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ECONOMIC RELIABILITY GROUP ACCEPTANCE SAMPLING PLANS FOR LIFETIMES FOLLOWING A GENERALIZED EXPONENTIAL DISTRIBUTION

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Abstract: *Economic reliability test plan (ERTP) is developed considering that the lifetime of the submitted items follow generalized exponential distribution. For a given group size, specified acceptance number and producer's risk, test termination time is determined. Comparison of the proposed plan has been made with the existing plan developed by Rao [17]. Tables and examples illustrate the approach developed in the paper.*

Keywords: *Reliability test plan, group size, producer's risk, generalized exponential distribution.*

1. Introduction

In the global business market, quality determination plays an important role to recognize the visible characteristic values of the various manufactured products. We must therefore understand its necessity and the available techniques for the determination of the quality and ensure future improvement in this regard. For this purpose statistical process control and statistical product control are two modes available in the form of acceptance sampling plans to ensure the quality control. These are considered essential for the inspection of the final product. The advantage of these plans is to accept or reject the assembled products on the basis of the quality of the sample inspected. The ordinary acceptance sampling scheme is used when the experimenter has the

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facility to test only one item at a time. In this way the experimenter needs more time and testers to inspect the products. On the other hand, group acceptance sampling plans (GASPs) are used when the tester has the facility to install more than one item at the same time in a single tester. By doing so, substantial testing time and cost can be reduced if the tester to accommodate a multi number of items are utilized for testing purpose. The other importance of GASPs is that it provides strict inspection before the product is sent for the consumer's use. As such GASPs perform better than ordinary plans in terms of reduction of the time, strict inspection, cost, energy, minimize the risks and labor.

Reliability sampling test are considered necessary and pre-requisite to establish the trustworthiness of a product with regard to its lifetime. Its objects are to find out a confident and perfect limit with regard to the mean/median life of an item. In this way, it becomes easier to arrive at a definite decision as to whether the submitted lot may be accepted or rejected on the basis of the mean/median life of the product. The submitted lot is accepted in case the mean life of a product is found above the desired standard and if not, the same is rejected. Such tests are therefore deemed to be very economical.

Some ordinary acceptance sampling plans have been developed by Epstein [10], Goode and Kao [11], Gupta and Groll [14], Baklizi [7], Kantam *et al.* [15], Tsai and Wu [21], Balakrishnan *et al.* [8], Rosaiah *et al.* [18, 19]. In ordinary acceptance sampling plan single item is examined, where as in group acceptance sampling plan a multiple number of items are observed by the availability of the testers. Vlcek *et al.* [22] discussed the Monte Carlo simulation of sudden death ball bearing testing. More recently, Aslam and Jun [2], Rao [16, 17] and Aslam *et al.* [3, 4, 5] proposed group acceptance sampling plans for failure time distributions. In this paper, an economic reliability test plan is proposed for generalized exponential distribution assuming that the life time of a product follows this distribution with known shape parameter. The objective of this study is to find minimum termination time in view of given design parameters and satisfying both the consumer's and the producer's risks. Risks can be defined as, the probability of rejection of a good item is the producer's risk and the probability of acceptance of a bad item is the consumer's risk.

2. Design of ERTP

The group acceptance sampling plan based on truncated life tests proposed by Aslam *et al.* [2] is:

- Select a random sample of size n from the lot and distribution r items into g groups.
- Determine the acceptance number c and specify the termination time of the life test t_0
- Accept the lot if at most c failed items are found in each and every group by the termination time. Truncate the life test and reject the lot if more than c failures are found in any group.

Recently, it is found that the generalized exponential distribution (GED) has been used to analyze the lifetime data. In many cases it is found that it provides a better fit than the Weibull, gamma and lognormal, for more detail, reader may refer to Gupta and Kundu [12, 13] and Aslam *et al.* [3]. According to Gupta and Kundu [12, 13] that GED mean is not in a compact form, but the median is in a compact form. It is important to note that for a symmetric distribution, mean is preferable to use as a quality parameter in to design acceptance sampling plans. Since the GED is a skewed distribution we prefer to use the median as the quality parameter. The objective of this paper is to propose ERTP for life times following the two-parameter GED, which is deceived by Gupta and Kundu [12, 13]. For more detail about GED, reader may refer to Gupta and Kundu [12, 13], Bain and Engelhardt [6] and Cohen and Whitten [9]. The density and distribution functions of GED are

$$g_{T\{(\delta,\lambda)\}}(t)=\frac{\delta}{\lambda}e^{-\frac{t}{\lambda}}\left(1-e^{-\frac{t}{\lambda}}\right) \quad ,t>0 \tag{1}$$

$$G_{T\{(\delta,\lambda)\}}(t)=\left(1-e^{-\frac{t}{\lambda}}\right)^{\delta} \quad ,t>0 \tag{2}$$

where δ and λ are shape and scale parameters respectively. The median of this distribution for $\delta=2$ is $\theta=1.2279\lambda$. Aslam and Shahbaz [1] also developed an ERTP for ordinary acceptance sampling plan. Stephens [20] justified to use of binomial distribution in acceptance sampling study. The lot acceptance probability for the plan given by Aslam and Jun [2] is:

$$L(p)=\left[\sum_{i=0}^{c}\binom{r}{i}p^{i}(1-p)^{r-i}\right]^{g} \tag{3}$$

where p is the probability that a product fails before the termination time. The probability p for the GED with $\delta=2$ is:

$$p=G_{T\{2,\lambda\}}(t_0)=\left(1-e^{-1.2279a/(\mu/\mu_0)}\right)^{\delta} \quad t>0. \tag{4}$$

The minimum values of g is found by satisfying the following inequality,

$$\left[\sum_{i=0}^{c}\binom{r}{i}p^{i}(1-p)^{r-i}\right]^{g}\le\beta \tag{5}$$

Now, we find the minimum termination time for given producer's risk, sample size ($n=r\times g$) and the acceptance number c, when the following inequality is satisfied,

$$\left[\sum_{i=0}^{c}\binom{r}{i}p^{i}(1-p)^{r-i}\right]^{g}\ge 1-\alpha \tag{6}$$

Table 1. Test termination time for $\delta = 2$.

α =0.10								
c	r \ g	2g	3g	4g	5g	6g	7g	8g
0	1	0.2091	0.1673	0.1431	0.1270	0.1152	0.1061	0.0989
1	2	0.5262	0.3768	0.3076	0.2657	0.2369	0.2157	0.1991
2	3	*	0.6884	0.5117	0.4243	0.3696	0.3313	0.3025
3	4	*	*	0.8178	0.6241	0.5241	0.4598	0.4139
4	5	*	*	*	0.9255	0.7206	0.6112	0.5395
5	6	*	*	*	*	1.0180	0.8051	0.6885
6	7	*	*	*	*	*	1.0989	0.8803
7	8	*	*	*	*	*	*	1.1711
α =0.05								
c	r \ g	2g	3g	4g	5g	6g	7g	8g
0	1	0.1410	0.1130	0.0975	0.0867	0.0788	0.0727	0.0678
1	2	0.4145	0.2998	0.2461	0.2132	0.1906	0.1738	0.1607
2	3	*	0.5755	0.4316	0.3596	0.3141	0.2821	0.258
3	4	*	*	0.7057	0.5429	0.4577	0.4026	0.3631
4	5	*	*	*	0.8145	0.6389	0.5439	0.4812
5	6	*	*	*	*	0.9081	0.7233	0.6208
6	7	*	*	*	*	*	0.9905	0.7985
7	8	*	*	*	*	*	*	1.0639
α =0.01								
c	r \ g	2g	3g	4g	5g	6g	7g	8g
0	1	0.0596	0.0483	0.0417	0.0372	0.0340	0.0313	0.0293
1	2	0.2513	0.1850	0.1531	0.1335	0.1199	0.1095	0.1014
2	3	*	0.3980	0.3030	0.2540	0.2230	0.2011	0.1844
3	4	*	*	0.5210	0.4070	0.3455	0.3050	0.2761
4	5	*	*	*	0.6270	0.4981	0.4270	0.3794
5	6	*	*	*	*	0.7200	0.5800	0.5010
6	7	*	*	*	*	*	0.8020	0.6540
7	8	*	*	*	*	*	*	0.8750

*(note): * shows termination time does not exist.*

Table 1 represents the termination at various values of α , *g* and *c*. From Table 1, we can see the different behavior of termination time. For example, when the values of α 's, decreases the test termination time is also decreases. From Table 1, the entry against *c*=2, *r*=5, g=3, α =0.05, $\delta = 2$ under the column 5g is 0.3596. Let specified median life (θ_0) is 500 hours, the table value say that $t/\theta_0 = 0.3596$ so that t=0.3596×500=179.4=179 hours (approximately). It is interesting to note that as the group size (sample size) decreases for fixed value of c the termination time increases.

3. Example

Suppose software manufacturer would like to know if the median life of their software product is longer than the specified life, θ_0=1000 cycles. Aslam *et al.* [3] showed that lifetime of this product follows the GED with δ =2(approximately). The design parameters of Rao [17] plan are $(g, r, c, t/\theta_0)$= (3, 6, 2, 1.0) for $\alpha = 0.10$ and δ =2. In practice, the manufacture needs to select 18 items from the lot and allocate 6 items to 3 groups on the life test. The lot is accepted if no more than 2 failed items is found in 1000 hours in each and every group. Otherwise, the lot is rejected. For all the same quantities, the design parameters of proposed approach from Table 1 are $(g, r, c, t/\theta_0)$= (3, 6, 2, 0.3696) for $\alpha = 0.10$. According to proposed approach, select 18 items from a lot and run the experiment for 369 hours. Accept the lot if number of failures are not larger than 2 during 369 hours. So by adopting the proposed approach the manufacturer can reach on same decision about the software product in 369 hours than 1000 hours. Hence the proposed approach is more efficient in saving the time and cost of the experiment.

4. Comparative Study

We compare the termination time from the proposed approach and Rao [17] approach. The upper values of cells denoting the proposed plan termination time. From Table 2, it is clear that for the same values of c, α, g and δ, the termination time from the present approach is less than the existing plan. For example, when c=0 and g=4, the termination time from the present approach is 143 hours and from the existing approach it is 1000 hours.

Table 2. Comparisons of test termination time for generalized exponential distribution when for δ =2.

α =0.10								
c	r / g	2g	3g	4g	5g	6g	7g	8g
0	1			0.1431 1.0000				
1	2				0.2657 1.0000			
2	3					0.3696 1.0000		
3	4						0.4598 1.0000	

α =0.05								
c	r / g	2g	3g	4g	5g	6g	7g	8g
0	1				0.0867 1.0000			
1	2					0.1906 1.0000		
2	3						0.2821 1.0000	
3	4							
α =0.01								
c	r / g	2g	3g	4g	5g	6g	7g	8g
0	1						0.0313 1.0000	
1	2							0.1014 1.0000
2	3							
3	4							

5. Conclusion

In this paper, termination time is determined for the specified values of producer's risk assuming that the lifetime of the product follows the generalized exponential distribution. The termination time obtained by proposed approach is less than the exiting approach. So, it recommended to use the termination time for the inspection of the product to save time and cost of the experiments.

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