Equilibria in Social Networks with Heterogeneous Agents

Giuseppe De Marco,
Dipartimento di Statistica e Matematica per la Ricerca Economica
Università di Napoli Parthenope
giuseppe.demarco@uniparthenope.it

Abstract: *Empirical literature shows that ex-ante asymmetries across players arise quite naturally in social network formation. In this work, a very general kind of heterogeneity is considered, and two different models of network formation are introduced corresponding to different kind of disutility of establishing direct connections. These models are games with vector valued payoffs which are here investigated by using the concept of Pareto-Nash equilibrium and its refinements.*

Keywords: social network formation, one sided networks, heterogeneity, multicriteria games, Pareto Nash equilibrium, refinements.

The processes of information diffusion within a group of individuals and their implication on how they affect social learning have attracted increasing interest in the economics literature. The analysis of these issues has developed in various directions; for instance, a first approach considers non-strategic agents while, in the literature on social networks, individuals are identified with the vertices of a graph and create strategically relationships (links) within the others in such a way that the level of connectedness of the network determines the benefits accruing them. The basic assumption behind models of network formation is that establishing and maintaining connections with other individuals is costly. As a consequence, individuals limit the number or the intensity of their connections and then network structures develop from agents' comparison of disutility (costs) versus benefits of connection. In the seminal papers by Jackson and Wolinski (1996) and by Dutta and Mutuswami (1997) two-sided link formation has been investigated, in other words a situation in which two agents must agree on the decision to form a mutual link, but defection by one agent is sufficient to break the link; such notion of *pairwise stability* has been analyzed in a cooperative theoretic game framework. In Bala and Goyal (2000), one-sided link formation has been studied, this is the case where an individual can form links with the others autonomously and incurring in the cost of connection. This situation allows a description in terms of non cooperative theoretic game models and network stability has been studied in terms of Nash equilibrium concept and its refinements. In their paper, both the one-way and two-way flow of benefits (which correspond to directed and not directed graphs) are considered, and agents are supposed to be symmetric (homogeneous) and maximizing real valued payoff functions depending on two variables: the number of people (directly or indirectly) accessed and the number of links the agent forms himself. Moreover payoffs are assumed to be strictly increasing in the first variable and strictly decreasing in the second one.
The first important result in Bala and Goyal is that a *Nash network* is either empty or *"tw-connected"*, i.e., there is a path between any couple of players, and satisfying the *"no cycles"* property. However, it turns out that there is a great number of Nash network so, as stated in Bala and Goyal, *"multiplicity of equilibria motivates an examination of a stronger equilibrium concept"*. They focus on the concept of *strict Nash equilibrium* which is characterized by the uniqueness of the best reply correspondences in equilibrium. They find out that in the one-way flow model the unique not empty (no links) strict Nash network is the *wheel*, that is, a connected network in which each player creates and receives one link. In the two-way flow model the unique not empty strict Nash network is the *center-sponsored star*, that is, a network in which one player, the center, forms links with all the other players.

A common assumption in this first literature of network formation is homogeneity, in the sense that all decision makers conjecture that others receive information or establish links of equivalent value. Empirical literature shows that ex-ante asymmetries across players arise quite naturally in reality. For instance, agents might suppose the others are informed differently (information has different values) or differ in communication and social skills (forming links is cheaper for some individuals as compared to others). A first step in the investigation of the effects of agents heterogeneity in the network formation literature has been introduced by Johnson and Gilles (2000) in which ``spatial costs" are considered, representing geographical, social or individual differences and extending the two-sided network formation model of Jackson and Wolinsky (1996). Other papers investigate the impact of ex ante player heterogeneity on one sided network formation. In particular, in Galeotti et al. (2006), it is considered heterogeneity (in terms of the costs of linking and the values of accessing other players) in shaping the equilibrium architecture of one-sided two flow networks.

In this paper a different kind of heterogeneity is taken into account: agents are not able to valuate a priori the type of information coming from the others (for instance this can be caused by uncertainty with unknown distributions); this implies that there is no a-priori opinion on the relative importance of benefits that each player may get from connection with the others. In other words, it is studied the situation in which each player has an utility function associated to each other agent and which depends (increasingly) on the intensity of connection; moreover, these payoffs are not a-priori comparable. Finally, the cost of forming a link may differ qualitatively. In fact, on one hand, it can be relative to the benefits coming from the agent linked and we refer to these costs as the *rd-networks* (relative disutilities). This is the case, for instance, of costs depending on the same uncertainty of the benefits coming from the agent linked. Another approach is to consider *ad-networks* (absolute disutilities) in which costs are comparable each other but not comparable with benefits of connection.

Summarizing, agents are endowed with vector valued payoff functions. In the rd-networks these payoffs have as many components as it is the number of each player's opponents. For every player *i*, the component of the payoff associated to player *j* is increasing in the level of connection between *i* and *j* and decreasing in the level of investment of player *i* in the direct link with player *j*. In the ad-networks payoffs have as many components as it is the number of each player's opponents plus one. For every player *i* the component of the payoff associated to player *j* is increasing in the level of connection between *i* and *j* while the additional component represents costs and it is given by the (weighted) sum of levels of investment of player *i* in the direct links with the others. Stability of network structures is analyzed in terms of Pareto Nash equilibria (also called Multicriteria Nash equilibria) and their refinements (see the seminal paper by Shapley (1957), or also further results and references in Borm et al. (1999), Wang (1990) and De Marco and Morgan (2007)). Firstly, it is considered the classical assumption that players can choose only between forming links or not. It turns out that in the rd-networks the properties of "tw connectedness" and "no cycles" *("minimally connectedness")* partially characterize Pareto Nash equilibria (similarly to the classical two-way model in Bala and Goyal). Morever, the center sponsored star is here completely characterized by a refinement of Pareto Nash equilibria called *ideal equilibrium* (see Voornerveld et al. (2000)) in which the strategy of each player realizes the maximum of each component of his payoff function (also this result is in line with the corresponding result in Bala and Goyal). In the ad-networks, the results found are substantially different in fact, on one hand, Pareto Nash characterize only the "no cycles" property and simple examples show that a Pareto Nash can be disconnected. Morever examples show that ideal equilibria and *"strong Nash"*-like refinements (cooperative solutions) are ineffective in ad-networks. However, it turns out that the "tw-connectedness" is characterized by an extension to multicriteria games of the *friendliness equilibrium* concept (investigated in De Marco and Morgan (2008;a,b)). Recall also that the general property called *friendly behavior* has been defined and used by Rusinowska (2002) for equilibrium selection in some 2-players bargaining models. Friendliness equilibria are based on a property of robustness of the equilibrium with respect to a particular class of deviations: a player is supposed to move away from the equilibrium even only to guarantee a better payoff to the others and feasible deviations are unilateral and only towards Nash equilibria.

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