		0.7119	

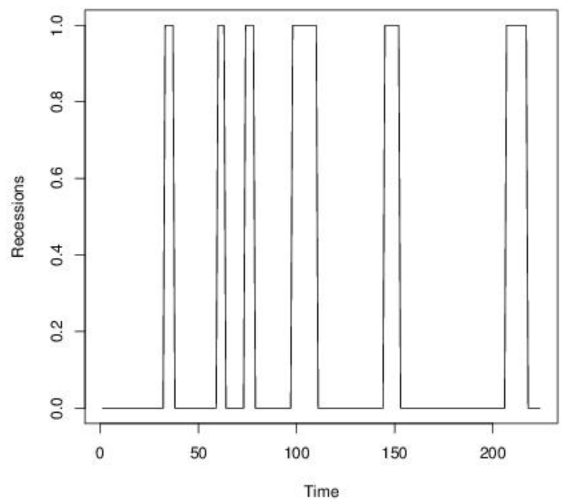


Figure 5: Italian quarterly recession data (1956.1-2011.4).

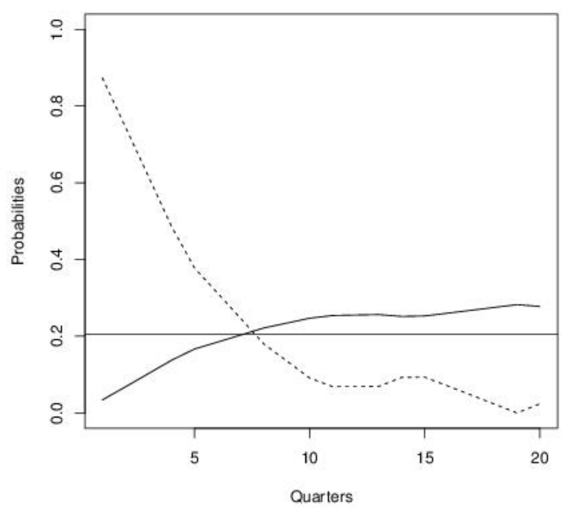


Figure 6: $APG^0(k)$ (solid line) and $APG^1(k)$ (dotted line) for Italian quarterly recession data (1956.1-2011.4).

together with the consideration that German market is the main end market of Italian exports, such that the good state of the German economy positively influences the Italian

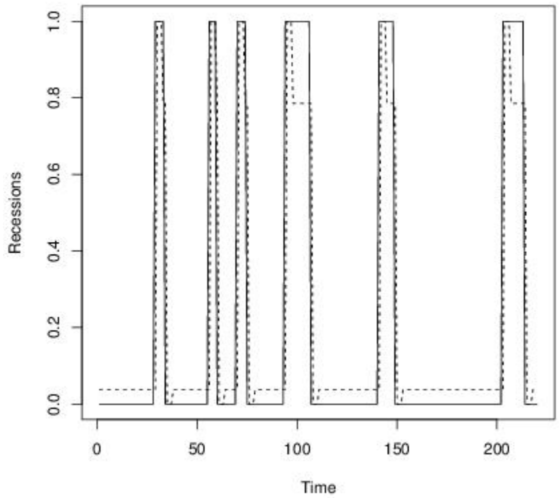


Figure 7: Italian quarterly recession data (solid line) and fitted data from the BHAR model, $k = 0$ (dotted line), 1960.1 to 2011.4.

economy. So it is plausible to consider Italian recessions to be dependent from past German recessions.

Let us consider the Germany as a leading economy which externally affects Italian economic activity. The BHR model proposed in Section 4 is a candidate to improve the results of the estimated BHAR model.

Firstly, let us examine the XPG in Figure 9. The graphs suggest that a relationship between present Italian recession and past German recessions is plausible. Furthermore, we note that the shape of the XPG has still long-memory characteristics, that is far lags of German recessions appear to affect Italian recession. Taking into account twenty lags, there is no convergence to the marginal probability of recession.

The BHR model is estimated with the usual range for k (from 0 to 3) and the results are reported in Table 6. According to the AIC the selected model has $k = 2$, that is the equation for η_t^F is given by

$$\eta_t{}^F = \phi_0^F + \phi_1^F X_{t-1} + \gamma_0^F X_{t-1}^{(1)} + \gamma_1^F X_{t-1}^{(2)} + \gamma_2^F X_{t-1}^{(4)}$$

The significance of the parameter γ_2^F confirms the goodness of the choice of k. However, the parameter ϕ_1^F with a p-value of almost 0.90 involves that Italian recessions do not depend just on the one-lagged value of the German time series of recessions, but only on some cumulative lags. In other words, German recession at quarter $t-1$ is not explicative of Italian recession at time t.

The goodness-of-fit-measures are not very good (see Table 7). All the MAE values are higher than the corresponding values of the BHAR model and the statistics R_M^2

Table 6: Parameter estimates (p-values) of the BHR models, $k = 0, 1, 2, 3$. Italian quarterly recession data, 1956.1-2011.4

Parameter	$k = 0$	$k = 1$	$k = 2$	$k = 3$
ϕ_0^F	-2.297 (0.000)	-2.142 (0.000)	-1.672 (0.000)	-2.433 (0.000)
ϕ_1^F	0.865 (0.217)	0.074 (0.928)	-0.131 (0.874)	-0.157 (0.854)
γ_0^F	0.446 (0.024)	1.114 (0.007)	0.747 (0.083)	0.720 (0.102)
γ_1^F	–	-0.312 (0.062)	0.253 (0.353)	0.332 (0.228)
γ_2^F	–		-0.334 (0.012)	-0.454 (0.001)
γ_3^F	–	–	–	0.133 (0.020)
AIC	0.7453	0.7468	0.7370	0.7450

Table 7: MAE^0, MAE^1 and MAE, BHR model, $k = 2$. Italian quarterly recession data, 1956.1 - 2011.4.

n	MAE^0	MAE^1	MAE
4	0.0148	0.0119	0.0134
8	0.0108	0.0131	0.0119
12	0.0111	0.0106	0.0109
16	0.0131	0.0120	0.0125
R_M^2		0.2664	
R_p^2		0.2463	

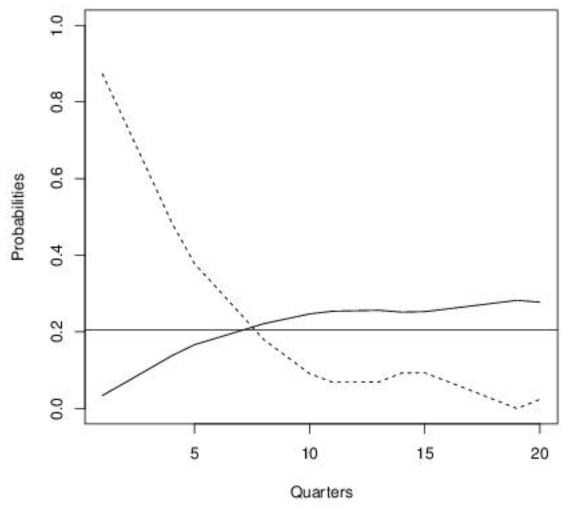


Figure 8: $\text{APG}^0(\text{k})$ (solid line) and $\text{APG}^1(\text{k})$ (dotted line) for Italian quarterly recession data, 1956.1 to 2011.4 (left), $\text{APF}^0(\text{k})$ (solid line) and $\text{APF}^1(\text{k})$ (dotted line) from the BHAR model, $k = 0$, for Italian quarterly recession data (right).

and R_p^2 assume very small values insomuch that the improvement with respect a simple constant model is definitively poor. Looking at Figure 10 it is easy to see that in some expansion periods the model provides high probabilities of recession and, vice versa, it cannot capture some recessions.

At this point, we estimate the GBHAR model. From Tables 4 and 6 we can observe

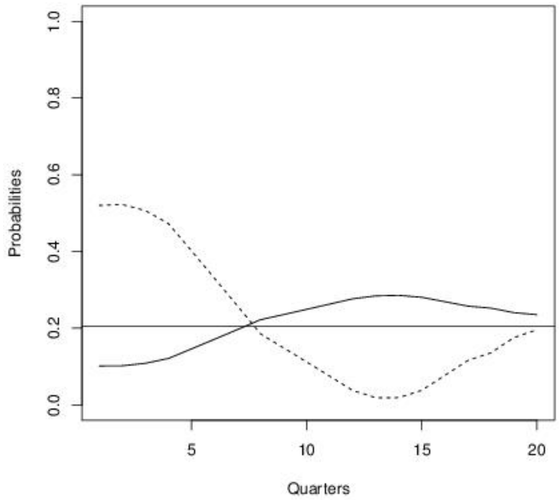


Figure 9: XPG^0(k) (solid line) and XPG^1(k) (dotted line) for Italian quarterly recession data (1956.1-2011.4).

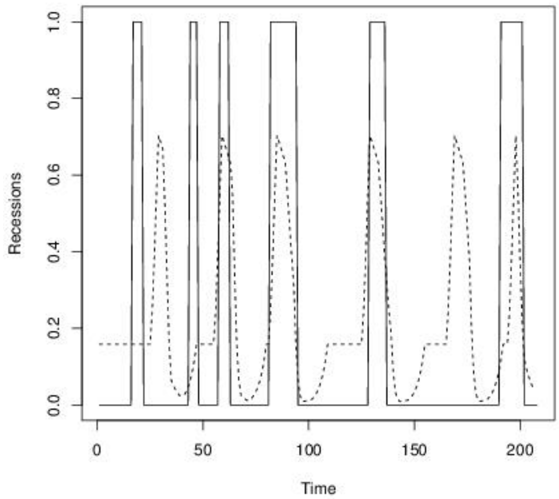


Figure 10: Italian quarterly recession data (solid line) and fitted data (dotted line), BHR model, $k = 2$, 1960.1 to 2011.4.

that the most important initial lags are those of the time series of Italian recessions,

Table 8: Parameter estimates (p-values) of the GBHAR model, $k = 2$. Italian quarterly recession data, 1956.1-2011.4.

Parameter	$k = 2$
ϕ_0^F	-2.929 (0.000)
ϕ_1^F	50.473 (0.000)
γ_0^F	-11.660 (0.000)
γ_1^L	0.482 (0.049)
γ_2^L	-0.328 (0.070)
R_M^2	0.6746
R_p^2	0.7251

while the significant cumulative lagged terms belong to Germany. So we estimate the following GBHAR model:

$$\eta_t^F = \phi_0^F + \phi_1^F Y_{t-1} + \gamma_0^F Y_{t-1}^{(1)} + \gamma_1^L X_{t-1}^{(2)} + \gamma_2^L X_{t-1}^{(4)}. \tag{2}$$

The estimates can be found in Table 8. The parameters are all significant taking into account their p-values lower than 0.10.

Finally, the statistics R_M^2 (0.6746) and R_p^2 (0.7251) are higher than the corresponding statistics of the BHAR and BHR models. As a result, we can state that a favorable mix of components from the BHAR and the BHR models allows to specify a new model, the GBHAR model, able to capture the salient features of a specific time-series. Figure 11 depicts the fitted values of the model.

An interesting interpretation of the components of the model can be provided in terms of short-run and long-run effects. In Equation (2), Y_{t-1} and $Y_{t-1}^{(1)}$ can be seen as the short-run components which refer to a time horizon of one year, while $X_{t-1}^{(2)}$ and $X_{t-1}^{(4)}$ are the long-run components, embracing a time span of two and four years, respectively. So, in the time series of Italian quarterly recession data the short term effect is endogenous, while the long term effect is exogenous.

6 Conclusions

In this paper we have analyzed the time series of German and Italian recessions using a novel approach for binary time series. For the German recessions a suitable Binomial Heterogeneous Autoregressive model has been successfully applied. For the Italian recessions we have had to extend the model in such a way to take into account the recessions of an external country (Germany) which has the role of driving the economic performance of a follower country. The best candidate for describing the Italian recessions has

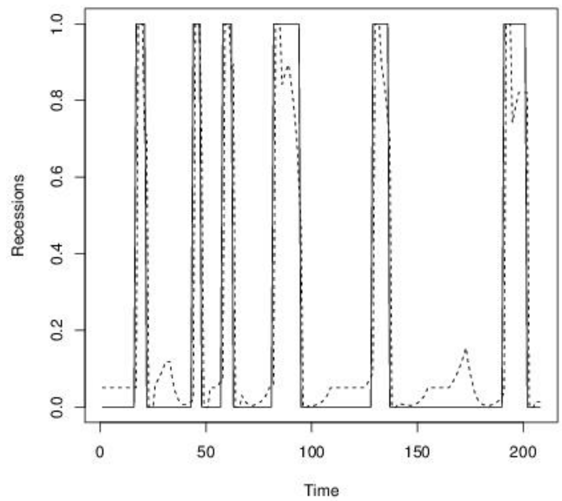


Figure 11: Italian quarterly recession data (solid line) and fitted data (dotted line) from the GBHAR model, 1960.1 to 2011.4.

revealed the Generalized Binomial Heterogeneous Autoregressive which presents a mix of components from the Binomial Heterogeneous Autoregressive and the simple Binomial Heterogeneous Regression models.

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